Principles of Mathematics 12

April 2000 Provincial Examination

ANSWER KEY / SCORING GUIDE

CURRICULUM:

Organizers		**Sub-Organizers**
1. Problem Solving	A	Problem Set
2. Patterns and Relations	B	Sequences and Series
	C	Polynomials
	D	Logarithms and Exponents
	E	Quadratic Relations
	F	Quadratic Systems
3. Shape and Space	G	Trigonometry
	H	Geometry

Part A: Multiple Choice

Q	K	C	CO	PLO
1.	A	K	2	C4
2.	C	U	2	C5
3.	B	U	2, 1	C6, A7
4.	B	U	2	C4
5.	C	U	2	C3
6.	B	H	2	C1
7.	C	K	2	E5
8.	A	U	2	F5
9.	A	U	2	E4
10.	D	U	2	E2
11.	B	U	2	E6
12.	B	U	2	F1
13.	C	U	2	E7
14.	A	H	2	F1
15.	D	K	2	D5
16.	D	U	2	D5
17.	A	U	2	D2
18.	D	U	2	D5
19.	D	U	2, 1	D5, A7
20.	A	H	2	D1
21.	C	H	2	D5
22.	B	K	2	B6
23.	C	U	2	B4
24.	D	U	2	B4
25.	D	U	2	B2
26.	B	U	2	B4
27.	D	U	2	B2
28.	B	H	2	B4
29.	C	U	3	G1
30.	D	K	3	G5
31.	D	U	3	G5
32.	A	U	3	G5
33.	C	U	3	G9
34.	A	U	3, 1	G3, A7
35.	B	U	3	G2
36.	D	U	3	G8
37.	C	H	3	G7, G8
38.	A	H	3	G7
39.	C	U	3	H2
40.	B	U	3	H2
41.	A	U	3	H4
42.	C	H	3	H3, G10
43.	B	U	1	A3
44.	D	H	1	A3
45.	D	H	1	A3

Multiple Choice = 45 marks

Part B: Written Response

Q	B	C	S	CO	PLO
1.	1	U	3	2	E4
2.	2	U	3	2, 1	C9, A7
3.	3	U	3	3, 1	G6, A7, A1
4.	4	U	3	2	F3
5.	5	U	3	2	D3, D6
6.	6	H	3	3	H2
7.	7	U	3	1	A3
8.	8	H	4	3	H2

Written Response = 25 marks

Multiple Choice = 45 (45 questions)
Written Response = 25 (8 questions)
EXAMINATION TOTAL = 70 marks

LEGEND:
Q = Question Number
K = Keyed Response
C = Cognitive Level
B = Score Box Number
S = Score
CO = Curriculum Organizer
PLO = Prescribed Learning Outcome

PART B: WRITTEN RESPONSE

Value: 25 marks **Suggested Time: 45 minutes**

INSTRUCTIONS:	Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided. If, in a justification, you refer to information produced by the calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem, it is important to sketch the graph, showing its general shape and indicating the appropriate window dimensions. When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution. **Full marks will NOT be given for the final answer only.**

1. A hyperbola has vertices at $(1,\ -4)$ and $(1,\ 8)$. If the asymptotes have slopes ± 2, determine the equation of the hyperbola in standard form. **(3 marks)**

