Principles of Mathematics 12

April 2001 Provincial Examination

ANSWER KEY / SCORING GUIDE

CURRICULUM:

Organizers	**Sub-Organizers**
1. Problem Solving	A Problem Set
2. Patterns and Relations	B Sequences and Series
	C Polynomials
	D Logarithms and Exponents
	E Quadratic Relations
	F Quadratic Systems
3. Shape and Space	G Trigonometry
	H Geometry

Part A: Multiple Choice

Q	K	C	S	CO	PLO
1.	D	K	1.5	2	C5
2.	A	K	1.5	2	C2
3.	B	U	1.5	2, 1	C2, A7
4.	D	U	1.5	2	C3
5.	C	H	1.5	2	C1
6.	B	H	1.5	2	C1
7.	D	U	1.5	2	E2
8.	D	K	1.5	2	E5
9.	B	U	1.5	2	E6
10.	D	U	1.5	2	E4
11.	C	U	1.5	2	F5
12.	B	U	1.5	2	F1
13.	A	H	1.5	2	E7
14.	A	H	1.5	2	F1
15.	A	K	1.5	2	D5
16.	B	U	1.5	2	D1
17.	D	U	1.5	2	D5
18.	A	U	1.5	2	D5, A7
19.	D	U	1.5	2	D5
20.	D	H	1.5	2	D5
21.	C	H	1.5	2	D2
22.	D	K	1.5	2	B1
23.	C	U	1.5	2	B2
24.	D	U	1.5	2	B5
25.	C	U	1.5	2	B4
26.	B	U	1.5	2	B6
27.	C	H	1.5	2	B4, B2
28.	C	H	1.5	2	B4
29.	B	U	1.5	3	G1
30.	A	U	1.5	3	G2
31.	C	K	1.5	3	G5
32.	D	U	1.5	3	G5
33.	A	U	1.5	3	G5
34.	A	U	1.5	3, 1	G3, A7
35.	C	U	1.5	3	G9
36.	B	U	1.5	3	G7
37.	B	H	1.5	3	G2
38.	B	U	1.5	3	H2
39.	A	U	1.5	3	H2, H3
40.	C	U	1.5	3	H4
41.	C	U	1.5	3	H3, H4
42.	A	U	1.5	1	A1
43.	C	U	1.5	1	A1
44.	D	H	1.5	1	A1

Multiple Choice = 66 marks

Part B: Written Response

Q	B	C	S	CO	PLO
1.	1	U	4	2, 1	C9, A7
2.	2	U	4	2	F3
3.	3	U	4	2	D3, D6
4.	4	U	5	2	E4
5.	5	U	4	3	G8
6.	6	H	4	3, 1	H3, A3
7.	7	U	4	1	A1, A7
8.	8	H	5	3	H4

Written Response = 34 marks

Multiple Choice = 66 (44 questions)
Written Response = 34 (8 questions)
EXAMINATION TOTAL = 100 marks

LEGEND:

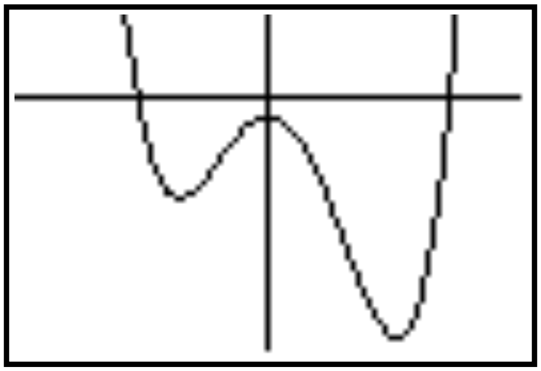
Q = Question Number
K = Keyed Response
C = Cognitive Level
B = Score Box Number
S = Score
CO = Curriculum Organizer
PLO = Prescribed Learning Outcome

1. Solve the following using a graphing calculator. **(4 marks)**

$$x^4 - x^3 \geq 8x^2 + 2$$

Sketch the graph in the viewing window below and indicate appropriate window dimensions. State the function(s) used in your graph. Ensure that the relative maximum and relative minimum points of the function(s) are visible within the viewing window.

