Principles of Mathematics 12

June 2001 Provincial Examination

ANSWER KEY / SCORING GUIDE

CURRICULUM:

Organizers	**Sub-Organizers**
1. Problem Solving	A Problem Set
2. Patterns and Relations	B Sequences and Series
	C Polynomials
	D Logarithms and Exponents
	E Quadratic Relations
	F Quadratic Systems
3. Shape and Space	G Trigonometry
	H Geometry

Part A: Multiple Choice

Q	**K**	**C**	**S**	**CO**	**PLO**
1.	B	K	1.5	2	C1
2.	D	U	1.5	2	C3
3.	D	U	1.5	2	C5
4.	D	U	1.5	2	C9
5.	D	U	1.5	2, 1	C6, A7
6.	A	H	1.5	2	C4
7.	B	K	1.5	2	E1
8.	A	U	1.5	2	F5
9.	C	K	1.5	2	E5
10.	A	U	1.5	2	F1
11.	B	U	1.5	2	E6
12.	A	U	1.5	2	E4
13.	B	H	1.5	2	E2
14.	A	H	1.5	2	F1
15.	C	K	1.5	2	D4
16.	D	K	1.5	2	D1
17.	A	K	1.5	2	D5
18.	C	U	1.5	2	D5, D4
19.	A	U	1.5	2	D6
20.	D	H	1.5	2	D5
21.	A	H	1.5	2	D5
22.	B	U	1.5	2	B5
23.	C	K	1.5	2	B1
24.	A	U	1.5	2	B4
25.	B	U	1.5	2	B4
26.	C	U	1.5	2	B4
27.	B	U	1.5	2	B6
28.	D	H	1.5	2	B4
29.	B	U	1.5	3	G1
30.	D	K	1.5	3	G2
31.	D	U	1.5	3	G7
32.	B	U	1.5	3	G3
33.	B	U	1.5	3	G5
34.	C	U	1.5	3,1	G9, A7
35.	D	H	1.5	3	G2, G7
36.	C	U	1.5	3,1	G5, A7
37.	C	U	1.5	3	G8
38.	C	U	1.5	3	H2
39.	B	U	1.5	3	H2
40.	C	U	1.5	3	H3
41.	B	H	1.5	3	H3
42.	C	U	1.5	1	A3
43.	C	U	1.5	1	A3
44.	A	H	1.5	1	A1

Multiple Choice = 66 marks

Part B: Written Response

Q	B	C	S	CO	PLO
1.	1	U	4	2	C7
2.	2	U	4	2	F2
3.	3	U	4	3	G7
4a.	4	U	3	2	E4
4b.	5	U	2	2	E4
5.	6	U	4	2, 1	D5, A7
6.	7	U	4	1	A1, A7
7.	8	U	4	3	H3
8.	9	H	5	3	H4

Written Response = 34 marks

Multiple Choice = 66 (44 questions)
Written Response = 34 (8 questions)
EXAMINATION TOTAL = 100 marks

LEGEND:

Q = Question Number
K = Keyed Response
C = Cognitive Level
B = Score Box Number
S = Score
CO = Curriculum Organizer
PLO = Prescribed Learning Outcome

1. A cubic polynomial function has a double zero at -2 and a single zero at 3. If this function passes through the point $(4,\ -24)$, determine an equation of the function. Answer may be left in factored form. **(4 marks)**

Solution

1 mark for *a*

$$y = \overset{\downarrow}{a}(x+2)^2(x-3) \quad \leftarrow \textbf{1 mark} \text{ for factors}$$

$$-24 = a(4+2)^2(4-3) \quad \leftarrow \textbf{1 mark} \text{ for substitution}$$

$$-24 = 36a$$

$$a = -\frac{2}{3} \quad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$y = -\frac{2}{3}(x+2)^2(x-3) \quad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

2. Solve the following system algebraically. Express answers as ordered pairs. **(4 marks)**

$$3x^2 - 2y^2 = 38$$
$$x^2 + y^2 = 21$$

Solution

$$\begin{aligned} 3x^2 - 2y^2 &= 38 \\ x^2 + y^2 &= 21 \end{aligned} \quad \Rightarrow \quad \begin{aligned} 3x^2 - 2y^2 &= 38 \\ 2x^2 + 2y^2 &= 42 \end{aligned}$$