Solution

centre = $(1,\ 2)$

$\frac{1}{2}$ **mark** $\frac{1}{2}$ **mark**

$\downarrow$ $\downarrow$

$$\frac{(x-1)^2}{9} - \frac{(y-2)^2}{36} = -1$$

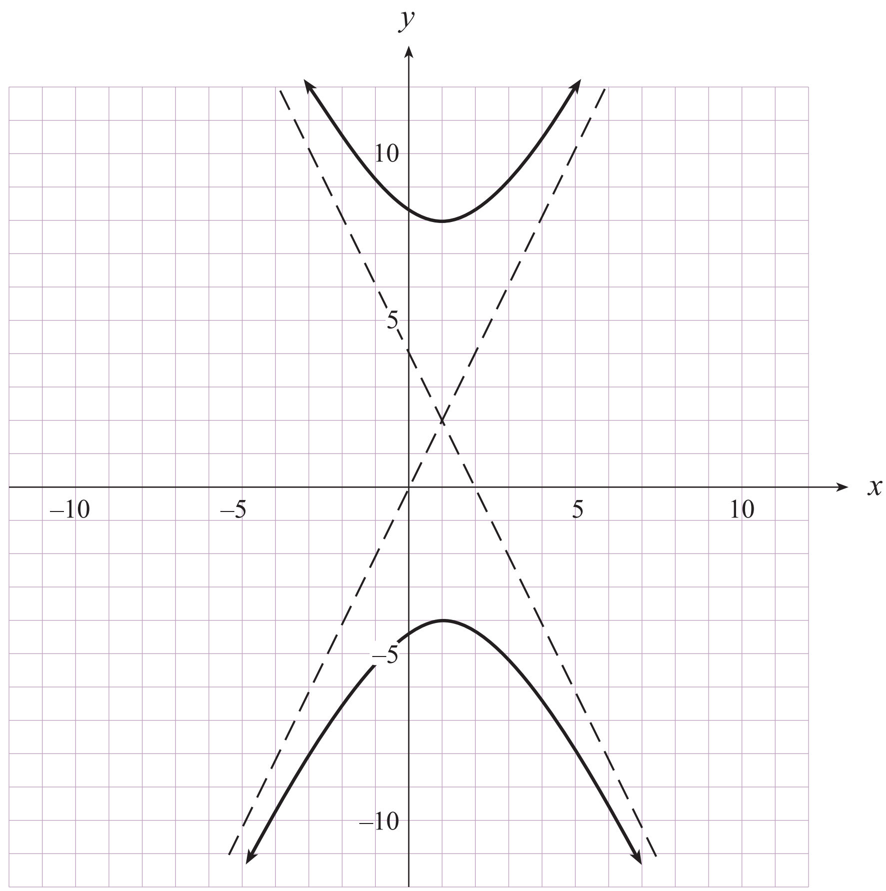
$\uparrow$ $\uparrow$

1 mark $\frac{1}{2}$ **mark**

$\frac{1}{2}$ **mark** for form

or

$$\frac{(y-2)^2}{36} - \frac{(x-1)^2}{9} = 1$$

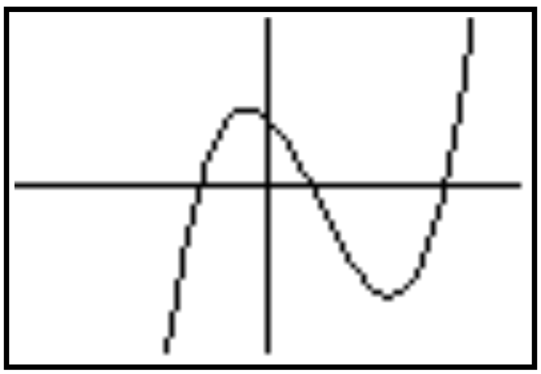


2. Solve the following inequality using a graphing calculator. **(3 marks)**

$$x^3 - 3x^2 - x > 2x - 4$$

Sketch the graph in the viewing window below and indicate appropriate window dimensions. State the function(s) used in your graph. Ensure that the relative maximum and relative minimum points of the function(s) are visible within the viewing window. The solution may be given in algebraic form or shown on a number line.

