Principles of Mathematics 12

January 2001 Provincial Examination

ANSWER KEY / SCORING GUIDE

CURRICULUM:

Organizers	Sub-Organizers
1. Problem Solving	A Problem Set
2. Patterns and Relations	B Sequences and Series
	C Polynomials
	D Logarithms and Exponents
	E Quadratic Relations
	F Quadratic Systems
3. Shape and Space	G Trigonometry
	H Geometry

Part A: Multiple Choice

Q	K	C	S	CO	PLO	Q	K	C	S	CO	PLO
1.	A	K	1.5	2	C4	23.	D	H	1.5	2	D5
2.	B	K	1.5	2	C5	24.	C	U	1.5	2	B4
3.	C	U	1.5	2	C3	25.	A	U	1.5	2	B4
4.	C	U	1.5	2	C4	26.	B	U	1.5	2	B5
5.	C	H	1.5	2	C9	27.	B	U	1.5	2	B6, B4
6.	B	K	1.5	2	E6	28.	B	H	1.5	2	B3
7.	A	U	1.5	2	E2	29.	C	U	1.5	3	G1
8.	B	U	1.5	2	F5	30.	A	K	1.5	3	G5
9.	A	K	1.5	2	E4	31.	C	U	1.5	3	G3
10.	A	U	1.5	2	F1	32.	B	U	1.5	3	G2
11.	C	U	1.5	2	F2	33.	B	U	1.5	3	G8
12.	D	U	1.5	2	E4	34.	C	U	1.5	3, 1	G3, A7
13.	D	U	1.5	2	F3	35.	D	U	1.5	3	G7
14.	C	U	1.5	2	E4	36.	A	H	1.5	3	G5, G7
15.	D	H	1.5	2	F1	37.	D	H	1.5	3	G6
16.	B	H	1.5	2	E7	38.	B	U	1.5	3	H2
17.	B	U	1.5	2	D5	39.	A	U	1.5	3	H4
18.	D	U	1.5	2	D5	40.	B	H	1.5	3	H3
19.	A	K	1.5	2	D2	41.	D	U	1.5	3	H2
20.	C	U	1.5	2	D4	42.	C	U	1.5	1	A1
21.	A	U	1.5	2	D3	43.	D	U	1.5	1	A1
22.	D	U	1.5	2, 1	D2, A7	44.	C	U	1.5	1	A1, A7

Multiple Choice = 66 marks

Part B: Written Response

Q	B	C	S	CO	PLO
1.	1	U	4	2, 1	C2, A7
2.	2	U	5	2	E6
3.	3	U	4	2	B4
4.	4	U	4	3	G8
5.	5	U	4	2, 1	D2, A7
6.	6	U	4	1	A1
7.	7	H	4	3	H1
8.	8	H	5	3	H2

Written Response = 34 marks

Multiple Choice = 66 (44 questions)
Written Response = 34 (8 questions)
EXAMINATION TOTAL = 100 marks

LEGEND:

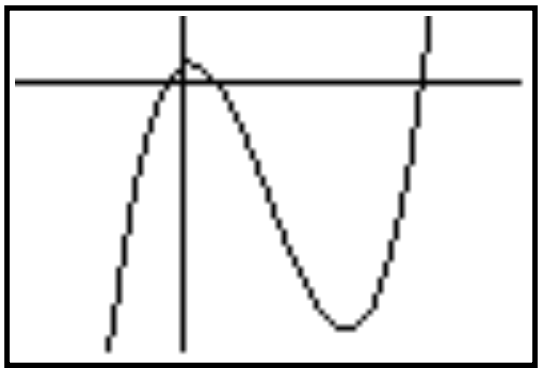
Q = Question Number
K = Keyed Response
C = Cognitive Level
B = Score Box Number
S = Score
CO = Curriculum Organizer
PLO = Prescribed Learning Outcome

1. Solve the following equation using a graphing calculator. **(4 marks)**

$$x^3 - 15x^2 = -10x - 30$$

Sketch the graph in the viewing window below and indicate appropriate window dimensions. State the function(s) used in your graph. Ensure that the relative maximum and relative minimum points of the function(s) are visible within the viewing window.