← $\frac{1}{2}$ **mark** for setup (same for substitution)

$$5x^2 = 80$$

$$x^2 = 16$$

$$x = \pm 4$$

← $\frac{1}{2}$ **mark** for ±, $\frac{1}{2}$ **mark** for 4

$$x^2 + y^2 = 21$$

$$16 + y^2 = 21$$

← $\frac{1}{2}$ **mark** for substitution

$$y^2 = 5$$

$$y = \pm\sqrt{5}$$

← $\frac{1}{2}$ **mark** for ±, $\frac{1}{2}$ **mark** for $\sqrt{5}$

$\therefore$ solutions are: $\left(4,\ \sqrt{5}\right)$, $\left(4,\ -\sqrt{5}\right)$, $\left(-4,\ \sqrt{5}\right)$, $\left(-4,\ -\sqrt{5}\right)$ ← $\frac{1}{2}$ **mark** for **any 2** ordered pairs.
Further $\frac{1}{2}$ **mark** for the remaining
2 ordered pairs.

3. Prove: **(4 marks)**

$$\frac{\sin\theta\cos\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\tan\theta}$$

Solution

LEFT SIDE	RIGHT SIDE
$\frac{\sin\theta\cos\theta}{1+\cos\theta}$	$\frac{1-\cos\theta}{\tan\theta}$
1 mark → $= \frac{\sin\theta\cos\theta}{1+\cos\theta}\cdot\frac{(1-\cos\theta)}{(1-\cos\theta)}$	
½ mark → **½ mark →** $= \frac{\sin\theta\cos\theta(1-\cos\theta)}{1-\cos^2\theta}$	
½ mark → $= \frac{\sin\theta\cos\theta(1-\cos\theta)}{\sin^2\theta}$	
½ mark → $= \frac{\cos\theta(1-\cos\theta)}{\sin\theta}$	
½ mark → $= \cot\theta(1-\cos\theta)$	
½ mark → $= \frac{1-\cos\theta}{\tan\theta}$	

LS = RS

3. Prove: **(4 marks)**

$$\frac{\sin\theta\cos\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\tan\theta}$$

Alternate Solution

LEFT SIDE	RIGHT SIDE
$\frac{\sin\theta\cos\theta}{1+\cos\theta}$	$\frac{1-\cos\theta}{\tan\theta}$
	$= \frac{(1-\cos\theta)}{\tan\theta}\;\frac{(1+\cos\theta)}{(1+\cos\theta)} \quad \leftarrow \textbf{1 mark}$
	$= \frac{1-\cos^2\theta}{\frac{\sin\theta}{\cos\theta}\;(1+\cos\theta)} \quad \begin{matrix}\leftarrow \frac{1}{2}\textbf{ mark}\\ \leftarrow \frac{1}{2}\textbf{ mark}\end{matrix}$
	$= \left.\frac{\overset{\overset{\frac{1}{2}\textbf{ mark}}{\downarrow}}{\left(\sin^2\theta\right)}}{\left(\frac{\sin\theta}{\cos\theta}\;(1+\cos\theta)\right)}\;\frac{\cos\theta}{\cos\theta}\;\right\} \leftarrow \frac{1}{2}\textbf{ mark}$
	$= \frac{\sin^2\theta\cos\theta}{\sin\theta(1+\cos\theta)} \quad \leftarrow \frac{1}{2}\textbf{ mark}$
	$= \frac{\sin\theta\cos\theta}{1+\cos\theta} \quad \leftarrow \frac{1}{2}\textbf{ mark}$

LS = RS

Note: this question has two parts, a) and b).
A grid is provided for rough work only.

4. An ellipse which has vertices at $(-2,\ 2)$ and $(8,\ 2)$ is tangent to the x-axis.

a) Determine an equation of this ellipse. **(3 marks)**

Solution

$$\underset{\substack{\uparrow\\ \frac{1}{2}\textbf{ mark}}}{\frac{(x-3)^2}{25}} + \underset{\substack{\uparrow\\ \textbf{1 mark}}}{\frac{(y-2)^2}{4}} = 1$$

1 mark for centre $(3,\ 2)$
$\frac{1}{2}$ **mark** for form of equation

b) If $(6,\ y)$ is a point on the ellipse, determine all possible values for y. **(2 marks)**

Solution

$$\frac{(6-3)^2}{25} + \frac{(y-2)^2}{4} = 1$$ ← $\frac{1}{2}$ **mark** for substituting $(6,\ y)$ into an equation

$$\frac{9}{25} + \frac{(y-2)^2}{4} = 1$$

$$\frac{(y-2)^2}{4} = \frac{16}{25}$$ ← $\frac{1}{2}$ **mark**

$$(y-2)^2 = \frac{64}{25}$$

$$y-2 = \pm\frac{8}{5} = \pm 1.60$$

$$\therefore\ y = \underset{\substack{\uparrow\\ \frac{1}{2}\textbf{ mark}}}{2+1.6 = 3.60} \qquad \text{or} \qquad y = \underset{\substack{\uparrow\\ \frac{1}{2}\textbf{ mark}}}{2-1.6 = 0.40}$$