Solution



$Y_1 = x^4 - x^3 - 8x^2 - 2$ ← $\frac{1}{2}$ **mark** for equation

← **1 mark** for graph

$x\ [-4.7,\ 4.7]$ $y\ [-30,\ 10]$

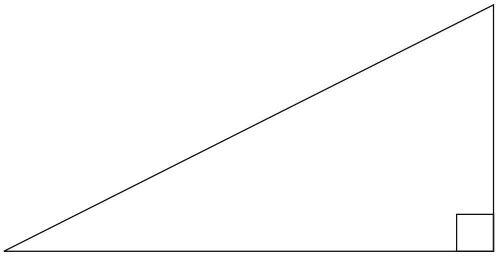
← $\frac{1}{2}$ **mark** for window dimensions **(deducted if missing, but not given if there alone.)**

zeros are: $-2.43,\ 3.40$

← **1 mark** for zeros

$$\underset{\frac{1}{2}\ \textbf{mark}}{\underset{\uparrow}{x \leq -2.43}} \quad \text{or} \quad \underset{\frac{1}{2}\ \textbf{mark}}{\underset{\uparrow}{x \geq 3.40}}$$

2. A right triangle has a perimeter of 56 cm. If the hypotenuse measures 25 cm, determine the lengths of the other two sides of the triangle. (Solve algebraically.) **(4 marks)**



Solution

$\frac{1}{2}$ **mark** → $x^2 + y^2 = 25^2$ $x + y + 25 = 56$ ← $\frac{1}{2}$ **mark**

$x + y = 31$

$y = \sqrt{25^2 - x^2}$ ← $\frac{1}{2}$ **mark or** → $y = (31 - x)$

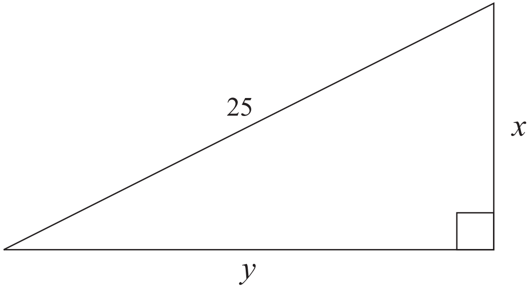
$x^2 + (31 - x)^2 = 25^2$ ← $\frac{1}{2}$ **mark or** → $x + \sqrt{25 - x^2} = 31$

$x^2 + 961 - 62x + x^2 - 625 = 0$ ← $\frac{1}{2}$ **mark** for correct squaring

$2x^2 - 62x + 336 = 0$

$x^2 - 31x + 168 = 0$

$(x - 24)(x - 7) = 0$ ← $\frac{1}{2}$ **mark**



$x = 24$ or $x = 7$

↑ $\frac{1}{2}$ **mark**   ↑ $\frac{1}{2}$ **mark**

$\therefore\ y = 7$ or $y = 24$

∴ the sides measure 7 cm and 24 cm

3. The population of a certain type of bacteria doubles every 20 hours. At this rate, in how many hours will the original population of 100 bacteria grow to 450 bacteria?
(Answer in hours, accurate to at least 2 decimal places.) **(4 marks)**

Solution

$$\underset{\substack{\nwarrow \quad \nearrow \\ \frac{1}{2}\textbf{ mark}}}{450 = 100}(\overset{\substack{\frac{1}{2}\textbf{ mark} \\ \downarrow}}{2})^{\overset{\swarrow\!\!- \, \frac{1}{2}\textbf{ mark}}{\frac{t}{20}}} \qquad \leftarrow \tfrac{1}{2}\textbf{ mark for form of equation}$$

$$\left.\begin{aligned} \log_2 4.5 &= \frac{t}{20} \\ t &= 20 \log_2 4.5 \end{aligned}\right\} \leftarrow \textbf{1 mark}$$

$$= 43.40 \text{ hours} \qquad \leftarrow \textbf{1 mark}$$

3. The population of a certain type of bacteria doubles every 20 hours. At this rate, in how many hours will the original population of 100 bacteria grow to 450 bacteria? (Answer in hours, accurate to at least 2 decimal places.) **(4 marks)**

