Principles of Mathematics 12

August 2000 Provincial Examination

ANSWER KEY / SCORING GUIDE

CURRICULUM:

Organizers		Sub-Organizers	
1.	Problem Solving	A	Problem Set
2.	Patterns and Relations	B	Sequences and Series
		C	Polynomials
		D	Logarithms and Exponents
		E	Quadratic Relations
		F	Quadratic Systems
3.	Shape and Space	G	Trigonometry
		H	Geometry

Part A: Multiple Choice

Q	K	C	CO	PLO	Q	K	C	CO	PLO
1.	A	K	2	C1	24.	B	U	2	B4
2.	B	U	2	C5	25.	B	K	2	B6
3.	A	U	2	C6	26.	A	U	2	B4
4.	B	U	2, 1	C1, A7	27.	A	H	2	B2
5.	D	U	2	C9	28.	C	H	2	B5
6.	A	K	2	E2	29.	B	U	3	G1
7.	D	K	2	E4	30.	B	K	3	G5
8.	C	U	2	F5	31.	B	U	3	G5
9.	C	U	2	E5	32.	A	U	3	G5
10.	D	U	2	E6	33.	D	U	3	G2
11.	A	U	2	F2	34.	B	U	3	G7
12.	A	U	2	F1	35.	C	U	3, 1	G3, A7
13.	D	U	2	E5	36.	A	U	3, 1	G3, A7
14.	C	H	2	F3	37.	C	H	3	G7
15.	D	H	2	E4	38.	B	H	3	G8
16.	D	H	2	F1	39.	B	U	3	H2
17.	A	K	2	D4	40.	D	U	3	H2
18.	A	U	2	D5	41.	B	U	3	H2
19.	D	U	2	D2	42.	C	H	3	H1
20.	C	U	2	D2	43.	C	U	1	A3
21.	D	U	2	D5	44.	C	U	1	A3
22.	C	H	2	D6	45.	D	H	1	A3
23.	B	H	2	D2					

Multiple Choice = 45 marks

Part B: Written Response

Q	B	C	S	CO	PLO
1.	1	U	3	2	C7
2.	2	U	3	2	B4
3.	3	U	3	2, 1	D5, A7
4.	4	U	3	2	E7
5.	5	U	3	3	G8
6.	6	H	3	3	H4
7.	7	U	3	1	A3, A7
8.	8	H	4	3	H2

Written Response = 25 marks

Multiple Choice = 45 (45 questions)
Written Response = 25 (8 questions)
EXAMINATION TOTAL = 70 marks

LEGEND:

Q = Question Number
K = Keyed Response
C = Cognitive Level
B = Score Box Number
S = Score
CO = Curriculum Organizer
PLO = Prescribed Learning Outcome

PART B: WRITTEN RESPONSE

Value: 25 marks **Suggested Time: 45 minutes**

INSTRUCTIONS: Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

If, in a justification, you refer to information produced by the calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem, it is important to sketch the graph, showing its general shape and indicating the appropriate window dimensions.

When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.

Full marks will NOT be given for the final answer only.

1. Determine the cubic polynomial function with zeros $1, 2,$ and ± 3 that passes through $(3, -10)$. (Answer may be left in factored form.) **(3 marks)**

Solution

$$y = \overset{\overset{\frac{1}{2}\textbf{ mark}}{\downarrow}}{a}(x-1)(x-2)(x+3) \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$-10 = a(3-1)(3-2)(3+3) \qquad \leftarrow \textbf{1 mark} \quad \left(\tfrac{1}{2}\textbf{ mark}\text{ each for substitution of } -10 \text{ for } y \text{ and } 3 \text{ for } x\right)$$

$$-10 = a(2)(1)(6)$$

$$-10 = 12a$$

$$-\frac{10}{12} = a \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$-\frac{5}{6} = a$$

$$\therefore \; y = -\frac{5}{6}(x-1)(x-2)(x+3) \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