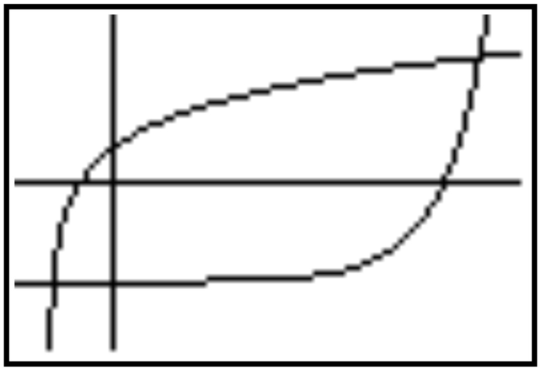
5. Solve the following system using a graphing calculator. **(4 marks)**

$$y = 2^{x-9} - 3$$

$$y = \log_2(x+2)$$

Sketch the graph in the viewing window below. State the function(s) that you entered to obtain your graph and your solution. Indicate the dimensions of the viewing window that will show enough of the graph so that recognizable characteristics of the function(s) and all intersection points are visible.

Solution



$$\left.\begin{aligned} Y_1 &= 2^{x-9} - 3 \\ Y_2 &= \frac{\log(x+2)}{\log 2} \end{aligned}\right\} \leftarrow \tfrac{1}{2}\ \textbf{mark} \text{ for equations}$$

← **1 mark** for graph

$x\ [-3,\ 13]$ $y\ [-5,\ 5]$ ← $\frac{1}{2}$ **mark** for window dimensions

$(-1.87,\ -3.00)$ ← **1 mark**

$(11.76,\ 3.78)$ ← **1 mark**

1 $\frac{1}{2}$ marks for x-values only in solution.

3 marks if y on the paper but committed to x-values only.

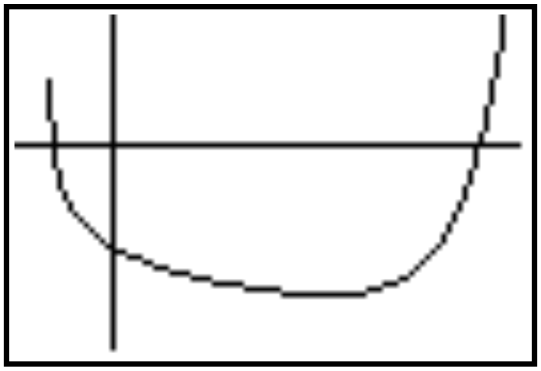
Cap at **3 marks** if **2** solutions are correct but equations are written incorrectly.

5. Solve the following system using a graphing calculator. **(4 marks)**

$$y = 2^{x-9} - 3$$
$$y = \log_2(x+2)$$

Sketch the graph in the viewing window below. State the function(s) that you entered to obtain your graph and your solution. Indicate the dimensions of the viewing window that will show enough of the graph so that recognizable characteristics of the function(s) and all intersection points are visible.

Alternate Solution



$Y_1 = 2^{x-9} - 3 - \dfrac{\log(x+2)}{\log 2}$ ← $\frac{1}{2}$ **mark** for equation

← **1 mark** for graph

$x\ [-3,\ 13]$ $y\ [-8,\ 5]$ ← $\frac{1}{2}$ **mark** for window dimensions

$(-1.87,\ -3.00)$ ← **1 mark**

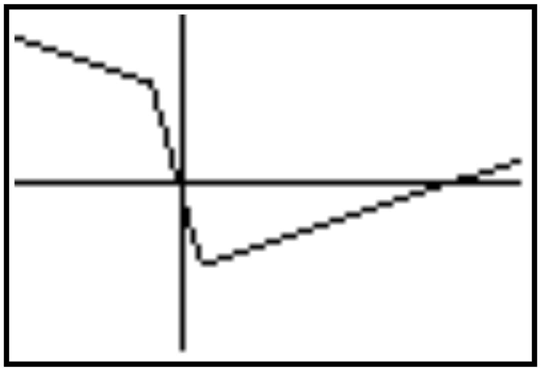
$(11.76,\ 3.78)$ ← **1 mark**

6. Solve the following equation using a graphing calculator. **(4 marks)**

$$1.2|x-1| = |x+2|$$

Sketch the graph in the viewing window below and indicate appropriate window dimensions. State the function(s) used in your graph. Ensure that the relative maximum and relative minimum points of the function(s) are visible within the viewing window.

