

APRIL 1999

PROVINCIAL EXAMINATION

MINISTRY OF EDUCATION

PRINCIPLES OF MATHEMATICS 12

GENERAL INSTRUCTIONS

1. Insert the stickers with your Student I.D. Number (PEN) in the allotted spaces above and on the **back** cover of this booklet. **Under no circumstance is your name or identification, other than your Student I.D. Number, to appear on this booklet.**
2. Ensure that in addition to this examination booklet, you have an **Examination Response Form**. Follow the directions on the front of the Response Form.
3. **Disqualification** from the examination will result if you bring books, paper, notes or unauthorized electronic devices into the examination room.
4. All multiple-choice answers must be entered on the Response Form using an **HB pencil**. Multiple-choice answers entered in this examination booklet will **not** be marked.
5. For each of the written-response questions, write your answer in the space provided in this booklet.
6. When instructed to open this booklet, **check the numbering of the pages** to ensure that they are numbered in sequence from page one to the last page, which is identified by

END OF EXAMINATION.

7. At the end of the examination, place your Response Form inside the front cover of this booklet and return the booklet and your Response Form to the supervisor.

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PRINCIPLES OF MATHEMATICS 12 PROVINCIAL EXAMINATION

- | | Value | Suggested Time |
|---|------------------------|--------------------|
| 1. This examination consists of two parts: | | |
| PART A: 45 multiple-choice questions | 45 | 75 |
| PART B: 8 written-response questions | 25 | 45 |
| | Total: 70 marks | 120 minutes |
- Aside from an approved calculator, electronic devices including dictionaries and pagers are **not** permitted in the examination room.
 - The last **three** pages inside the back cover contain **A Summary of Basic Identities and Formulae, Rough Work for Graphing, and Rough Work for Multiple-Choice**. These pages may be detached for convenient reference prior to writing this examination.
 - You will not be provided with any additional paper since rough-work space for the written-response questions has been incorporated into the space allowed for answering each question. You may not need all of the space provided to answer each question.
 - A graphing calculator is essential for the Principles of Mathematics 12 Provincial Examination.** The calculator must be a hand-held device designed primarily for mathematical computations involving logarithmic and trigonometric functions as well as for graphing functions. Computers, calculators with a QWERTY keyboard, and electronic writing pads will not be allowed. Students must not bring any external devices to support calculators such as manuals, printed or electronic cards, printers, memory expansion chips or cards, or external keyboards. Students may have more than one calculator available during the examination of which one may be a scientific calculator. Calculators may not be shared and must not have the ability to either transmit or receive electronic signals. In addition to an approved calculator, students will be allowed to use rulers, compasses, and protractors during the examination.
 - If, in a justification, you refer to information produced by the calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem, it is important to sketch the graph, showing its general shape and indicating the appropriate window dimensions.
 - When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.
 - This examination is designed to be completed in **two hours**. Students may, however, take up to **20 minutes** of additional time to finish.

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PART A: MULTIPLE CHOICE

Value: 45 marks

Suggested Time: 75 minutes

INSTRUCTIONS: For each question, select the **best** answer and record your choice on the Response Form provided. Using an HB pencil, completely fill in the circle that has the letter corresponding to your answer.

