Principles of Mathematics 12

June 2002 Provincial Examination

ANSWER KEY / SCORING GUIDE

CURRICULUM:

Organizers	Sub-Organizers	
1. Problem Solving	A	Problem Set
2. Patterns and Relations	B	Geometric Sequences and Series
	C/D	Logarithms and Exponents
	C/D	Trigonometry
3. Shape and Space	E	Conics
	F	Transformations
4. Statistics and Probability	G	Combinatorics
	G	Probability
	G	Statistics

Part A: Multiple Choice

Q	K	C	S	CO	PLO	Q	K	C	S	CO	PLO
1.	D	U	1.5	2	C3	23.	B	K	1.5	3	E2
2.	A	U	1.5	2	C7	24.	B	K	1.5	3	E2
3.	D	U	1.5	2	C4	25.	C	U	1.5	3	E2
4.	A	U	1.5	2	D6	26.	B	H	1.5	3	E2
5.	D	U	1.5	2	D6	27.	D	U	1.5	3	F2
6.	A	U	1.5	2	C4, C5	28.	C	U	1.5	3	F1
7.	A	U	1.5	2	C5	29.	C	U	1.5	3	F4
8.	C	U	1.5	2	C8	30.	B	H	1.5	3	F3
9.	D	H	1.5	2	C6	31.	A	H	1.5	3	F6
10.	D	H	1.5	2	D7	32.	C	K	1.5	4	G7
11.	B	K	1.5	2	B1	33.	C	U	1.5	4	G8
12.	C	U	1.5	2	B1	34.	C	U	1.5	4	G11
13.	C	U	1.5	2	B1	35.	D	U	1.5	4	G13
14.	B	U	1.5	2	B1	36.	D	U	1.5	4	G11
15.	B	U	1.5	2	B1	37.	B	K	1.5	4	G1
16.	D	K	1.5	2	D2	38.	A	U	1.5	4	G1
17.	Deleted					39.	A	U	1.5	4	G2
18.	A	U	1.5	2	C1	40.	B	U	1.5	4	G3
19.	A	U	1.5	2	D3	41.	A	U	1.5	4	G3
20.	B	U	1.5	2	D3	42.	A	U	1.5	1	A1
21.	C	U	1.5	2	D1	43.	B	U	1.5	1	A1
22.	D	H	1.5	2	D4	44.	C	H	1.5	1	A1

Multiple Choice = 64.5 marks

Part B: Written Response

Q	B	C	S	CO	PLO
1	1	U	5	2	C2
2a.	2	H	3	3	F6
2b.	3	H	2	3	F6
3.	4	U	5	3	E3
4a.	5	U	2	4	G5
4b.	6	U	1	4	G5
4c.	7	U	1	4	G5
5.	8	U	5	2	C8
6a.	9	U	1	4	G13
6b.	10	U	2	4	G13
6c.	11	U	2	4	G13
7a.	12	H	2	1	A1
7b.	13	H	3	1	A1

Written Response = 34 marks

Multiple Choice = 64.5 (43 questions)
Written Response = 34 (7 questions)
EXAMINATION TOTAL = 98.5 marks

LEGEND:

Q = Question Number
K = Keyed Response
C = Cognitive Level
B = Score Box Number
S = Score
CO = Curriculum Organizer
PLO = Prescribed Learning Outcome