2. The sum of the first three terms of an arithmetic series is -12 and the 8th term is 32. Determine the 4th term of this series. **(3 marks)**

Solution

$$\begin{aligned}
3a + 3d &= -12 && \leftarrow \tfrac{1}{2}\ \textbf{mark} \\
a + 7d &= 32 && \leftarrow \tfrac{1}{2}\ \textbf{mark} \\
a + d &= -4 && \\
6d &= 36 && \\
d &= 6 && \leftarrow \tfrac{1}{2}\ \textbf{mark} \\
a &= -10 && \leftarrow \tfrac{1}{2}\ \textbf{mark} \\
t_4 &= -10 + 3(6) && \leftarrow \tfrac{1}{2}\ \textbf{mark} \\
t_4 &= 8 && \leftarrow \tfrac{1}{2}\ \textbf{mark}
\end{aligned}$$

3. Solve the following system using a graphing calculator. **(3 marks)**

$$y^2 - x^2 = 16$$

$$y = 4\log(x+3)$$

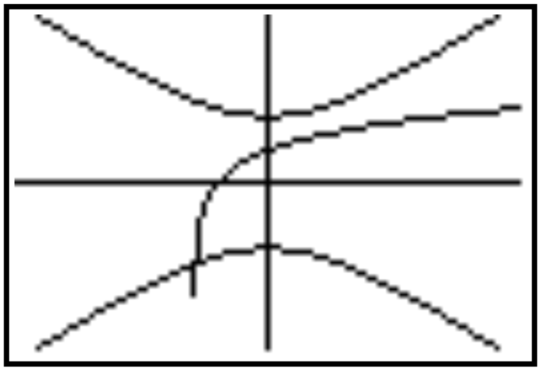
Sketch the graph in the viewing window below. State the function(s) that you entered to obtain your graph and your solution. Indicate the dimensions of the viewing window that will show enough of the graph so that recognizable characteristics of the function(s) and all intersection points are visible.

Solution

For changing relation to $y =$ $\leftarrow \frac{1}{2}$ **mark**

$$\left.\begin{array}{l} Y_1 = \sqrt{x^2+16} \\ Y_2 = -\sqrt{x^2+16} \\ Y_3 = 4\log(x+3) \end{array}\right\} \leftarrow \tfrac{1}{2}\ \textbf{mark}\ \text{for equations}$$

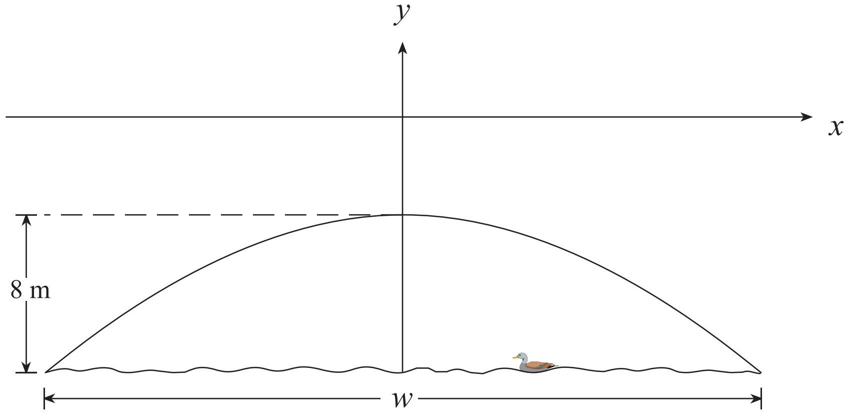
$\leftarrow$ **1 mark** for graph $\left(\begin{array}{l} \frac{1}{2}\ \textbf{mark}\ \text{for log} \\ \frac{1}{2}\ \textbf{mark}\ \text{for hyperbola} \end{array}\right)$