Solution



$Y_1 = x^3 - 15x^2 + 10x + 30$ ← **½ mark** for equation

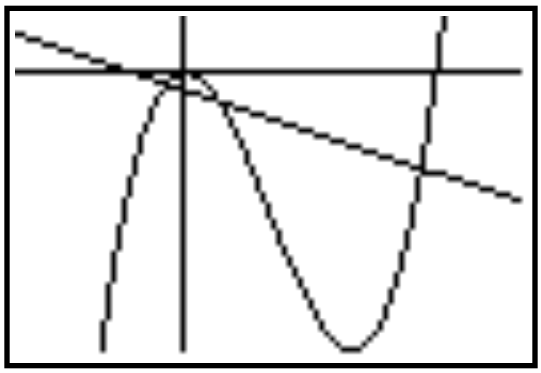
← **1 mark** for graph

$x\ [-10,\ 20]$ $\quad y\ [-400,\ 100]$ ← **½ mark** for window dimensions

$-1.09,\ 1.95,\ 14.14$ ← **2 marks**

↑ **½ mark** ↑ **½ mark** ↑ **1 mark**

Alternate Solution



$Y_1 = x^3 - 15x^2$
$Y_2 = -10x - 30$ } ← **½ mark** for equations

← **1 mark** for graph

$x\ [-10,\ 20]$ $\quad y\ [-500,\ 100]$ ← **½ mark** for window dimensions

$-1.09,\ 1.95,\ 14.14$ ← **2 marks**

↑ **½ mark** ↑ **½ mark** ↑ **1 mark**

2. Change the following equation to standard form. **(5 marks)**

$$4x^2 - 32x - 9y^2 - 36y - 116 = 0$$

Solution

$$4x^2 - 32x - 9y^2 - 36y - 116 = 0$$

$\frac{1}{2}$ $\frac{1}{2}$

$$4\left(x^2 + 8x\right) - 9\left(y^2 + 4y\right) = 116 \qquad \leftarrow \textbf{1 mark}$$

$$4\left(x^2 - 8x + 16\right) - 9\left(y^2 + 4y + 4\right) = 116 + 64 - 36$$

$\uparrow$ $\uparrow$ $\uparrow$ $\uparrow$

$\frac{1}{2}$ **mark** $\frac{1}{2}$ **mark** $\frac{1}{2}$ **mark each for correct signs**

$\frac{1}{2}$ **mark finishing binomial squares**

$\downarrow$ $\downarrow$

$$4(x-4)^2 - 9(y+2)^2 = 144 \qquad \leftarrow \tfrac{1}{2} \textbf{ mark correct sum}$$

$$\frac{4(x-4)^2}{144} - \frac{9(y+2)^2}{144} = \frac{144}{144} \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$\frac{(x-4)^2}{36} - \frac{(y+2)^2}{16} = 1 \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