1. Solve algebraically: $\log_2 x + \log_2(x-7) = 3$ **(5 marks)**

solution

$\log_2\big(x(x-7)\big) = 3$ ← **1 mark**

$x^2 - 7x = 8$

↑

1 mark

$x^2 - 7x - 8 = 0$ ← **½ mark**

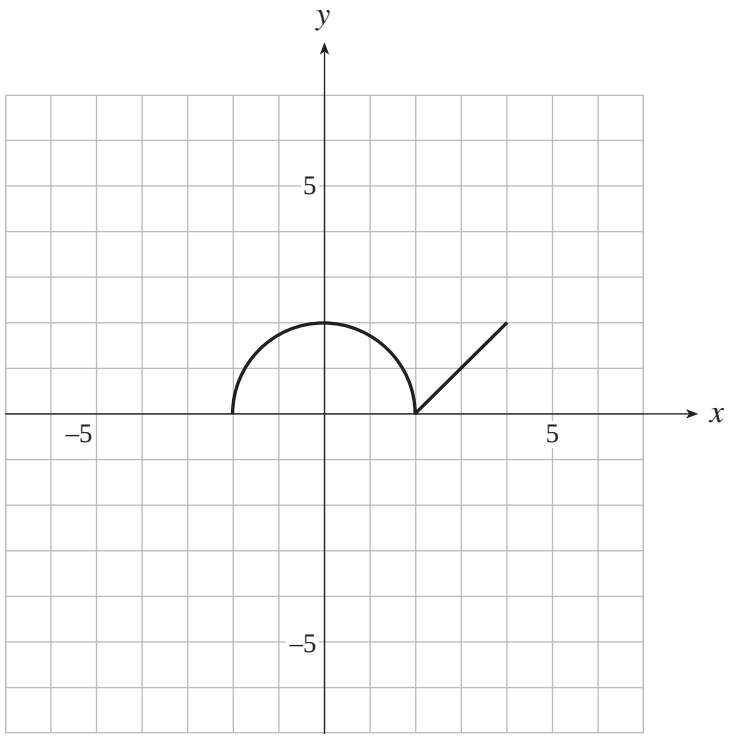
$(x-8)(x+1) = 0$

$x = 8$ $x = -1$ ← **½ mark**

↑

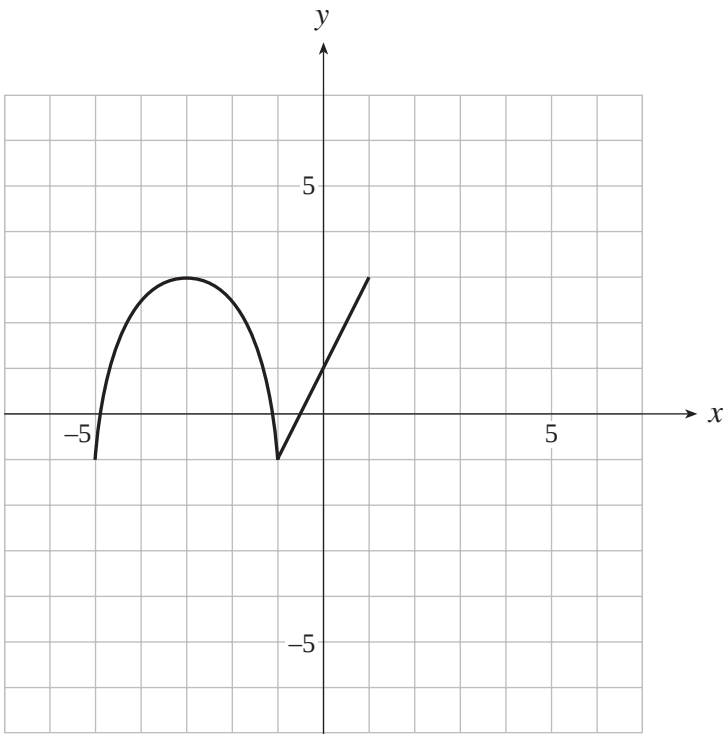
1 mark reject ← **1 mark**

2. The graph of $y = f(x)$ is shown below.



a) Graph $y = 2f(x+3)-1$ on the grid provided. **(3 marks)**

solution

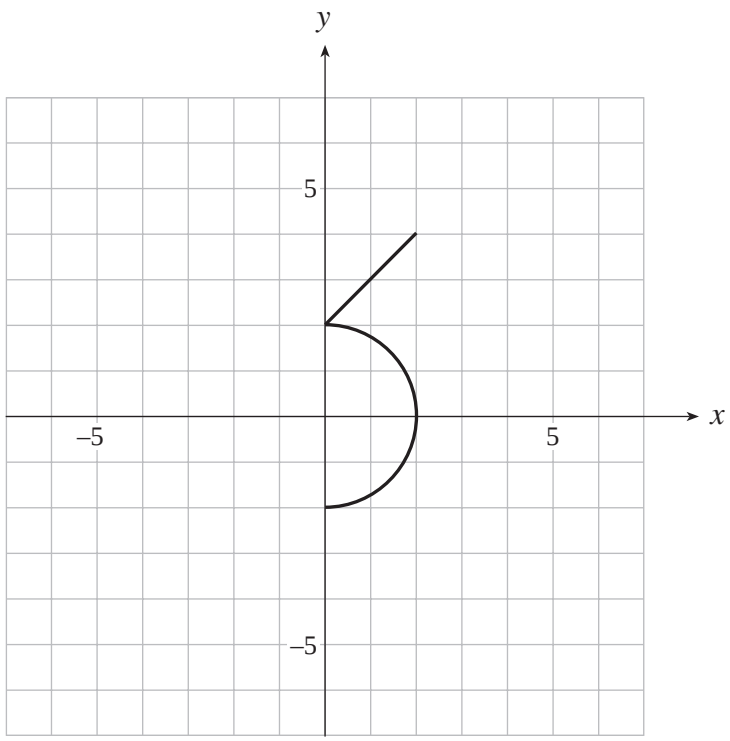


← **3 marks** for graph

1 mark for vertical expansion
1 mark for horizontal translation
1 mark for vertical translation