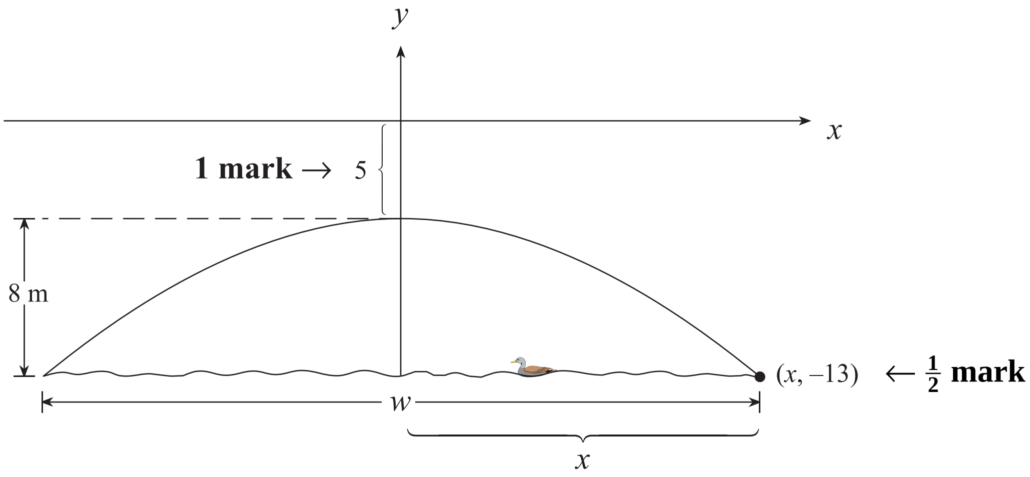
$x\ [-10,\ 10]$ $y\ [-10,\ 10]$

$(-2.94,\ -4.97)$ $\leftarrow \frac{1}{2}$ **mark each**

4. A bridge over a river is supported by an arch in the shape of a rectangular hyperbola as shown in the diagram. The equation of the arch is $y^2 - x^2 = 25$. If the maximum height of the arch above the water is 8 m, determine the width, w, of the river. **(3 marks)**



Solution



$$y^2 - x^2 = 25$$

$$(-13)^2 - x^2 = 25 \quad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$-x^2 = -144$$

$$x^2 = 144$$

$$x = 12 \quad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$\therefore$ width is 24 m $\leftarrow \frac{1}{2}$ **mark**

5. Prove the identity: **(3 marks)**

$$\frac{\cos 2\theta}{\sin\theta} \qquad = \qquad \frac{\cot^2\theta - 1}{\csc\theta}$$

Solution

$$\frac{\cos 2\theta}{\sin\theta} \qquad = \qquad \frac{\cot^2\theta - 1}{\csc\theta}$$

LEFT SIDE	RIGHT SIDE
	$= \dfrac{\dfrac{\cos^2\theta}{\sin^2\theta} - 1}{\dfrac{1}{\sin\theta}}$ ← **½ mark** ← **½ mark** $= \dfrac{\left(\dfrac{\cos^2\theta}{\sin^2\theta} - 1\right)}{\left(\dfrac{1}{\sin\theta}\right)} \quad \dfrac{\sin^2\theta}{\sin^2\theta}$ ← **½ mark** $= \dfrac{\cos^2\theta - \sin^2\theta}{\sin\theta}$ ← **½ mark** ← **½ mark** $= \dfrac{\cos 2\theta}{\sin\theta}$ ← **½ mark**