3. Find the sum of the arithmetic series: $20 + 26 + 32 + \ \ldots \ + 734$ **(4 marks)**

Solution

$20 + 26 + 32 + \ \ldots \ + 734$

$t_n = 20 + (n-1)6 = 734$ ← **1 mark**

$(n-1) = \dfrac{734 - 20}{6} = 119$

$n = 120$ ← **1 mark**

$S_n = \dfrac{n}{2}(a + \ell)$

$S_{120} = \dfrac{120}{2}(20 + 734)$ ← **1 mark**

$= 45\ 240$ ← **1 mark**

4. Prove the identity: **(4 marks)**

$$\frac{\cot\theta - 1}{1 - \tan\theta} = \frac{\csc\theta}{\sec\theta}$$

Solution

LEFT SIDE	RIGHT SIDE
$\dfrac{\cot\theta - 1}{1 - \tan\theta}$	$\dfrac{\csc\theta}{\sec\theta}$
$\frac{1}{2}$ **mark** $\rightarrow$ $= \dfrac{\frac{\cos\theta}{\sin\theta} - 1}{1 - \frac{\sin\theta}{\cos\theta}}$	$= \dfrac{\frac{1}{\sin\theta}}{\frac{1}{\cos\theta}}$ $\leftarrow \frac{1}{2}$ **mark**
$\frac{1}{2}$ **mark** $\rightarrow$ $= \dfrac{\left(\frac{\cos\theta}{\sin\theta} - 1\right)\sin\theta\cos\theta}{\left(1 - \frac{\sin\theta}{\cos\theta}\right)\sin\theta\cos\theta}$	$= \dfrac{\cos\theta}{\sin\theta}$ $\leftarrow \frac{1}{2}$ **mark**
$\frac{1}{2}$ **mark** $\rightarrow$ $= \dfrac{\cos^2\theta - \sin\theta\cos\theta}{\sin\theta\cos\theta - \sin^2\theta}$	
$\frac{1}{2}$ **mark** $\rightarrow$ $= \dfrac{\cos\theta(\cos\theta - \sin\theta)}{\sin\theta(\cos\theta - \sin\theta)}$	
$\frac{1}{2}$ **mark** $\rightarrow$ $= \dfrac{\cos\theta}{\sin\theta}$	

LS = RS

4. Prove the identity: **(4 marks)**

$$\frac{\cot\theta - 1}{1 - \tan\theta} = \frac{\csc\theta}{\sec\theta}$$

Alternate Solution 1

LEFT SIDE	RIGHT SIDE
$\frac{\cot\theta - 1}{1 - \tan\theta}$	$\frac{\csc\theta}{\sec\theta}$
$\frac{1}{2}$ **mark** $\rightarrow$ $= \frac{\frac{1}{\tan\theta} - 1}{1 - \tan\theta}$	$= \frac{\frac{1}{\sin\theta}}{\frac{1}{\cos\theta}}$ $\leftarrow \frac{1}{2}$ **mark**
$\frac{1}{2}$ **mark** $\rightarrow$ $= \frac{\left(\frac{1}{\tan\theta} - 1\right)\tan\theta}{(1 - \tan\theta)\tan\theta}$	$= \frac{\cos\theta}{\sin\theta}$ $\leftarrow \frac{1}{2}$ **mark**
$\frac{1}{2}$ **mark** $\rightarrow$ $= \frac{1 - \tan\theta}{(1 - \tan\theta)\tan\theta}$	$= \cot\theta$
$\frac{1}{2}$ **mark** $\rightarrow$ $= \frac{1}{\tan\theta}$	$= \frac{1}{\tan\theta}$ $\leftarrow \frac{1}{2}$ **mark for either**

LS = RS

4. Prove the identity: **(4 marks)**

$$\frac{\cot\theta - 1}{1 - \tan\theta} = \frac{\csc\theta}{\sec\theta}$$

Alternate Solution 2

LEFT SIDE	RIGHT SIDE
$\frac{\cot\theta - 1}{1 - \tan\theta}$	$\frac{\csc\theta}{\sec\theta}$
$\frac{1}{2}$ **mark** $\rightarrow \quad = \frac{\cot\theta - 1}{1 - \frac{1}{\cot\theta}}$	$= \frac{\frac{1}{\sin\theta}}{\frac{1}{\cos\theta}}$ $\leftarrow \frac{1}{2}$ **mark**
$\frac{1}{2}$ **mark** $\rightarrow \quad = \frac{(\cot\theta - 1)\ \cot\theta}{1 - \frac{1}{\cot\theta}\ \cot\theta}$	$= \frac{\cos\theta}{\sin\theta}$ $\leftarrow \frac{1}{2}$ **mark**
$\frac{1}{2}$ **mark** $\rightarrow$ $\frac{1}{2}$ **mark** $\rightarrow \quad = \frac{(\cot\theta - 1)\cot\theta}{\cot\theta - 1}$	$= \cot\theta$ $\leftarrow \frac{1}{2}$ **mark**
$\frac{1}{2}$ **mark** $\rightarrow \quad = \cot\theta$	

LS = RS

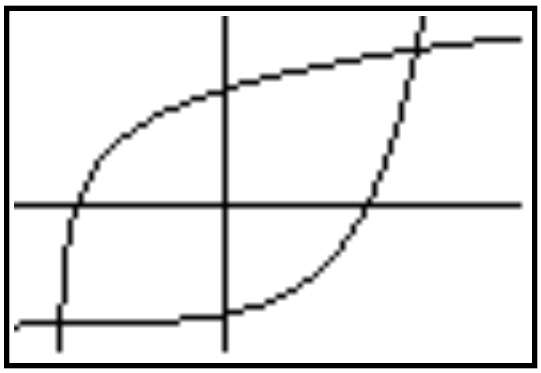
5. Solve the following system using a graphing calculator. Express all solutions as ordered pairs. **(4 marks)**

$$y = 5\log(x+4)+2$$

$$y = 2^{x-1} - 5$$

Sketch the graph in the viewing window below. State the function(s) that you entered to obtain your graph and your solution. Indicate the dimensions of the viewing window that will show enough of the graph so that recognizable characteristics of the function(s) and all intersection points are visible.