Alternate Solution 1

$$\overset{\frac{1}{2}\textbf{ mark}\ \swarrow}{}\qquad \overset{\frac{1}{2}\textbf{ mark}}{\downarrow}$$

$$450 = 100(2)^{\frac{t}{20}} \qquad \leftarrow \tfrac{1}{2}\textbf{ mark for form of equation}$$

$$\nwarrow\ \nearrow$$

$$\tfrac{1}{2}\textbf{ mark}$$

$$4.5 = 2^{\frac{t}{20}}$$

$$\log 4.5 = \log 2^{\frac{t}{20}}$$

$$\log 4.5 = \frac{t}{20}\log 2 \qquad \leftarrow \textbf{1 mark}$$

$$20\,\frac{\log 4.5}{\log 2} = t \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

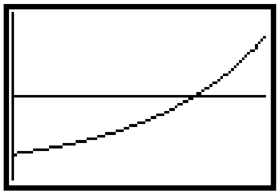
$$43.40 \approx t \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

Alternate Solution 2

$$\overset{\frac{1}{2}\textbf{ mark}}{\downarrow}\qquad \swarrow\textbf{1 mark}$$

$$450 = 100(2)^{\frac{t}{20}} \qquad \leftarrow \tfrac{1}{2}\textbf{ mark for function}$$

This equation may be solved graphically ← **1 mark for this solution**



$$Y_1 = 100(2)^{\frac{x}{20}} - 450$$

$x\ [0,\ 60] \qquad y\ [-500,\ 500]$

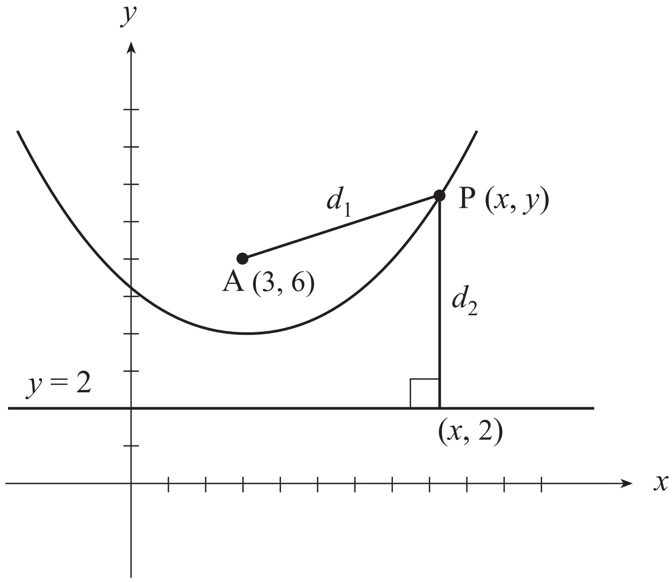
$$x = 43.3985$$

Therefore time = 43.40 hours ← **1 mark**

4. A point $P(x,\ y)$ moves such that it is the same distance from point $A(3,\ 6)$ as it is from the line $y = 2$. Determine the equation of this locus. Express your answer in standard form.

(5 marks)

Solution



$$d_1 = d_2 \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$\sqrt{(x-3)^2 + (y-6)^2} = \sqrt{(x-x)^2 + (y-2)^2} \qquad \leftarrow \textbf{2 marks}$$

$$(x-3)^2 + y^2 - 12y + 36 = y^2 - 4y + 4 \qquad \leftarrow \textbf{1 mark}$$

$$\left.\begin{aligned} (x-3)^2 &= 8y - 32 \\ 8y - 32 &= (x-3)^2 \end{aligned}\right\} \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$\left.\begin{aligned} y - 4 &= \frac{1}{8}(x-3)^2 \\ y &= \frac{1}{8}(x-3)^2 + 4 \end{aligned}\right\} \leftarrow \textbf{1 mark}$$

5. Prove the identity: **(4 marks)**

$$(1-\sin\theta)(\sec\theta+\tan\theta) \qquad = \qquad \frac{1}{\sec\theta}$$

Solution

	LEFT SIDE	RIGHT SIDE
	$(1-\sin\theta)(\sec\theta+\tan\theta)$	$\frac{1}{\sec\theta}$
1 mark →	$=\sec\theta+\tan\theta-\sin\theta\sec\theta-\sin\theta\tan\theta$	
1 mark →	$=\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}-\frac{\sin\theta}{\cos\theta}-\frac{\sin^2\theta}{\cos\theta}$	
$\frac{1}{2}$ mark →	$=\frac{1}{\cos\theta}-\frac{\sin^2\theta}{\cos\theta}$	
	$=\frac{1-\sin^2\theta}{\cos\theta}$	
$\frac{1}{2}$ mark →	$=\frac{\cos^2\theta}{\cos\theta}$	
$\frac{1}{2}$ mark →	$=\cos\theta$	
$\frac{1}{2}$ mark →	$=\frac{1}{\sec\theta}$	