LS = RS

5. Prove the identity: **(3 marks)**

$$\frac{\cos 2\theta}{\sin\theta} = \frac{\cot^2\theta - 1}{\csc\theta}$$

Alternate Solution 1

$$\frac{\cos 2\theta}{\sin\theta} = \frac{\cot^2\theta - 1}{\csc\theta}$$

LEFT SIDE	RIGHT SIDE
	$= \dfrac{\dfrac{\cos^2\theta}{\sin^2\theta} - \dfrac{\sin^2\theta}{\sin^2\theta}}{\dfrac{1}{\sin\theta}}$ ← $\frac{1}{2}$ **mark** ← $\frac{1}{2}$ **mark**
	$= \dfrac{\cos^2\theta - \sin^2\theta}{\sin^2\theta} \div \dfrac{1}{\sin\theta}$ ← $\frac{1}{2}$ **mark**
	$= \dfrac{\cos^2\theta - \sin^2\theta}{\sin^2\theta} \cdot \dfrac{\sin\theta}{1}$ ← $\frac{1}{2}$ **mark**
	$= \dfrac{\cos 2\theta}{\sin\theta}$ ← $\frac{1}{2}$ **mark** ← $\frac{1}{2}$ **mark**

LS = RS

5. Prove the identity: **(3 marks)**

$$\frac{\cos 2\theta}{\sin\theta} = \frac{\cot^2\theta - 1}{\csc\theta}$$

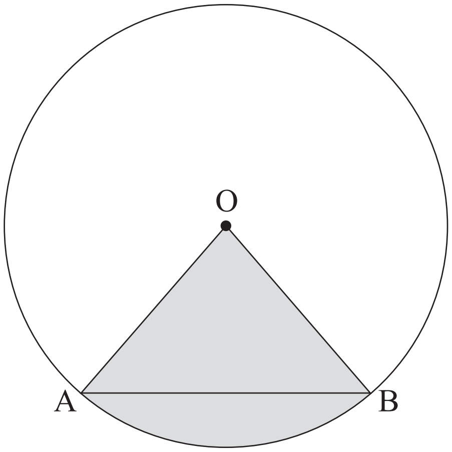
Alternate Solution 2

$$\frac{\cos 2\theta}{\sin\theta} = \frac{\cot^2\theta - 1}{\csc\theta}$$

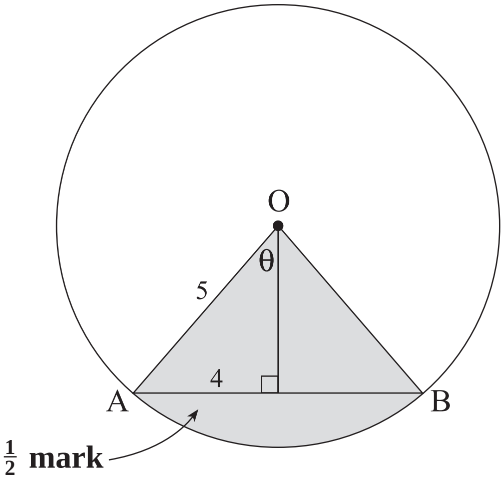
LEFT SIDE	RIGHT SIDE	
	$\frac{\csc^2\theta - 1 - 1}{\csc\theta}$	← $\frac{1}{2}$ **mark**
	$\frac{\csc^2\theta - 2}{\csc\theta}$	
	$\frac{\frac{1}{\sin^2\theta} - 2}{\frac{1}{\sin\theta}}$	← $\frac{1}{2}$ **mark**
	$\frac{\left(\frac{1}{\sin^2\theta} - 2\right)}{\left(\frac{1}{\sin\theta}\right)} \frac{\sin^2\theta}{\sin^2\theta}$	← $\frac{1}{2}$ **mark**
	$\frac{1 - 2\sin^2\theta}{\sin\theta}$	← $\frac{1}{2}$ **mark** ← $\frac{1}{2}$ **mark**
	$\frac{\cos 2\theta}{\sin\theta}$	← $\frac{1}{2}$ **mark**