b) Graph the inverse relation of $y = f(x)$. **(2 marks)**

solution



← **2 marks** for graph

3. Change to standard form: $4y^2+16y-9x^2+18x-29=0$ **(5 marks)**

solution

$$4y^2+16y-9x^2+18x-29=0$$

$$4\left(y^2+4y\right)-9\left(x^2-2x\right)=29$$

↑ **½ mark** ↑ **½ mark '–' sign** **½ mark '9'**

$$4\left(y^2+4y+4\right)-9\left(x^2-2x+1\right)=29+\underbrace{16-9}$$

↑ **½ mark** ↑ **½ mark** ↑ **½ mark**

$$4(y+2)^2-9(x-1)^2=36$$

↑ **½ mark** ↑ **½ mark**

$$\left.\begin{array}{c}\dfrac{(y+2)^2}{9}-\dfrac{(x-1)^2}{4}=1\\ \textbf{or}\\ \dfrac{(x-1)^2}{4}-\dfrac{(y+2)^2}{9}=-1\end{array}\right\}\leftarrow \textbf{1 mark}$$

4. A class has 30 students.

a) How many ways can a committee of 3 people be selected from the class? **(2 marks)**

solution

$$_{30}C_3 = \frac{30!}{3!\ 27!} = 4\ 060 \text{ ways}$$

↑ **1 mark** ↑ **$\frac{1}{2}$ mark** ↑ **$\frac{1}{2}$ mark**

alternate solution

$$\frac{30 \times 29 \times 28}{3!} = \frac{24\,360}{6} = 4\ 060 \text{ ways}$$

↑ **1 mark** ↑ **$\frac{1}{2}$ mark** ↑ **$\frac{1}{2}$ mark**

b) How many ways can an executive committee consisting of 3 people (president, vice-president, secretary) be selected from the class? **(1 mark)**

solution

$$\underbrace{_{30}P_3 = \frac{30!}{27!}} = 24\ 360 \text{ ways}$$

↑ **$\frac{1}{2}$ mark** ↑ **$\frac{1}{2}$ mark**

alternate solution

$$30 \times 29 \times 28 = 24\,360 \text{ ways}$$

↑ **$\frac{1}{2}$ mark** ↑ **$\frac{1}{2}$ mark**

c) If there are 10 boys and 20 girls in the class, how many ways can a committee of 3 people be selected from the class if the committee must contain 1 boy and 2 girls? **(1 mark)**

solution

$$\left({}_{10}C_1\right)\left({}_{20}C_2\right) = (10)(190) = 1\,900 \text{ ways}$$

$\uparrow$ **½ mark** $\uparrow$ **½ mark**

alternate solution

$$10 \times \frac{20 \times 19}{2!} = \frac{3800}{2} = 1\,900 \text{ ways}$$

$\uparrow$ **½ mark** $\uparrow$ **½ mark**

5. Prove the identity: **(5 marks)**

$$\sin 2x(\tan x + \cot x) \;=\; 2$$

solution

	LEFT SIDE	RIGHT SIDE
	$\sin 2x(\tan x + \cot x)$	2
2 marks →	$= 2\sin x \cos x\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)$	= 2
1 mark →	$= 2\sin^2 x + 2\cos^2 x$	= 2
1 mark → **or**	$= 2(\sin^2 x + \cos^2 x)$ $= 2(1)$	= 2
1 mark →	$= 2$	= 2

LS = RS

alternate solution

	LEFT SIDE	RIGHT SIDE
	$\sin 2x(\tan x + \cot x)$	2
2 marks →	$= 2\sin x \cos x\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)$	= 2
1 mark →	$= 2\sin x \cos x\left(\frac{\sin^2 x + \cos^2 x}{\cos x \sin x}\right)$	= 2
1 mark → **or**	$= 2\sin x \cos x\left(\frac{1}{\cos x \sin x}\right)$ $= 2(1)$	= 2
1 mark →	$= 2$	= 2

LS = RS

6. The probability of winning a game is 0.7. You play 3 games. (Answer all parts of the question accurate to at least 3 decimal places.)