Solution



$Y_1 = x^3 - 3x^2 - 3x + 4$ ← $\frac{1}{2}$ **mark** for equation

← $\frac{1}{2}$ **mark** for graph

$x\ [-5,\ 5]$ $y\ [-10,\ 10]$ ← $\frac{1}{2}$ **mark** for window dimensions

The zeros of the function Y_1 are: $x = -1.36,\ 0.83,\ 3.53$ ← $\frac{1}{2}$ **mark**

Therefore the solution is:

$-1.36 < x < 0.83$ or $x > 3.53$ ← **algebraic solution**

↑ ↑

$\frac{1}{2}$ **mark** $\frac{1}{2}$ **mark**

OR

↓ ↓

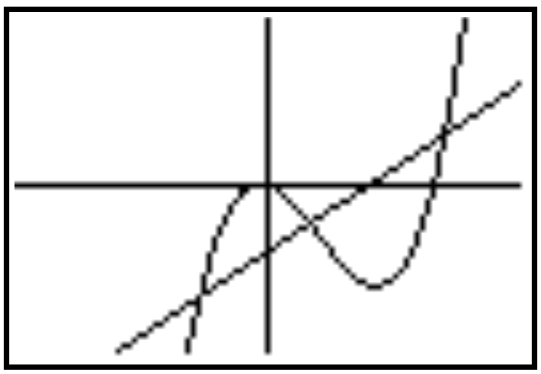
–1.36 0.83 3.53 ← **number line solution**

2. Solve the following inequality using a graphing calculator. **(3 marks)**

$$x^3 - 3x^2 - x > 2x - 4$$

Sketch the graph in the viewing window below and indicate appropriate window dimensions. State the function(s) used in your graph. Ensure that the relative maximum and relative minimum points of the function(s) are visible within the viewing window. The solution may be given in algebraic form or shown on a number line.

Alternate Solution



$Y_1 = x^3 - 3x^2 - x$
$Y_2 = 2x - 4$ ← $\frac{1}{2}$ **mark** for equations

← $\frac{1}{2}$ **mark** for graph

$x\ [-5,\ 5]$ $y\ [-10,\ 10]$ ← $\frac{1}{2}$ **mark** for window dimensions

The graphs intersect when $x = -1.36,\ 0.83,\ 3.53$ ← $\frac{1}{2}$ **mark**

Therefore the solution is:

$-1.36 < x < 0.83$ or $x > 3.53$ ← **algebraic solution**

↑ ↑

$\frac{1}{2}$ **mark** $\frac{1}{2}$ **mark**

OR

↓ ↓

← **number line solution**

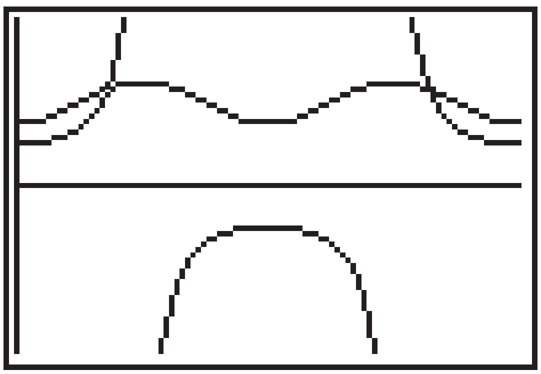
3. Solve the following system using a graphing calculator, when $0 \le x < 2\pi$. **(3 marks)**

$$y = \sec x$$

$$y = \sin^2 x + 1.5$$

Sketch the graph in the viewing window below. State the function(s) that you entered to obtain your graph and your solution. Indicate the dimensions of the viewing window that will show enough of the graph so that all intersection points are visible.

Solution



$x\ [0,\ 2\pi]$ $y\ [-4,\ 4]$

$Y_1 = (\sin(x))^2 + 1.5$

$Y_2 = \dfrac{1}{\cos(x)}$

← ½ **mark** for equations

← ½ **mark** for graph

← ½ **mark** for window dimensions

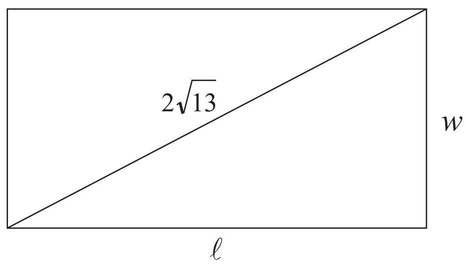
Note: ½ mark not given unless there was a graph drawn.