Solution



$Y_1 = 5\log(x+4)+2$
$Y_2 = 2^{x-1} - 5$
} ← **deduct $\frac{1}{2}$ mark if missing**

← **1 mark** for graph **($\frac{1}{2}$ mark for each plotted equation)**

x $[-5,\ 7]$ y $[-6,\ 8]$ ← **$\frac{1}{2}$ mark** for window dimensions

$(-3.96,\ -4.97)$ $(4.54,\ 6.66)$ ← **deduct 1 mark if missing *y* values ($\pm\frac{1}{2}$ for each)**

↑ **$1\frac{1}{2}$ marks** ↑ **1 mark**

Note: Full marks should be given if both solutions are shown. There is no penalty for not showing an intersection point in the third quadrant in the viewing screen.

Missing bracket on Exponential function or Logarithmic function ← **deduct 1 mark**

$x-4$ instead of $x+4$ } sign error
$x-1$ instead of $x+1$ } incorrect equation
} ← **deduct 1 mark**

Recognize existance of Quadrant III intersection point ← **$\frac{1}{2}$ mark**

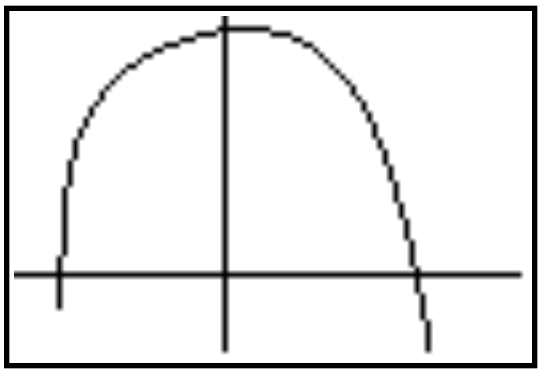
5. Solve the following system using a graphing calculator. Express all solutions as ordered pairs. **(4 marks)**

$$y = 5\log(x+4) + 2$$

$$y = 2^{x-1} - 5$$

Sketch the graph in the viewing window below. State the function(s) that you entered to obtain your graph and your solution. Indicate the dimensions of the viewing window that will show enough of the graph so that recognizable characteristics of the function(s) and all intersection points are visible.

Alternate Solution



$Y_1 = 5\log(x+4) + 2 - \left(2^{x-1} - 5\right)$ ← **$\frac{1}{2}$ mark**

← **$\frac{1}{2}$ mark** for graph

$x\ [-5,\ 7]$ $\quad y\ [-3,\ 10]$ ← **$\frac{1}{2}$ mark** for window dimensions