1. Determine the number of real roots of the equation $4x(x - 5)(x^2 + 9) = 0$.

- A. 1
- B. 2
- C. 3
- D. 4

2. Determine the quotient of the following division:

$$x^2 - 3x + 3 \overline{) x^3 - 4x^2 + 6x - 3}$$

- A. x
- B. $x - 7$
- C. $x - 1$
- D. $x + 1$

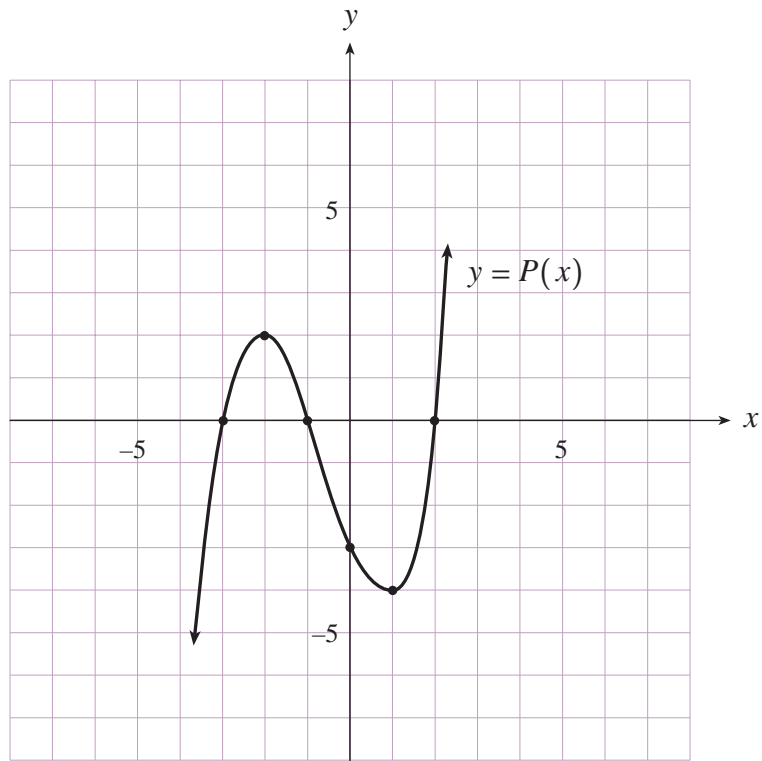
3. Solve: $x^3 - 6x^2 + 3x + 5 = 0$

- A. -0.53, 1.06, 5.23
- B. -0.64, 1.49, 5.32
- C. -0.67, 1.42, 5.25
- D. -0.85, 1.28, 5.11

4. When a polynomial $P(x)$ is divided by $x + 4$, the remainder is 5. Which point must be on the graph of the function $y = P(x)$?

- A. $(-4, 5)$
- B. $(5, -4)$
- C. $(-4, -5)$
- D. $(-5, -4)$

5. The graph of a polynomial function $y = P(x)$ is shown below. If $N(x) = 2P(x-1) + 1$, determine $N(1)$.

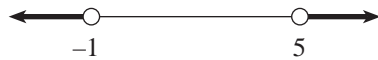


- A. -7
B. -6
C. -5
D. 1
6. Which conic is described by the equation $4x^2 - 5y^2 + 20 = 0$?
- A. circle
B. ellipse
C. parabola
D. hyperbola

7. Determine the midpoint of the line segment joining the points $(2, 9)$ and $(-4, 1)$.

- A. $(-1, 4)$
- B. $(-1, 5)$
- C. $(3, 4)$
- D. $(3, 5)$

8. Which absolute value inequality describes the solution shown?



- A. $|x - 2| < 3$
- B. $|x - 2| > 3$
- C. $|x - 3| < 2$
- D. $|x - 3| > 2$

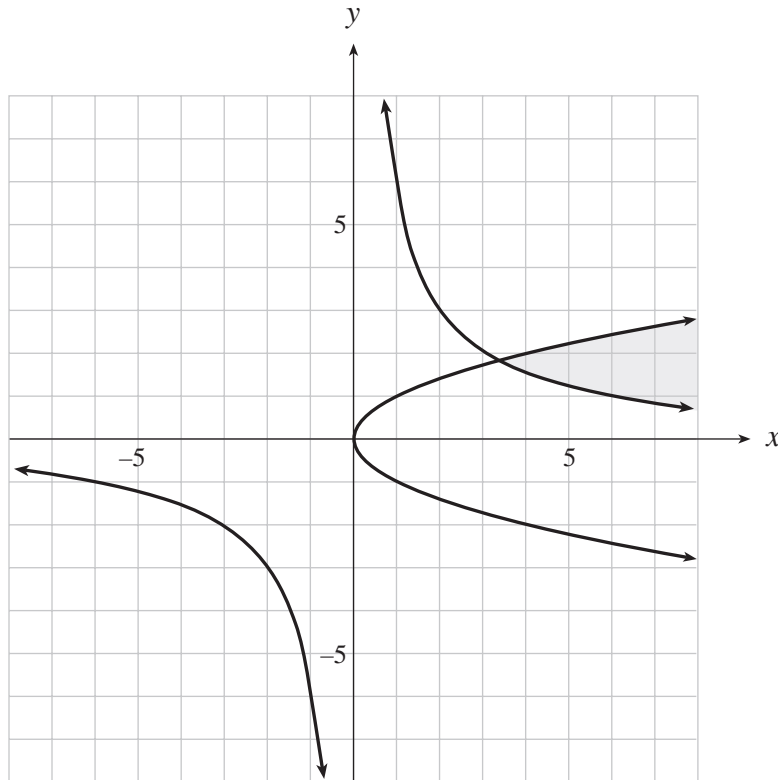
9. Determine the slopes of the asymptotes of $\frac{(x-1)^2}{4} - \frac{y^2}{16} = 1$.