Therefore the solution to the system is:

$(1.12,\ 2.31)$ $(5.16,\ 2.31)$ ← **1½ marks**

4. Determine the dimensions of a rectangle that has an area of 10 cm^2 and a diagonal of length $2\sqrt{13}$ cm. (Solve algebraically.) **(3 marks)**

Solution



$$\ell w = 10 \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$\ell^2 + w^2 = \left(2\sqrt{13}\right)^2 \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$w = \frac{10}{\ell}$$

$$\ell^2 + \left(\frac{10}{\ell}\right)^2 = 52 \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$\ell^2 + \frac{100}{\ell^2} = 52$$

$$\ell^4 + 100 = 52\ell^2$$

$$\ell^4 - 52\ell^2 + 100 = 0 \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$\left(\ell^2 - 2\right)\left(\ell^2 - 50\right) = 0$$

$$\ell = \sqrt{2} \qquad \ell = \sqrt{50}$$

$$\downarrow \qquad \downarrow$$

$$w = \sqrt{50} \qquad w = \sqrt{2}$$

$\therefore$ the dimensions of the rectangle are:

$$\sqrt{2}\text{ cm} \times \sqrt{50}\text{ cm} \qquad \leftarrow \textbf{1 mark}$$

$$\uparrow$$

or $5\sqrt{2}$

or 1.41 cm × 7.07 cm

5. The population of a culture of bacteria doubles every 4 hours. If the present population is 5 000 bacteria, how long will it take for the population to reach 70 000 bacteria? (Answer in hours, accurate to two decimal places.) **(3 marks)**

Solution

$$N = Ca^{\frac{t}{n}}$$

$\frac{1}{2}$ **mark each** (for 5 000 and 2)

$$N = 5\,000\left(2^{\frac{t}{4}}\right) \quad \leftarrow \tfrac{1}{2}\ \textbf{mark}\ (\text{for exponent})$$

$$70\,000 = 5\,000\left(2^{\frac{t}{4}}\right) \quad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$\frac{70\,000}{5\,000} = 2^{\frac{t}{4}}$$