LS = RS

5. Prove the identity: **(4 marks)**

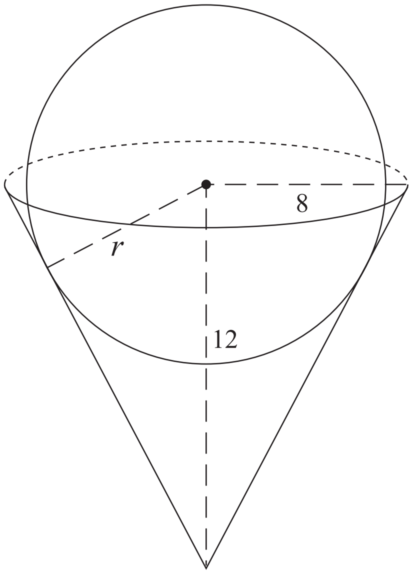
$$(1-\sin\theta)(\sec\theta+\tan\theta) \qquad = \qquad \frac{1}{\sec\theta}$$

Alternate Solution

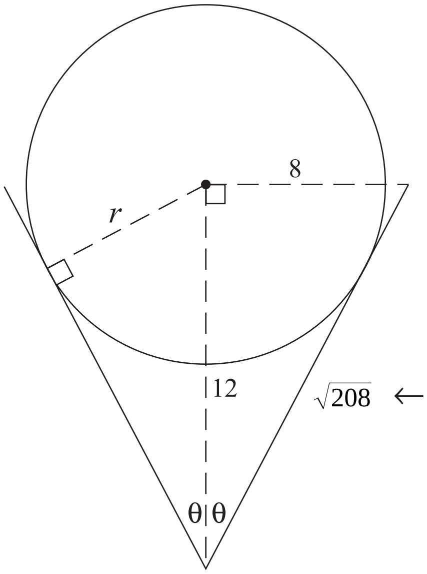
LEFT SIDE	RIGHT SIDE
$(1-\sin\theta)(\sec\theta+\tan\theta)$	$\frac{1}{\sec\theta}$
1 mark → $=(1-\sin\theta)\left(\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}\right)$	$=\frac{1}{\frac{1}{\cos\theta}}$
½ mark → $=(1-\sin\theta)\left(\frac{1+\sin\theta}{\cos\theta}\right)$	$=\cos\theta$ ← **½ mark**
1 mark → $=\frac{1-\sin^2\theta}{\cos\theta}$	
½ mark → $=\frac{\cos^2\theta}{\cos\theta}$	
½ mark → $=\cos\theta$	

LS = RS

6. A cone has radius 8 cm and height 12 cm. Determine the radius, r, of a sphere that will just fit into the cone so that its centre is level with the top of the cone, as shown in the diagram. **(4 marks)**