LS = RS

6. A circle with centre O has a radius of 5 cm. If chord AB = 8 cm, find the area of the shaded region. **(3 marks)**



Solution



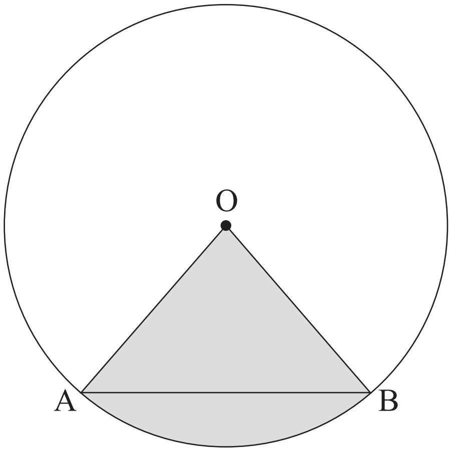
$$\sin\theta = \frac{4}{5} \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$\theta = 53.1301° \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

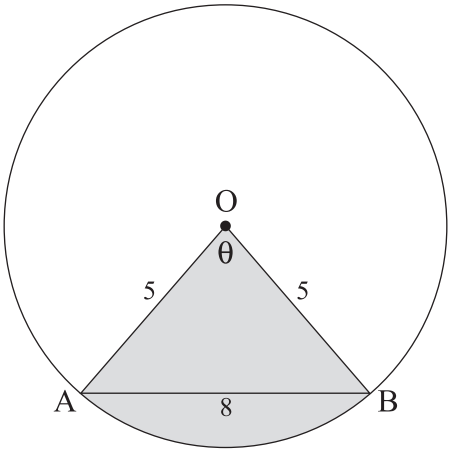
$$\therefore\ \text{area of sector} = \overbrace{\frac{106.2602°}{360°}}^{\tfrac{1}{2}\ \textbf{mark}\ \downarrow}\ \overbrace{\pi(25)}^{\tfrac{1}{2}\ \textbf{mark}\ \downarrow}$$

$$= 23.18\ \text{cm}^2 \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

6. A circle with centre O has a radius of 5 cm. If chord AB = 8 cm, find the area of the shaded region. **(3 marks)**



Alternate Solution



$$8^2 = 5^2 + 5^2 - 2(5)^2 \cos\theta \quad \leftarrow \textbf{1 mark}$$

$$14 = -50\cos\theta$$

$$\cos\theta = -\frac{7}{25}$$

$$\theta = 106.2602° \quad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$\therefore \text{ area of sector} = \overbrace{\frac{106.2602°}{360°}}^{\overset{\frac{1}{2}\textbf{ mark}}{\downarrow}} \; \overbrace{\pi(25)}^{\overset{\frac{1}{2}\textbf{ mark}}{\downarrow}}$$

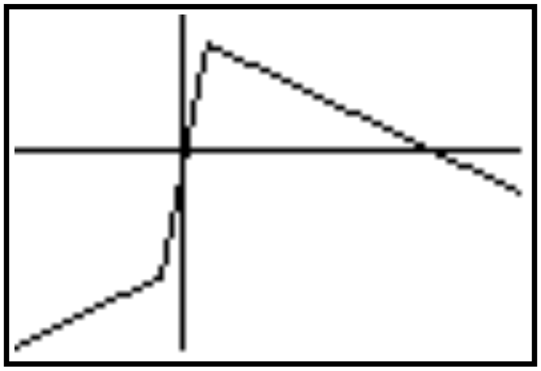
$$= 23.18 \text{ cm}^2 \quad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

7. Solve the following using a graphing calculator. **(3 marks)**

$$|x+4| = 1.2|x-4|$$

Sketch the graph in the viewing window below and indicate appropriate window dimensions. State the function(s) used in your graph. Indicate appropriate window dimensions that will provide enough of the graph so that all intersection points or all zeros are visible.