$$14 = 2^{\frac{t}{4}}$$

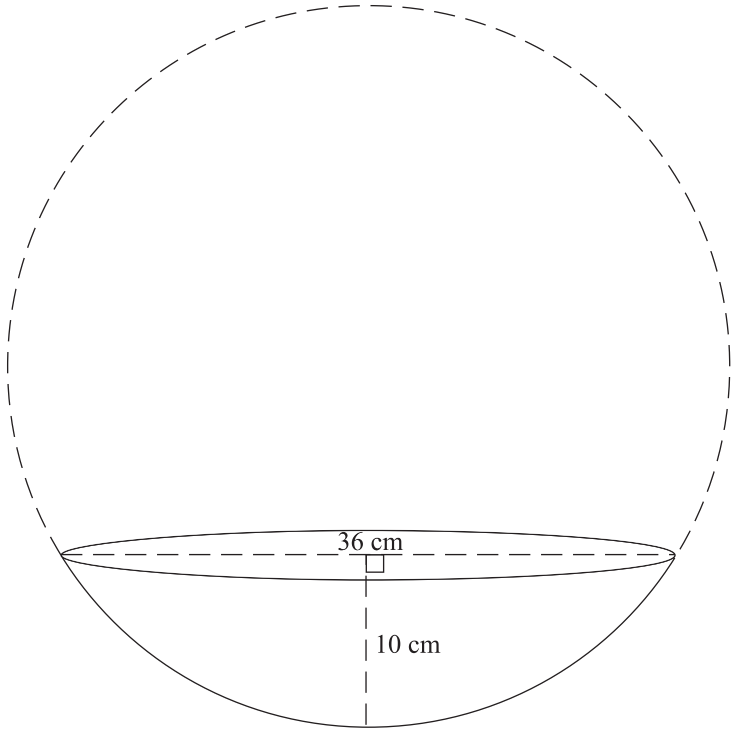
$$\log 14 = \frac{t}{4}\log 2$$

$$\frac{4\log 14}{\log 2} = t$$

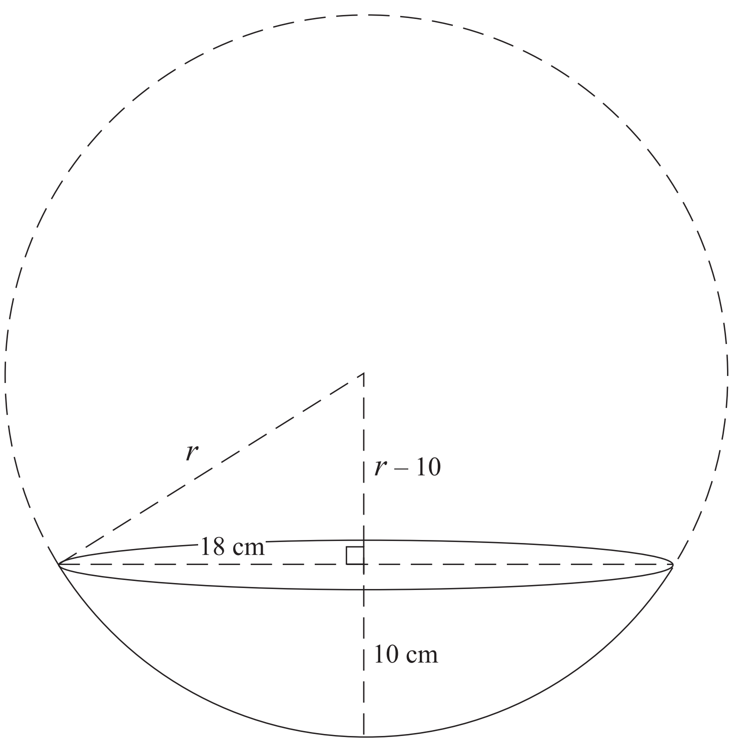
$$15.23 = t \quad \leftarrow \textbf{1 mark}$$

$\therefore$ it will take 15.23 hours

6. A hollow spherical ball has been sliced to form a bowl that measures 36 cm across the centre of the opening as shown in the diagram. If the bowl is 10 cm deep, determine the original radius of the spherical ball. **(3 marks)**



Solution



Drawing in radius

Labelling 18 and $r-10$

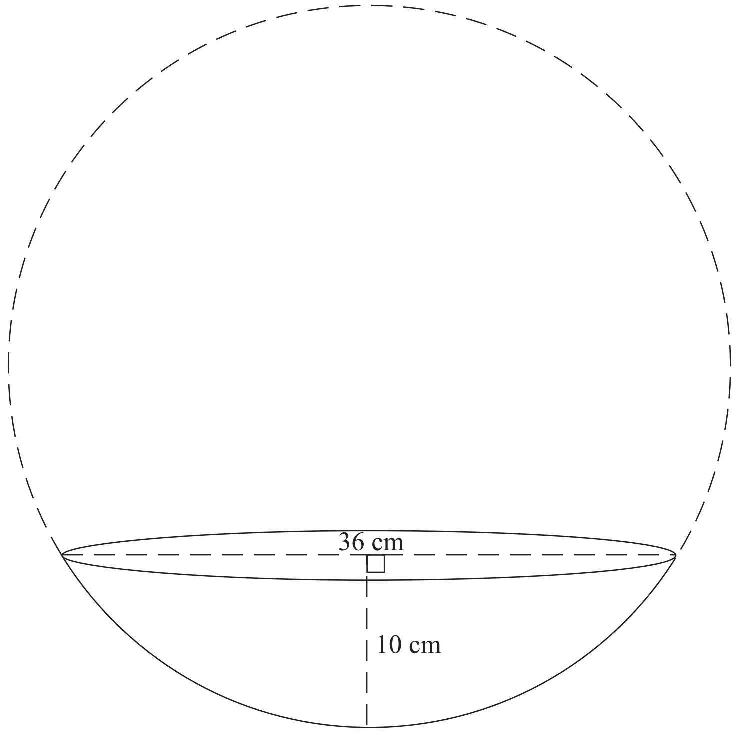
18 ← **½ mark**; $r-10$ ← **1 mark**

$$r^2 = (r-10)^2 + 18^2 \quad \leftarrow \tfrac{1}{2} \text{ mark}$$

$$\left.\begin{aligned} r^2 &= r^2 - 20r + 100 + 324 \\ 20r &= 424 \\ r &= 21.2\,\text{cm} \end{aligned}\right\} \leftarrow 1 \text{ mark}$$