Solution

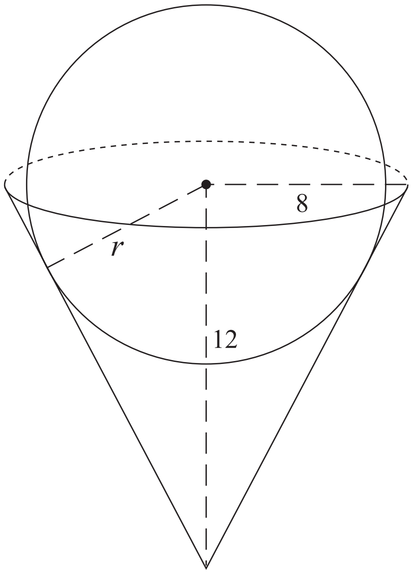


$\sqrt{208}$ ← **1 mark**

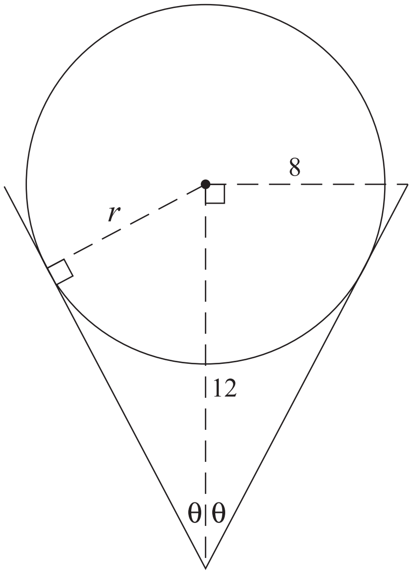
$$\frac{r}{12} = \frac{8}{\sqrt{208}}$$ ← **2 marks**

$r = 6.66$ cm ← **1 mark**

6. A cone has radius 8 cm and height 12 cm. Determine the radius, r, of a sphere that will just fit into the cone so that its centre is level with the top of the cone, as shown in the diagram. **(4 marks)**



Alternate Solution 1



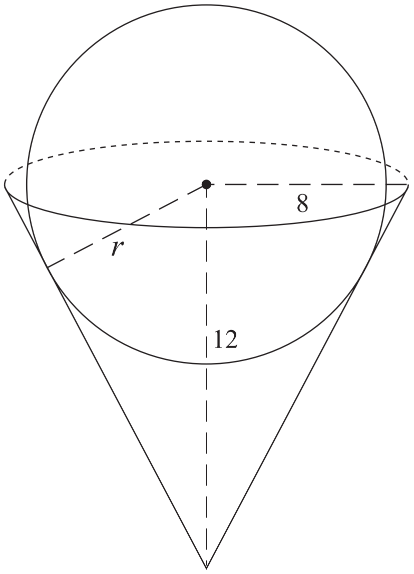
$\tan\theta = \frac{8}{12}$ ← **1 mark**

$\theta = 33.69°$ ← **1 mark**

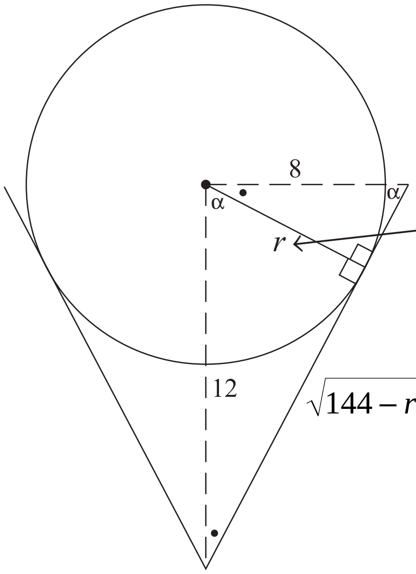
$\sin\theta = \frac{r}{12}$ ← **1 mark**

$r = 6.66$ cm ← **1 mark**

6. A cone has radius 8 cm and height 12 cm. Determine the radius, r, of a sphere that will just fit into the cone so that its centre is level with the top of the cone, as shown in the diagram. **(4 marks)**



Alternate Solution 2



$\frac{1}{2}$ **mark**

$\sqrt{144 - r^2}$ ← **1 mark**

$$\frac{r}{8} = \frac{\sqrt{144 - r^2}}{12} \qquad \leftarrow \textbf{1 mark}$$

$$12r = 8\sqrt{144 - r^2}$$

$$3r = 2\sqrt{144 - r^2}$$

$$9r^2 = 4\left(144 - r^2\right)$$

$$9r^2 = 576 - 4r^2$$

$$13r^2 = 576$$

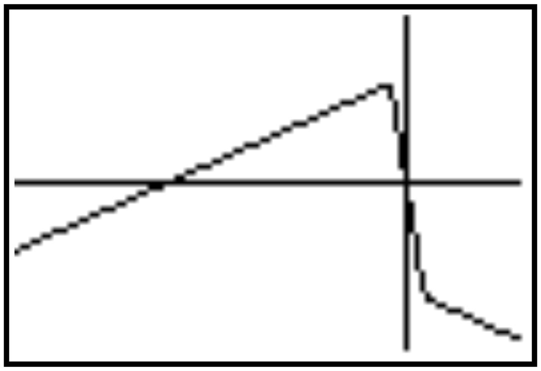
$$r = 6.66 \text{ cm} \qquad \leftarrow \textbf{1}\tfrac{\textbf{1}}{\textbf{2}}\textbf{ marks}$$

7. Solve the following equation using a graphing calculator. **(4 marks)**

$$|x-3| = 1.15|x+3|$$

Sketch the graph in the viewing window below and indicate appropriate window dimensions. State the function(s) used in your graph. Ensure that the relative maximum and relative minimum points of the function(s) are visible within the viewing window.