$x = -3.959596$ ← **1 mark**

$y = 2^{-3.959596-1} - 5$

$(-3.96,\ -4.97)$

↑

$\frac{1}{2}$ mark

$x = 4.5432631$ ← **$\frac{1}{2}$ mark**

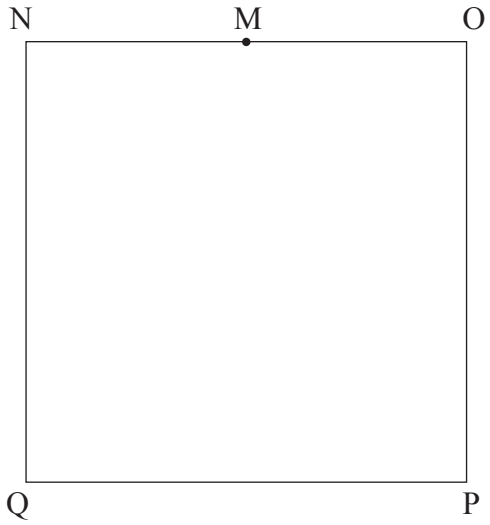
$y = 2^{4.5432631-1} - 5$

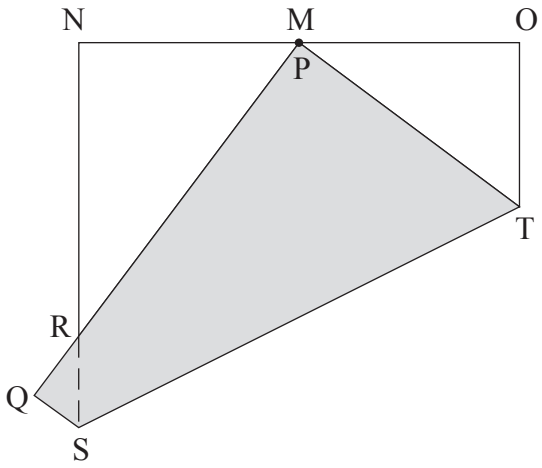
$(4.54,\ 6.66)$ ← **deduct 1 $\frac{1}{2}$ marks if missing *y* values**

↑

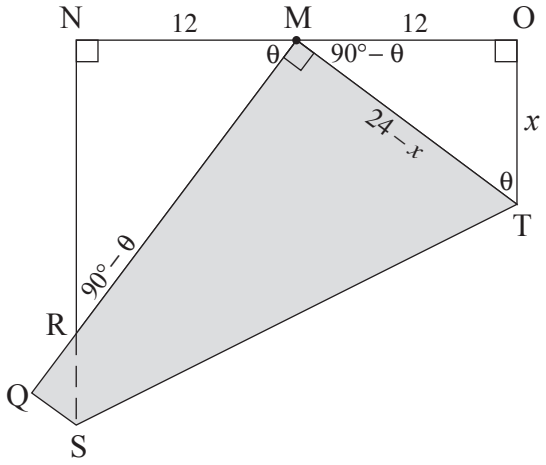
$\frac{1}{2}$ mark

6. A square piece of paper $24\text{ cm} \times 24\text{ cm}$ is folded in such a way that the lower right hand corner at P just touches the midpoint M of the top side, as shown in the diagram. In ΔMNR, determine the length of side NR. **(4 marks)**