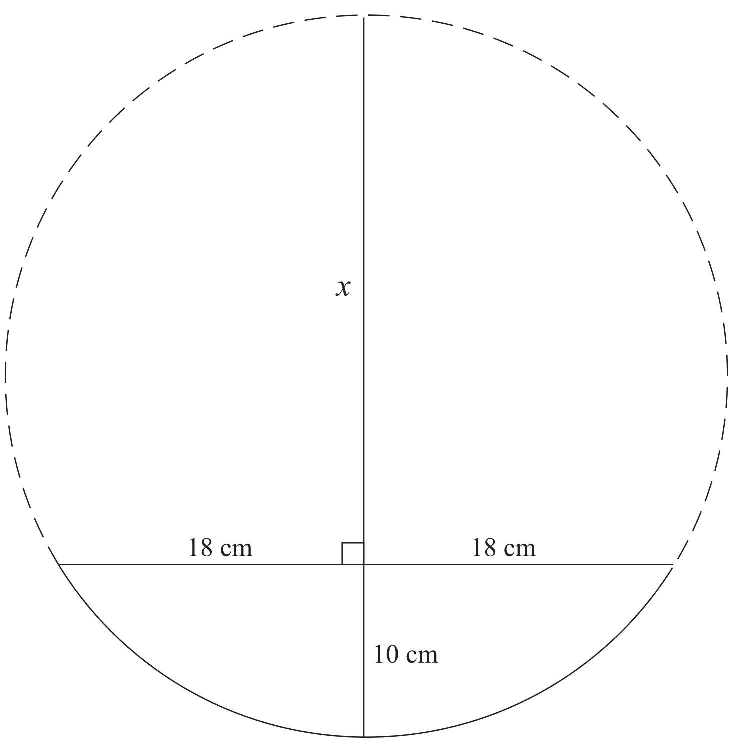
(May not see work for solving equation as graphing calculators may be used.)

6. A hollow spherical ball has been sliced to form a bowl that measures 36 cm across the centre of the opening as shown in the diagram. If the bowl is 10 cm deep, determine the original radius of the spherical ball. **(3 marks)**