Solution



$\mathrm{Y}_1 = |x+4| - 1.2|x-4|$ ← $\frac{1}{2}$ **mark** for equation

← $\frac{1}{2}$ **mark** for graph

$x\ [-30,\ 60]$ $y\ [-15,\ 10]$ ← $\frac{1}{2}$ **mark** for window dimensions

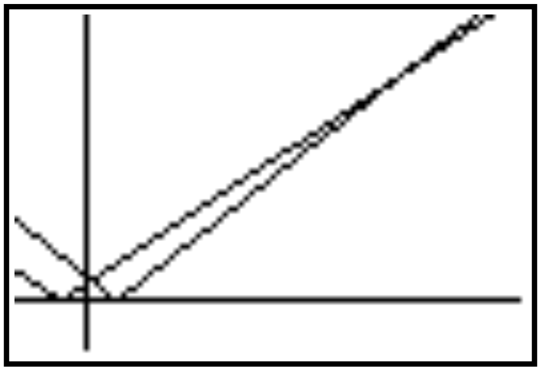
$x = 0.36$, $x = 44.00$ ← **1 $\frac{1}{2}$ marks**

7. Solve the following using a graphing calculator. **(3 marks)**

$$|x+4| = 1.2|x-4|$$

Sketch the graph in the viewing window below and indicate appropriate window dimensions. State the function(s) used in your graph. Indicate appropriate window dimensions that will provide enough of the graph so that all intersection points or all zeros are visible.

Alternate Solution



$$\left.\begin{array}{l} Y_1 = |x+4| \\ Y_2 = 1.2|x-4| \end{array}\right\} \leftarrow \tfrac{1}{2}\ \textbf{mark} \text{ for equations}$$

← $\frac{1}{2}$ **mark** for graph

x $[-10,\ 60]$ y $[-10,\ 60]$ ← $\frac{1}{2}$ **mark** for window dimensions

$x = 0.36,\quad x = 44.00$ ← **1 $\frac{1}{2}$ marks**

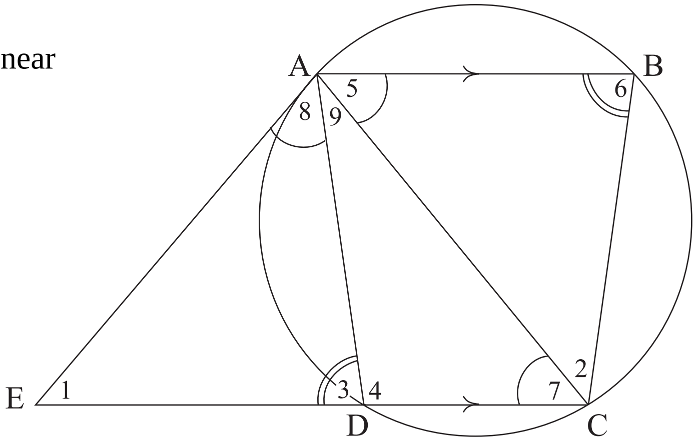
Students should choose one or the other method of proof.

8. Complete the proof. **(4 marks)**

Diagram clarification: E, D, C are collinear

Given: EA is tangent to the circle at A
AB || EC

Prove: ∠1 = ∠2



Solution

Paragraph proof method:

Since EA is tangent, ∠8 = ∠7 by tangent chord theorem (**1 mark**),

and since AB || EC, ∠5 = ∠7 ($\frac{1}{2}$ **mark**) by alternate interior ∠s.

∴ ∠5 = ∠8 by substitution ($\frac{1}{2}$ **mark**).

∠4 and ∠6 are supplementary because they are opposite ∠s in a cyclic quadrilateral ($\frac{1}{2}$ **mark**),

and ∠3 and ∠4 are supplementary ($\frac{1}{2}$ **mark**) because they are ∠s on a line.

∴ ∠3 = ∠6 because they are supplementary to same ∠ ($\frac{1}{2}$ **mark**).

Thus ∠1 = ∠2 by 3rd ∠s of Δs ($\frac{1}{2}$ **mark**).