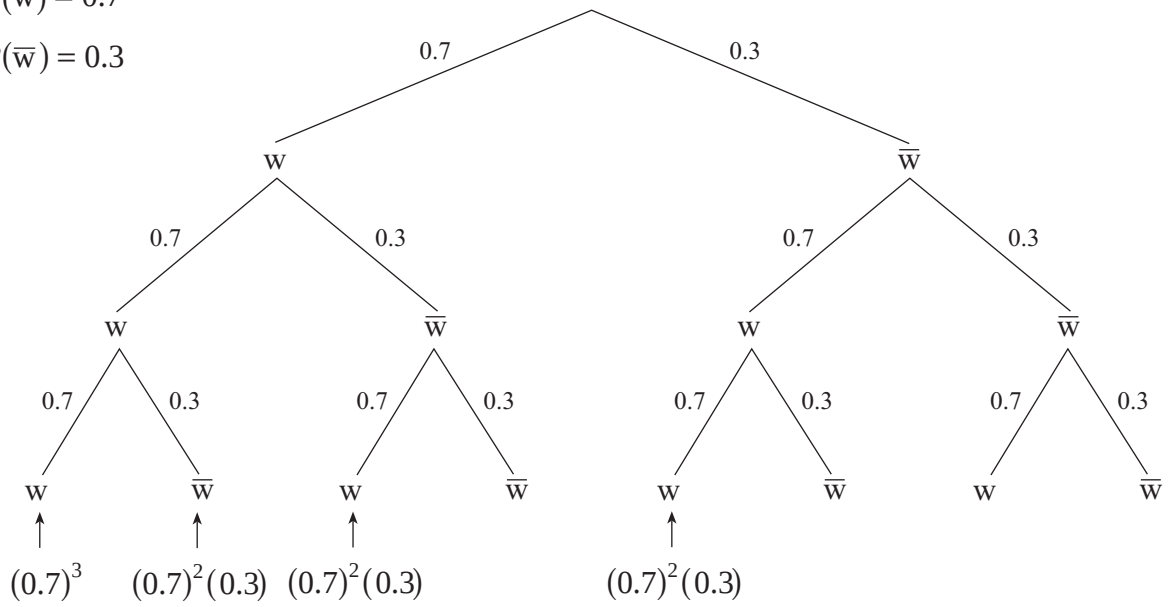
a) What is the probability that you win all 3 games? **(1 mark)**

solution

Let w = winning game

$P(\text{w}) = 0.7$

$P(\overline{\text{w}}) = 0.3$



$P(\text{3 wins}) = (0.7)^3 = 0.343$

↑ ½ **mark** ↑ ½ **mark**

b) What is the probability that you win at least twice? **(2 marks)**

$P(\text{at least 2 wins}) = \underbrace{(0.7)^3 + 3(0.7)^2(0.3)}_{\uparrow\ 1\frac{1}{2}\ \textbf{marks}} = 0.784 \quad \uparrow\ \tfrac{1}{2}\ \textbf{mark}$

c) If you win at least twice, what is the probability that you have 3 wins? **(2 marks)**

$$P(\text{3 wins} \mid \text{at least 2 wins})$$

$$= \frac{P(\text{3 wins } \textbf{and} \text{ at least 2 wins})}{P(\text{at least 2 wins})}$$

$$= \frac{(0.7)^3}{(0.7)^3 + 3(0.7)^2(0.3)} \leftarrow \textbf{1}\tfrac{\textbf{1}}{\textbf{2}} \textbf{ marks}$$

$$= 0.438 \text{ or } 0.4375 \quad \leftarrow \tfrac{\textbf{1}}{\textbf{2}} \textbf{ mark}$$

6. The probability of winning a game is 0.7. You play 3 games. (Answer all parts of the question accurate to at least 3 decimal places.)

a) What is the probability that you win all 3 games? **(1 mark)**

alternate solution 1

$$P(\text{3 wins}) = (0.7)^3 = 0.343$$

$\uparrow$ **$\frac{1}{2}$ mark** $\quad\uparrow$ **$\frac{1}{2}$ mark**

b) What is the probability that you win at least twice? **(2 marks)**

$$P(\text{at least 2 wins}) = (0.7)^3 + 3(0.7)^2(0.3)^1 \quad \leftarrow \textbf{1}\,\tfrac{1}{2}\textbf{ marks}$$

$$= 0.784 \quad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

c) If you win at least twice, what is the probability that you have 3 wins? **(2 marks)**

$$P(\text{3 wins} \mid \text{at least 2 wins})$$

$$= \frac{P(\text{3 wins } \textbf{and} \text{ at least 2 wins})}{P(\text{at least 2 wins})}$$

$$= \frac{0.343}{0.784} \quad \leftarrow \textbf{1}\,\tfrac{1}{2}\textbf{ marks}$$

$$= 0.4375 \quad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

6. The probability of winning a game is 0.7. You play 3 games. (Answer all parts of the question accurate to at least 3 decimal places.)