Solution



$x^2 + 12^2 = (24 - x)^2$ ← **1 mark**

$x^2 + 144 = 576 - 48x + x^2$ ← **½ mark**

$48x = 432$

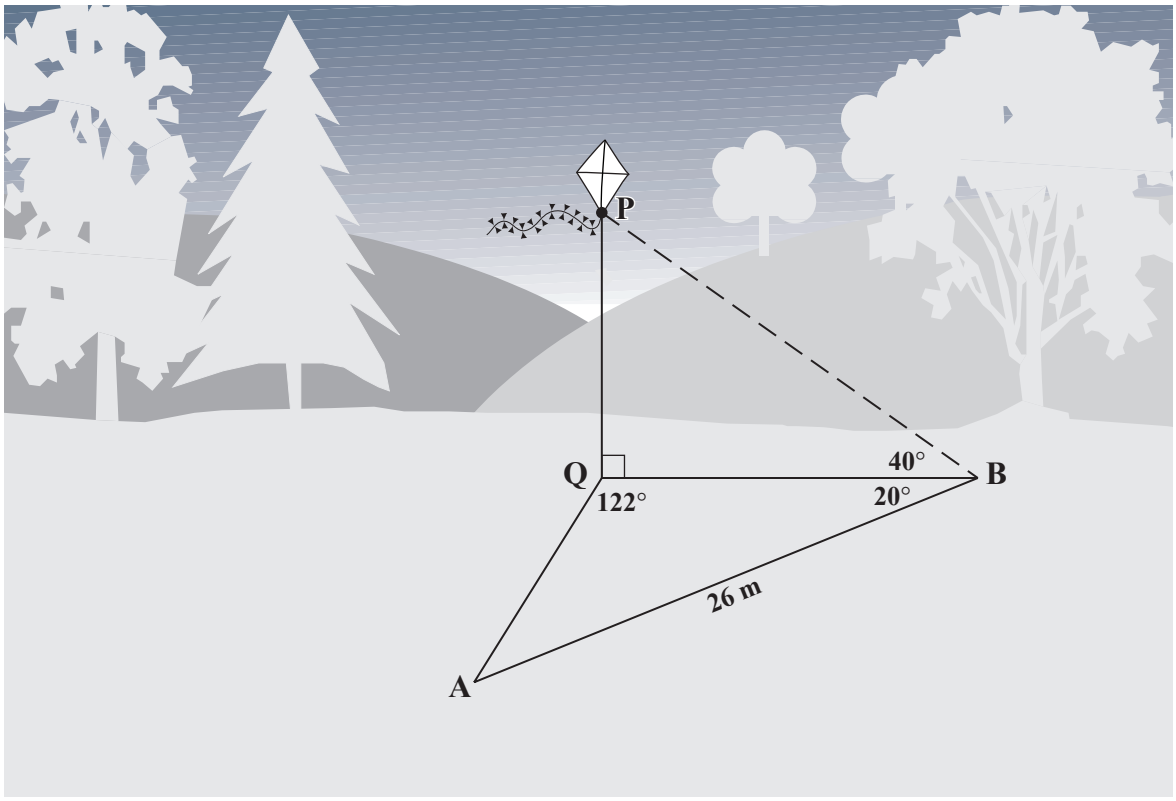
$x = 9$ ← **½ mark**

$\Delta MNR \sim \Delta TOM$

$\frac{NR}{12} = \frac{12}{9}$ } ← **1 ½ marks**

$NR = 12\left(\frac{12}{9}\right) = 16\text{ cm}$ ← **½ mark**

7. A kite is flying at a point P vertically above a point Q which is in the same horizontal plane as two observers standing at A and B. The distance between the observers is 26 metres. $\angle QBA = 20°$ and $\angle AQB = 122°$. The angle of elevation of the kite from the observer at B is $40°$. How high is the kite flying above point Q ? **(4 marks)**



Solution

$\angle A = 180° - (122° + 20°) = 38°$ ← **$\frac{1}{2}$ mark**

$\frac{\sin 38°}{QB} = \frac{\sin 122°}{26}$ ← **1 mark**

$QB = \frac{26 \sin 38°}{\sin 122°} = 18.8753$ ← **1 mark**

$\tan 40° = \frac{PQ}{18.8753}$ ← **$\frac{1}{2}$ mark**

$PQ = 18.8753 \tan 40°$

$PQ = 15.8383$

$= 15.84$ m ← **1 mark**

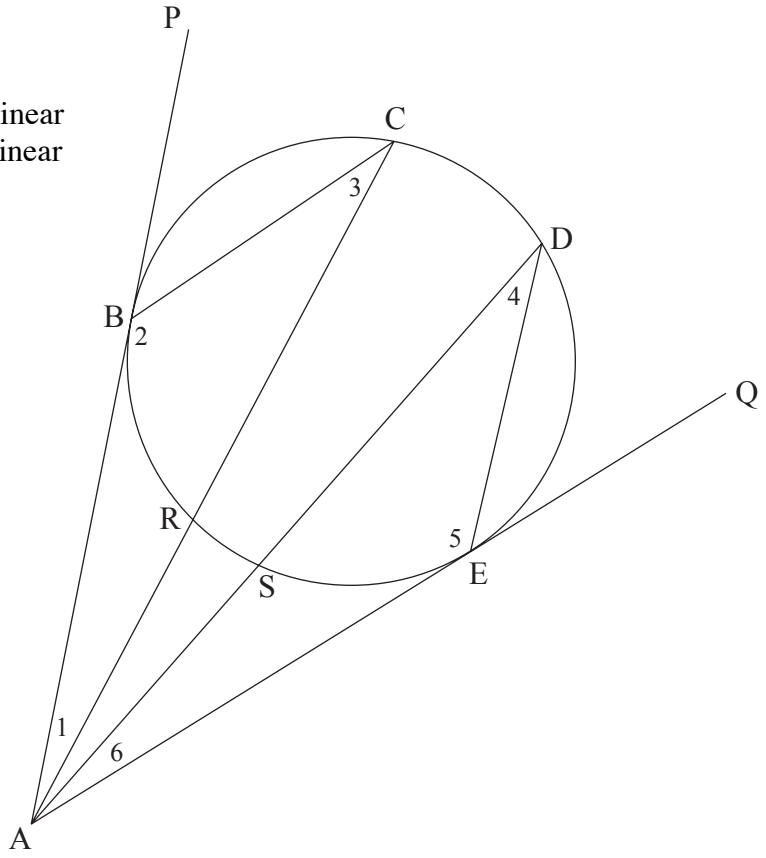
Students must choose one or the other method of proof.