Note: deduct $\frac{1}{2}$ **mark** if it is not mentioned: tangent / AB || EC

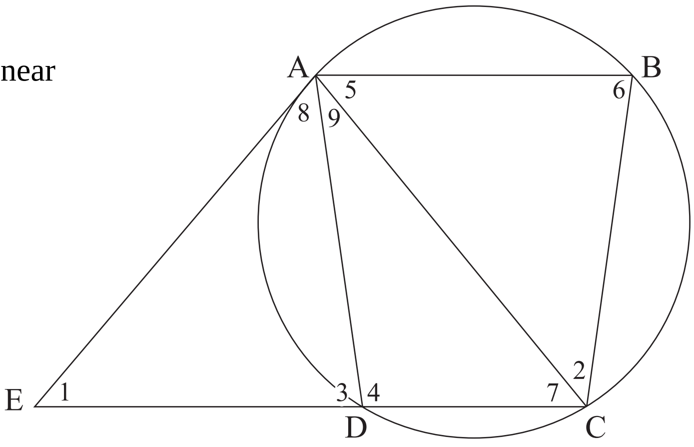
Students should choose one or the other method of proof.

8. Complete the proof. **(4 marks)**

Diagram clarification: E, D, C are collinear

Given: EA is tangent to the circle at A
AB || EC

Prove: ∠1 = ∠2



Solution

Two-column proof method:

Statement	Reason	
EA is tangent to the circle at A	given	
∠7 = ∠8	∠ between tangent and chord	← **1 mark**
AB || EC	given	
∠5 = ∠7	alternate interior ∠s are =	← $\frac{1}{2}$ **mark**
∠5 = ∠8	both = ∠7	← $\frac{1}{2}$ **mark**
$\angle 4 + \angle 6 = 180°$	opposite ∠s in cyclic quadrilateral	← $\frac{1}{2}$ **mark**
$\angle 3 + \angle 4 = 180°$	∠s on a line	← $\frac{1}{2}$ **mark**
∠3 = ∠6	supplementary to same angle	← $\frac{1}{2}$ **mark**
∠1 = ∠2	3rd ∠s of Δs are =	← $\frac{1}{2}$ **mark**

Note: deduct $\frac{1}{2}$ **mark** if givens missing.

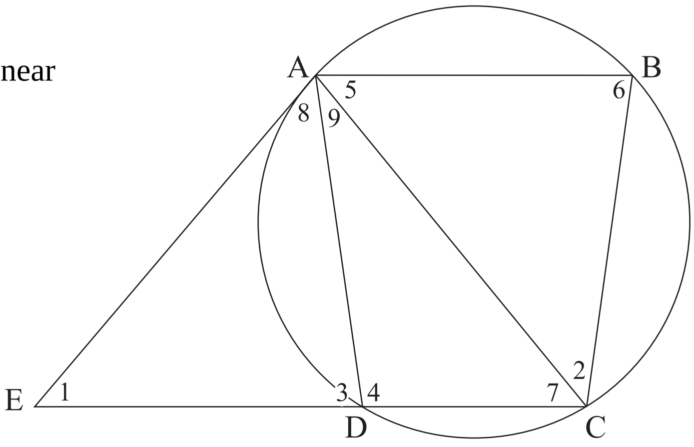
Students should choose one or the other method of proof.

8. Complete the proof. **(4 marks)**

Diagram clarification: E, D, C are collinear

Given: EA is tangent to the circle at A
AB││EC

Prove: ∠1 = ∠2



Alternate Solution

Two-column proof method:

	Reason	Statement	
	EA is tangent to the circle at A	given	
1 mark →	∠EAC = ∠6	∠ between tangent and chord	← **$1\frac{1}{2}$ marks**
cap at ↵ **$1\frac{1}{2}$ marks** for this line if nothing follows it.	AB││EC	given	
	∠7 = ∠5	alternate interior ∠s are =	← **$\frac{1}{2}$ mark**
	∠1 = ∠2	3rd ∠s of Δs are =	← **1 mark**

END OF KEY