Alternate Solution

Intersection chord theorem



$10x = 18^2$ ← **1 mark**

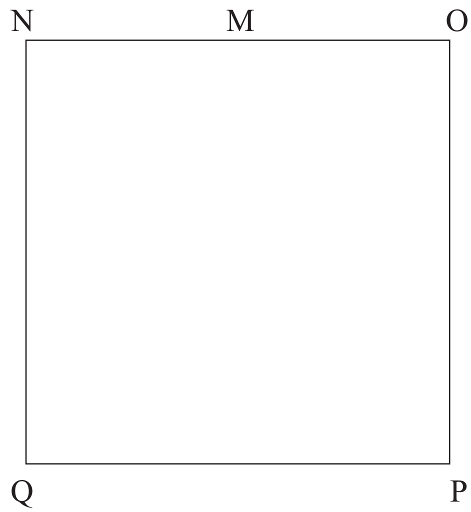
$x = 32.4$ ← **$\frac{1}{2}$ mark**

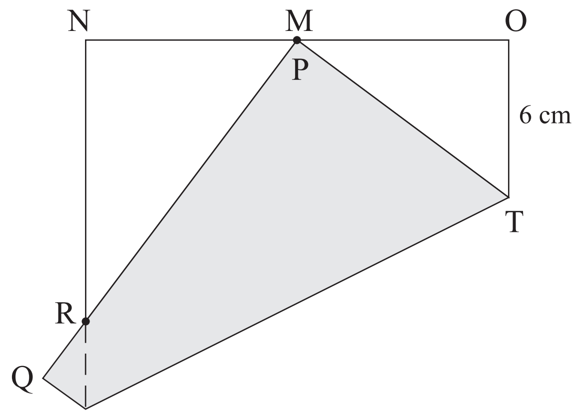
$\therefore\ d = 32.4 + 10$

$d = 42.4$ ← **1 mark**

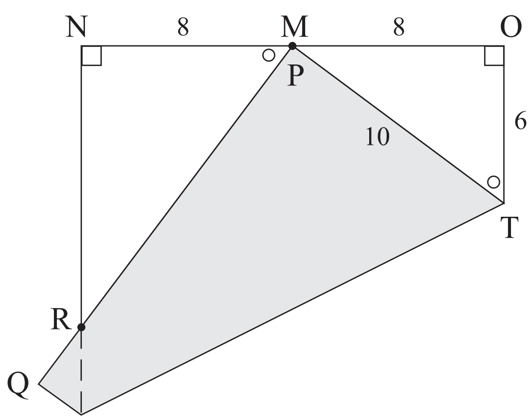
$r = 21.2$ cm ← **$\frac{1}{2}$ mark**

7. A square piece of paper 16 cm × 16 cm is folded in such a way that the lower right hand corner at P just touches the midpoint M of the top side, as shown in the diagram. If OT = 6 cm, determine the length of side QR. **(3 marks)**





Solution



By Pythagoras

$$MT = 10 \text{ cm}$$ ← **½ mark**

since $\Delta MOT \sim \Delta RNM$

$$\frac{6}{10} = \frac{8}{MR}$$ ← **1 mark**

$$6MR = 80$$

$$MR = \frac{40}{3}$$ ← **1 mark**

$$\therefore \; QR = 16 - \frac{40}{3} = \frac{8}{3} \text{ cm}$$ ← **½ mark**

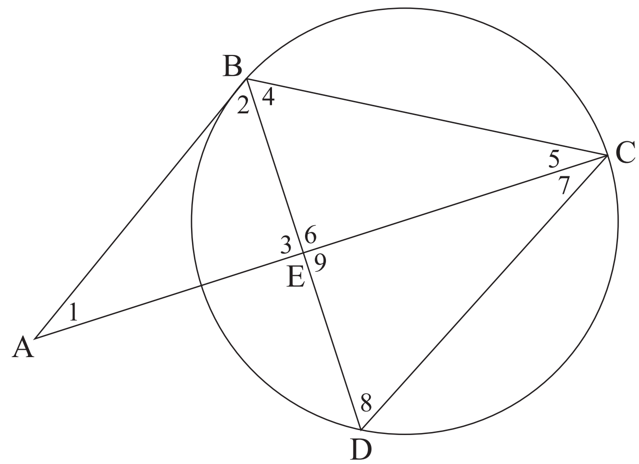
Students should choose one or the other method of proof.

8. Complete the proof. **(4 marks)**

Diagram clarification: A, E, C are collinear

Given: AB is a tangent
BD = CD
AB = BC

Prove: $\angle 3 = \angle 6$



Solution

Paragraph proof method:

Since AB is a tangent, then $\angle 2 = \angle BCD$ $\left(\frac{1}{2}\ \textbf{mark}\right)$ by tangent chord theorem $\left(\frac{1}{2}\ \textbf{mark}\right)$. Since BD = CD, then $\angle 4 = \angle BCD$ $\left(\frac{1}{2}\ \textbf{mark}\right)$ since $\angle$s opposite = sides are = $\left(\frac{1}{2}\ \textbf{mark}\right)$. Therefore, $\angle 2 = \angle 4$ by substitution $\left(\frac{1}{2}\ \textbf{mark}\right)$. Since AB = BC, then $\angle 1 = \angle 5$ $\left(\frac{1}{2}\ \textbf{mark}\right)$ since $\angle$s opposite = sides are = . Thus $\angle 3 = \angle 6$ by 3rd $\angle$s of Δs are equal $(\textbf{1 mark})$.