8. Complete the proof. **(5 marks)**

Diagram clarification: A, R, C are collinear
A, S, D are collinear

Given: AP and AQ are tangents
$\overset{\frown}{BR} = \overset{\frown}{SE}$
$\angle 2 = \angle 5$

Prove: BC = ED



Solution

Paragraph proof method:

Since AP and AQ are tangents, AB = AE **($\frac{1}{2}$ mark)** because tangents from an external point are equal **(1 mark)** and since $\overset{\frown}{BR} = \overset{\frown}{SE}$, $\angle 3 = \angle 4$ **(1 mark)** because inscribed $\angle$s on equal arcs are equal **(1 mark)**. Since $\angle 2 = \angle 5$, $\Delta ABC \cong \Delta AED$ **($\frac{1}{2}$ mark)** by AAS **($\frac{1}{2}$ mark)**. Thus BC = ED **($\frac{1}{2}$ mark)**.

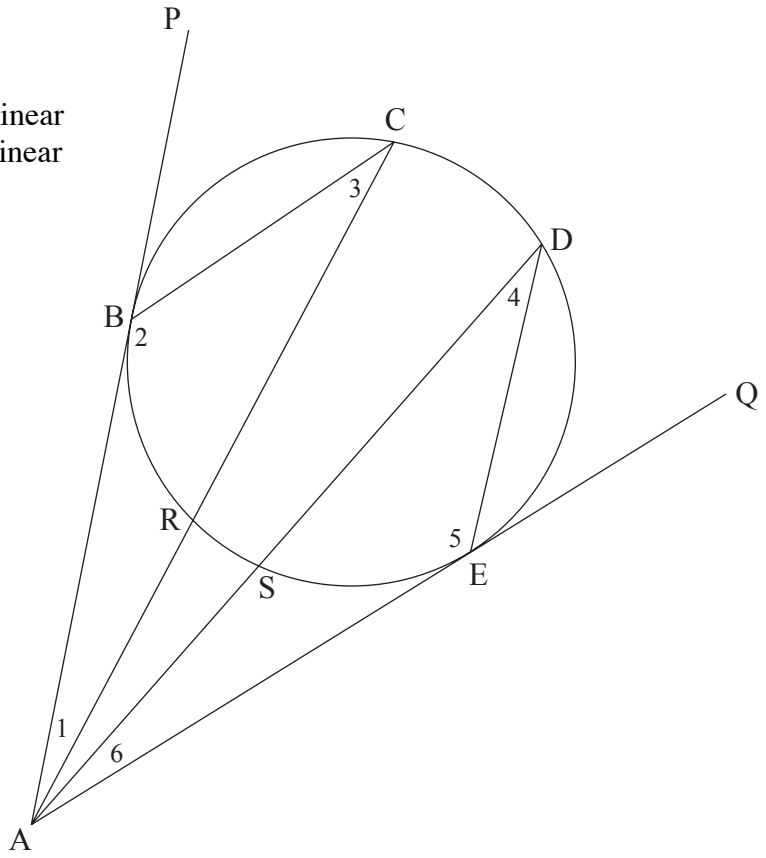
Deduct $\frac{1}{2}$ mark if givens, linking words are missing.