Solution



$Y_1 = 1.2|x-1| - |x+2|$ ← $\frac{1}{2}$ **mark** for equation

← **1 mark** for graph

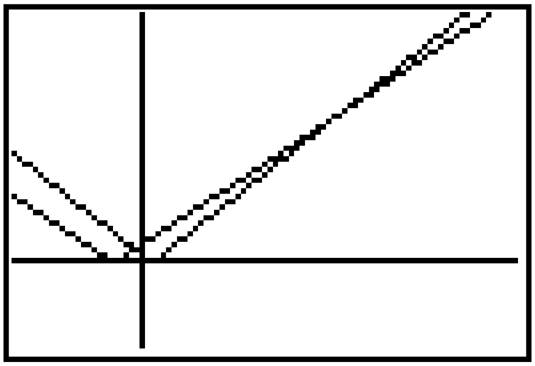
$x\ [-10,\ 20]$ $y\ [-6,\ 6]$ ← $\frac{1}{2}$ **mark** for window dimensions

$x = -0.36\ ,\ \ 16$

↑ ↑

1 mark 1 mark

Alternate Solution



$Y_1 = 1.2|x-1|$

$Y_2 = |x+2|$

} ← $\frac{1}{2}$ **mark** for equations

← **1 mark** for graph

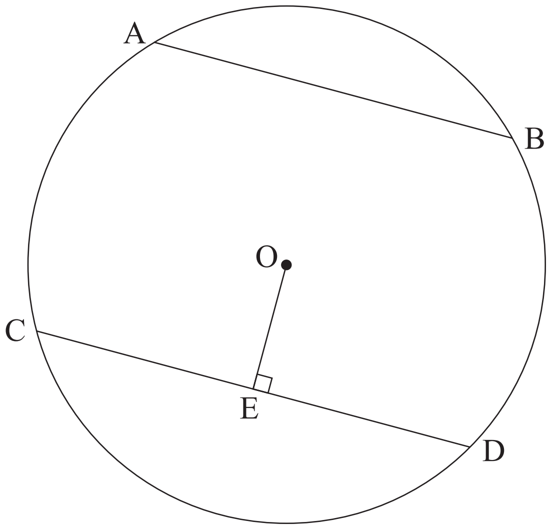
$x\ [-10,\ 30]$ $y\ [-10,\ 30]$ ← $\frac{1}{2}$ **mark** for window dimensions

$x = -0.36\ ,\ \ 16$

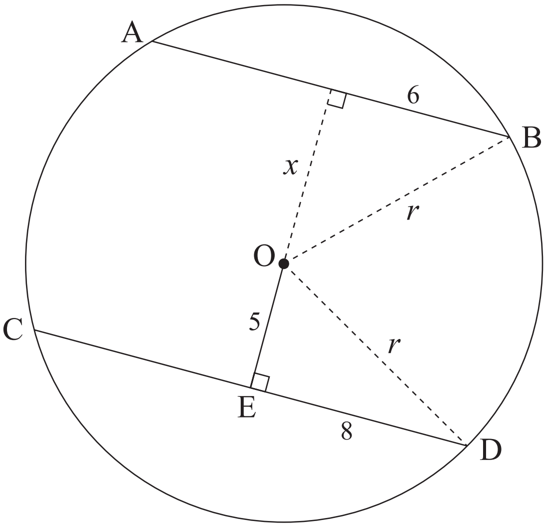
↑ ↑

1 mark 1 mark

7. A circle with centre O has parallel chords AB and CD. If AB = 12 cm, CD = 16 cm, OE = 5 cm and OE $\perp$ CD, determine the distance between the chords. **(4 marks)**