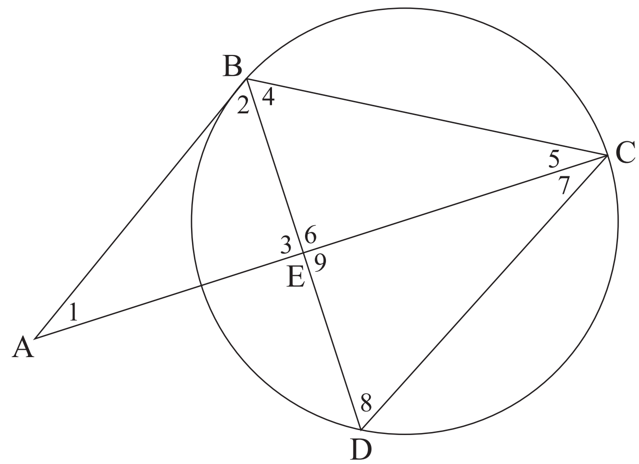
Students should choose one or the other method of proof.

8. Complete the proof. **(4 marks)**

Diagram clarification: A, E, C are collinear

Given: AB is a tangent
BD = CD
AB = BC

Prove: ∠3 = ∠6



Solution

Two-column proof method:

	STATEMENT	REASON	
	AB is a tangent	given	
½ mark →	∠2 = ∠BCD	∠ between tangent and chord	**← ½ mark**
	BD = CD	given	
½ mark →	∠4 = ∠BCD	∠s opposite = sides are =	
½ mark →	∠2 = ∠4	both = ∠BCD (substitution)	**← ½ mark**
	AB = BC	given	
½ mark →	∠1 = ∠5	∠s opposite = sides are =	
	∠3 = ∠6	3rd ∠s of Δs are =	**← 1 mark**

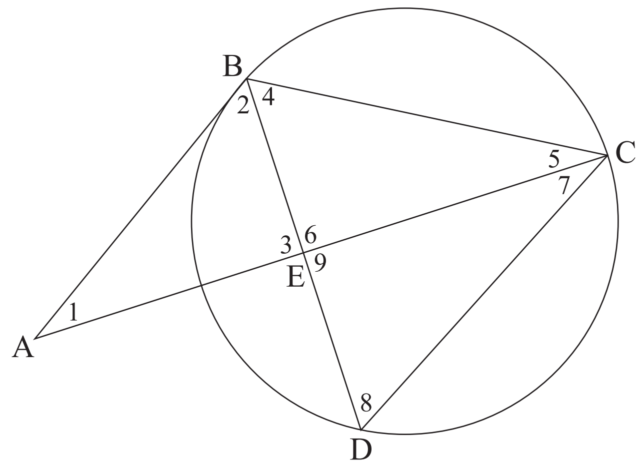
Students should choose one or the other method of proof.

8. Complete the proof. **(4 marks)**

Diagram clarification: A, E, C are collinear

Given: AB is a tangent
BD = CD
AB = BC

Prove: ∠3 = ∠6



Alternate Solution

Two-column proof method:

	STATEMENT	REASON	
	AB is a tangent	given	
$\frac{1}{2}$ mark →	∠2 = ∠BCD	∠ between tangent and chord	← **$\frac{1}{2}$ mark**
	BD = CD	given	
$\frac{1}{2}$ mark →	∠4 = ∠BCD	∠s opposite = sides are =	
$\frac{1}{2}$ mark →	∠2 = ∠4	both = ∠BCD (substitution)	← **$\frac{1}{2}$ mark**
	AB = BC	given	
$\frac{1}{2}$ mark →	∠1 = ∠5	∠s opposite = sides are =	
$\frac{1}{2}$ mark →	ΔABE ≅ ΔCBE	ASA	
	∠3 = ∠6	CPCTC	← **$\frac{1}{2}$ mark**

Note:

ΔABE ≅ ΔCBE can be proved using SAS with common side BE instead of ∠1 = ∠5 step.

END OF KEY