8. Complete the proof. **(5 marks)**

Diagram clarification: A, R, C are collinear
A, S, D are collinear

Given: AP and AQ are tangents
$\overset{\frown}{BR} = \overset{\frown}{SE}$
$\angle 2 = \angle 5$

Prove: BC = ED



Solution

Two-column proof method:

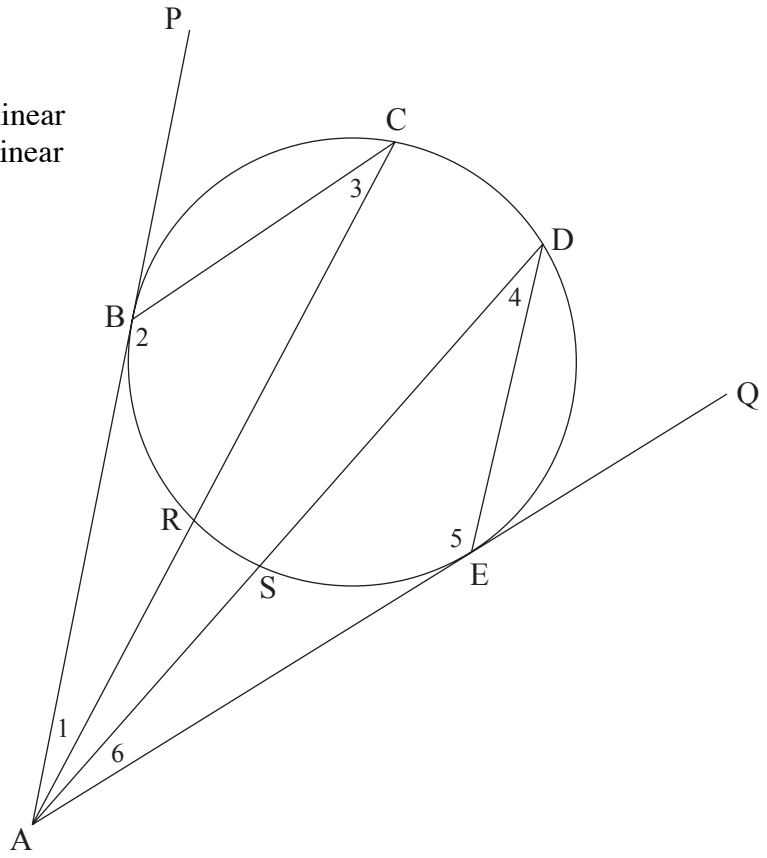
	STATEMENT	REASON
	AP and AQ are tangents	given
	AB = AE $\left(\frac{1}{2}\ \textbf{mark}\right)$	tangents from an external point are = $(\textbf{1 mark})$
3 $\frac{1}{2}$ marks →	$\overset{\frown}{BR} = \overset{\frown}{SE}$	given
	$\angle 3 = \angle 4$ $(\textbf{1 mark})$	inscribed angles on = arcs are = $(\textbf{1 mark})$
	$\angle 2 = \angle 5$	given
1 $\frac{1}{2}$ marks →	$\Delta ABC \cong \Delta AED$ $\left(\frac{1}{2}\ \textbf{mark}\right)$	AAS $\left(\frac{1}{2}\ \textbf{mark}\right)$
	BC = ED	CPCTC $\left(\frac{1}{2}\ \textbf{mark}\right)$

8. Complete the proof. **(5 marks)**

Diagram clarification: A, R, C are collinear
A, S, D are collinear

Given: AP and AQ are tangents
$\overset{\frown}{BR} = \overset{\frown}{SE}$
$\angle 2 = \angle 5$

Prove: BC = ED



Alternate Solution

Two-column proof method:

	STATEMENT	REASON
	AP and AQ are tangents	given
	BA = EA $\left(\frac{1}{2}\ \textbf{mark}\right)$	tangents from an external point are = $(\textbf{1 mark})$
3 $\frac{1}{2}$ marks →	$\overset{\frown}{BR} = \overset{\frown}{SE}$	given
	$\angle 3 = \angle 4$ $(\textbf{1 mark})$	inscribed angles on = arcs are = $(\textbf{1 mark})$
	$\angle 2 = \angle 5$	given
	$\angle 1 = \angle 6$	3rd angles of triangles are = $\left(\pm \frac{1}{2}\ \textbf{mark if missing}\right)$
1 $\frac{1}{2}$ marks →	$\Delta ABC \cong \Delta AED$ $\left(\frac{1}{2}\ \textbf{mark}\right)$	ASA $\left(\frac{1}{2}\ \textbf{mark}\right)$
	BC = ED	CPCTC $\left(\frac{1}{2}\ \textbf{mark}\right)$

END OF KEY