a) What is the probability that you win all 3 games? **(1 mark)**

alternate solution 2

$$P(3 \text{ wins}) = \underbrace{\text{binompdf}\ (3,\ 0.7,\ 3)}_{\uparrow\ \textbf{1}\frac{\textbf{1}}{\textbf{2}}\ \textbf{marks}} = 0.343 \quad \uparrow\ \tfrac{1}{2}\ \textbf{mark}$$

b) What is the probability that you win at least twice? **(2 marks)**

$$P(\text{at least 2 wins}) = \underbrace{1 - \text{binomcdf}\ (3,\ 0.7,\ 1)}_{\uparrow\ \textbf{1}\frac{\textbf{1}}{\textbf{2}}\ \textbf{marks}} = 0.784 \quad \uparrow\ \tfrac{1}{2}\ \textbf{mark}$$

c) If you win at least twice, what is the probability that you have 3 wins? **(2 marks)**

$$P(3 \text{ wins} \mid \text{at least 2 wins})$$

$$= \frac{P(3 \text{ wins } \textbf{and} \text{ at least 2 wins})}{P(\text{at least 2 wins})}$$

$$= \frac{\text{binompdf}\ (3,\ 0.7,\ 3)}{1 - \text{binomcdf}\ (3,\ 0.7,\ 1)} \qquad \leftarrow \textbf{1}\tfrac{\textbf{1}}{\textbf{2}}\ \textbf{marks}$$

$$= 0.438 \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

7. The Bullock and Brown organization surveyed 400 randomly chosen BC residents and found that 20% of the respondents have cellular phones.

   a) Determine the standard error for the sample proportion. **(2 marks)**

solution

$n = 400$ ← **½ mark**

$\hat{p} = 0.20$ ← **½ mark**

$\hat{q} = 0.80$

$$SE = \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$SE = \sqrt{\frac{(0.20)(0.80)}{400}}$$ ← **½ mark**

$$= 0.02$$ ← **½ mark**

b) Use the results from this sample to find a 95% confidence interval for the actual proportion of BC residents who have cellular phones. Clearly show the substitution into the confidence interval formula. **(3 marks)**

solution

$\frac{1}{2}$ **mark** $\rightarrow \quad \hat{p} - z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}}$ **1 mark** for $z_{\frac{\alpha}{2}} = 1.96$

$\frac{1}{2}$ **mark**

$\downarrow$

$$0.20 - 1.96\sqrt{\frac{(0.8)(0.2)}{400}} < p < 0.20 + 1.96\sqrt{\frac{(0.8)(0.2)}{400}}$$

$\frac{1}{2}$ **mark** $\rightarrow \left\{ \begin{array}{c} 0.16 < p < 0.24 \\ \text{or} \\ 16\% < p < 24\% \end{array} \right\} \leftarrow \frac{1}{2}$ **mark**

alternate solution

1 mark for $z_{\frac{\alpha}{2}} = 1.96$

$\frac{1}{2}$ **mark** $\rightarrow \{\ 0.20 - 1.96(\text{SE}) < p < 0.20 + 1.96(\text{SE})\ \} \leftarrow \frac{1}{2}$ **mark**

$$0.20 - 1.96(0.02) < p < 0.20 + 1.96(0.02)$$

$\frac{1}{2}$ **mark** $\rightarrow \left\{ \begin{array}{c} 0.16 < p < 0.24 \\ \text{or} \\ 16\% < p < 24\% \end{array} \right\} \leftarrow \frac{1}{2}$ **mark**

Note: If students write invNorm(0.975) for $z_{\frac{\alpha}{2}}$ in the formula, this is correct.

If students use $z_{\frac{\alpha}{2}} = 1.959963986$, this is also correct.

Note: Since the accuracy requirements are that answers be correct to at least two decimal places, the answer above has been rounded to two decimal places. Greater decimal place accuracy, such as $0.1608 < p < 0.2392$ is also acceptable.

END OF KEY