Solution



$Y_1 = |x-3| - 1.15|x+3|$ ← $\frac{1}{2}$ **mark** for equation

← **1 mark** for graph

$x\ [-70,\ 20]$ $y\ [-10,\ 10]$ ← $\frac{1}{2}$ **mark** for window dimensions

$x = -43,\ -0.21$

↑ ↑

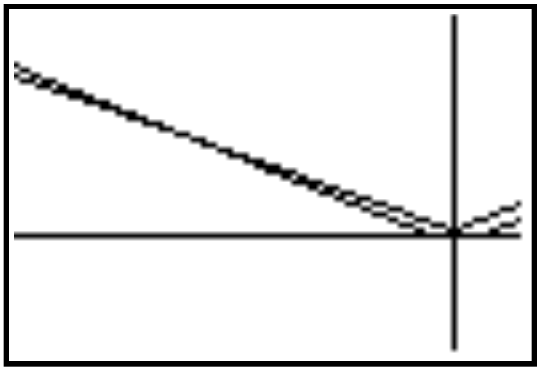
1 mark each

7. Solve the following equation using a graphing calculator. **(4 marks)**

$$|x-3| = 1.15|x+3|$$

Sketch the graph in the viewing window below and indicate appropriate window dimensions. State the function(s) used in your graph. Ensure that the relative maximum and relative minimum points of the function(s) are visible within the viewing window.

Alternate Solution



$\left.\begin{array}{l} Y_1 = |x-3| \\ Y_2 = 1.15|x+3| \end{array}\right\}$ ← $\frac{1}{2}$ **mark** for equations

← **1 mark** for graph

$x\ [-70,\ 10]$ $y\ [-50,\ 100]$ ← $\frac{1}{2}$ **mark** for window dimensions

$$\underset{\substack{\uparrow \\ \textbf{1 mark each}}}{x = -43,\ -0.21}$$

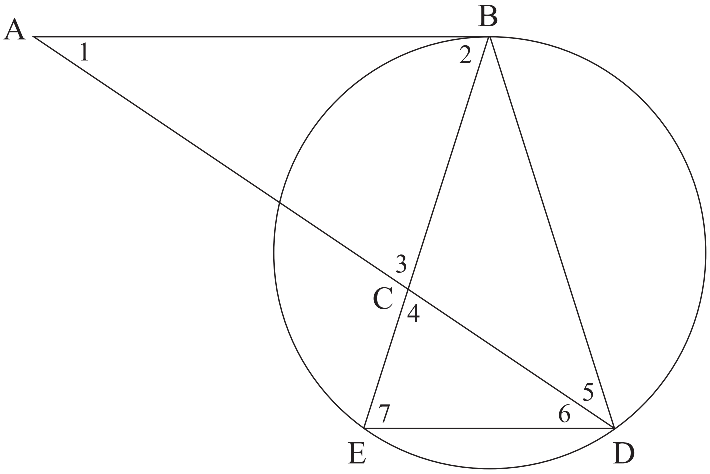
Students must choose one or the other method of proof.

8. Complete the proof. **(5 marks)**

Diagram clarification: A, C, D are collinear
B, C, E are collinear

Given: BE = BD
AB is tangent to the circle

Prove: $\angle 1 = \angle 6$



Solution

Paragraph proof method:

Since BE = BD, $\angle 7 = \angle BDE$ **($\frac{1}{2}$ mark)** because of isosceles Δ **($\frac{1}{2}$ mark)**. Also since AB is a tangent, $\angle 2 = \angle BDE$ **($\frac{1}{2}$ mark)** because of $\angle$ between tangent and chord **(1 mark)**. Therefore $\angle 7 = \angle 2$ **($\frac{1}{2}$ mark)** by substitution **(1 mark)**. However $\angle 3 = \angle 4$ **($\frac{1}{2}$ mark)** because they are vertically opposite $\angle$s, so $\angle 1 = \angle 6$ by third $\angle$s in Δs are equal **($\frac{1}{2}$ mark)**.