Solution



$\frac{1}{2}$ mark for diagram only

$r^2 = 8^2 + 5^2 = 89$ ← **1 mark**

$r = \sqrt{89}$ ← **$\frac{1}{2}$ mark**

$x^2 = r^2 - 6^2$ ← **1 mark**

$x^2 = 89 - 36$

$x^2 = 53$

$x = \sqrt{53}$ ← **$\frac{1}{2}$ mark**

distance $= 5 + \sqrt{53} = 12.2801\ldots$ ← **$\frac{1}{2}$ mark**

↑ **$\frac{1}{2}$ mark**

$= 12.28$ cm

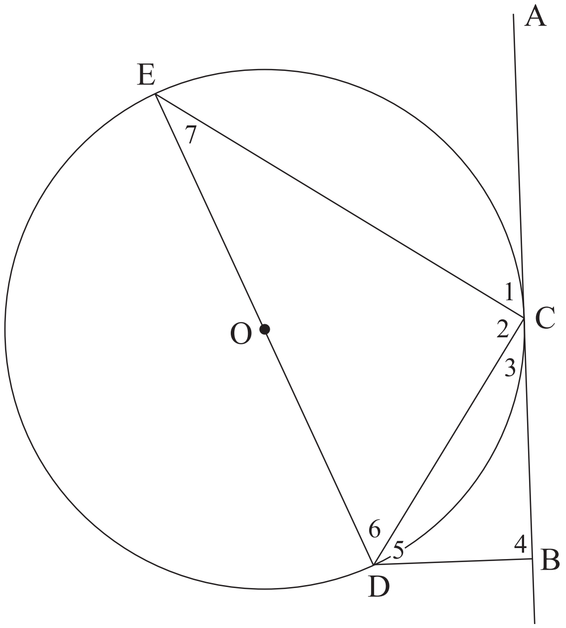
Students must choose one or the other method of proof.

8. Complete the proof. **(5 marks)**

Diagram clarification: O is the centre of the circle

Given: AB is tangent to the circle at C
DC bisects $\angle EDB$

Prove: $DB \perp AB$



Solution

Paragraph proof method:

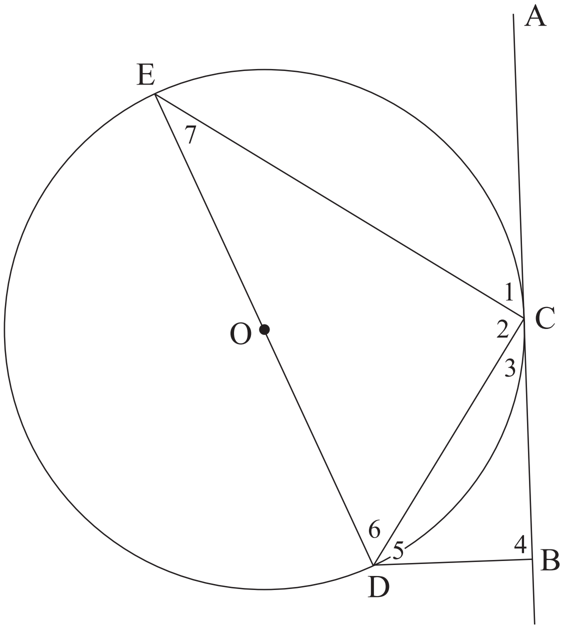
Since AB is a tangent, $\angle 3 = \angle 7$ (**$\frac{1}{2}$ mark**) by $\angle$ between tangent and chord (**1 mark**) and since DC bisects $\angle EDB$, $\angle 5 = \angle 6$ (**$\frac{1}{2}$ mark**) by definition of an angle bisector. Therefore $\angle 4 = \angle 2$ (**$\frac{1}{2}$ mark**) by 3rd $\angle$s of Δs (**$\frac{1}{2}$ mark**), but $\angle 2 = 90°$ (**$\frac{1}{2}$ mark**) since it is an inscribed $\angle$ on the diameter (**$\frac{1}{2}$ mark**), so $\angle 4 = 90°$ (**$\frac{1}{2}$ mark**).
Thus $DB \perp AB$ by definition of perpendicular (**$\frac{1}{2}$ mark**).

8. Complete the proof. **(5 marks)**

Diagram clarification: O is the centre of the circle

Given: AB is tangent to the circle at C
DC bisects ∠EDB

Prove: DB ⊥ AB



Solution

Two-column proof method:

	STATEMENT	REASON	
	AB is tangent to the circle	given	
½ mark →	∠3 = ∠7	∠ between tangent and chord	**← 1 mark**
	DC bisects ∠EDB	given	
½ mark →	∠5 = ∠6	definition of ∠ bisector	
½ mark →	∠4 = ∠2	3rd ∠s of Δs are =	**← ½ mark**
½ mark →	∠2 = 90°	inscribed ∠ on diameter = 90°	**← ½ mark**
½ mark →	∠4 = 90°	substitution	
	DB ⊥ AB	definition of ⊥	**← ½ mark**

END OF KEY