8. Complete the proof. **(5 marks)**

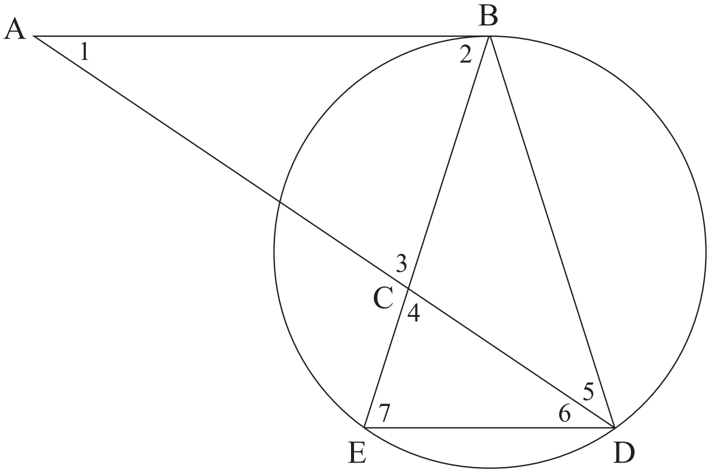


Diagram clarification: A, C, D are collinear
B, C, E are collinear

Given: BE = BD
AB is tangent to the circle

Prove: ∠1 = ∠6

Solution

Two-column proof method:

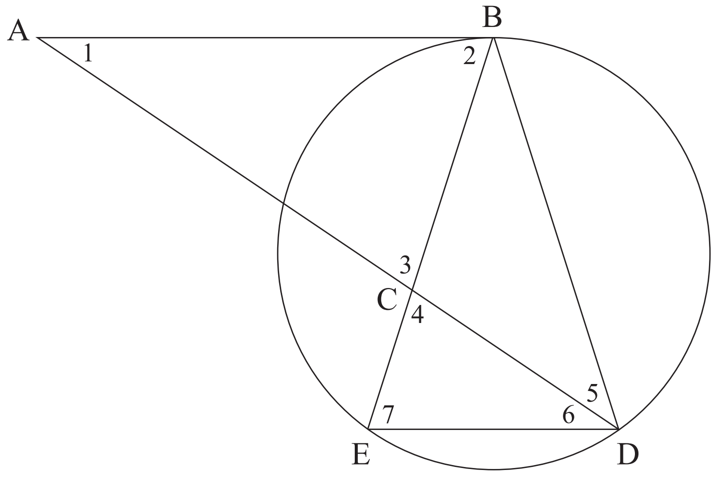
	STATEMENT	REASON	
	BE = BD	given	
$\frac{1}{2}$ mark →	∠7 = ∠BDE	∠s opposite = sides are = (inscribed ∠s on = chords are =)	← **$\frac{1}{2}$ mark**
	AB is tangent to the circle	given	
$\frac{1}{2}$ mark →	∠2 = ∠BDE	∠ between tangent and chord	← **1 mark**
$\frac{1}{2}$ mark →	∠7 = ∠2	substitution (both equal to ∠BDE)	← **1 mark**
$\frac{1}{2}$ mark →	∠3 = ∠4	vertically opposite ∠s are =	
	∠1 = ∠6	3rd ∠s in Δs are =	← **$\frac{1}{2}$ mark**

8. Complete the proof. **(5 marks)**

Diagram clarification: A, C, D are collinear
B, C, E are collinear

Given: BE = BD
AB is tangent to the circle

Prove: ∠1 = ∠6



Alternate Solution

Two-column proof method:

	STATEMENT	REASON	
	BE = BD	given	
½ mark →	∠7 = ∠BDE	∠s opposite = sides are =	← **½ mark**
	AB is a tangent	given	
½ mark →	∠2 = ∠BDE	∠ between tangent and chord	← **1 mark**
½ mark →	∠7 = ∠2	both equal to ∠BDE	← **1 mark**
½ mark →	AB││ED	alternate interior ∠s are =	
	∠1 = ∠6	alternate interior ∠s are =	← **½ mark**

END OF KEY