- A. $\pm \frac{1}{4}$
- B. $\pm \frac{1}{2}$
- C. ± 2
- D. ± 4

10. Change $3y^2 + 6y - x - 3 = 0$ to standard form.

- A. $x = 3(y-1)^2$
- B. $x = 3(y+1)^2 - 4$
- C. $x = 3(y+1)^2 - 6$
- D. $x = 3(y+1)^2 - 9$

11. Which system of inequalities represents the shaded region?



- | | |
|----------------|----------------|
| A. $xy \leq 6$ | B. $xy \geq 6$ |
| $x \leq y^2$ | $x \leq y^2$ |
| C. $xy \leq 6$ | D. $xy \geq 6$ |
| $x \geq y^2$ | $x \geq y^2$ |

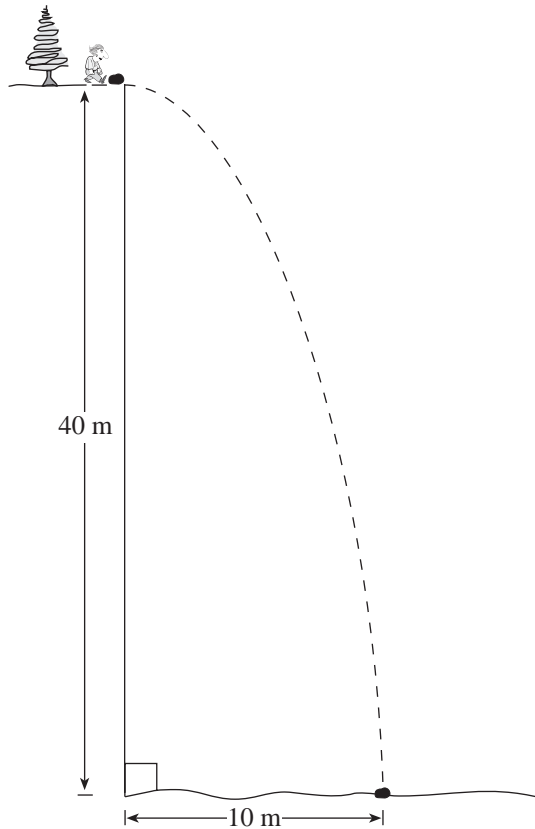
12. Determine the number of real solutions for the following system:

$$y = x^2 - 3x + 2$$

$$y = 2.1x - 4.7$$

- A. 0
- B. 1
- C. 2
- D. 3

13. A rock is kicked off a vertical cliff and falls in a parabolic path to the water below. The cliff is 40 m high and the rock hits the water 10 m from the base of the cliff. What is the horizontal distance of the rock from the cliff face when the rock is at a height of 30 m above the water?



- A. 5.00
- B. 6.32
- C. 7.07
- D. 8.66

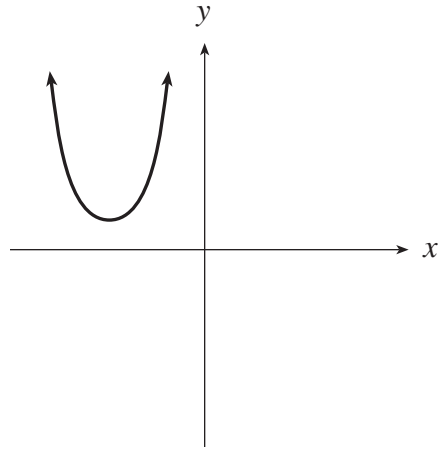
14. Evaluate: $\log_6 12$

- A. 0.18
- B. 0.30
- C. 1.08
- D. 1.39

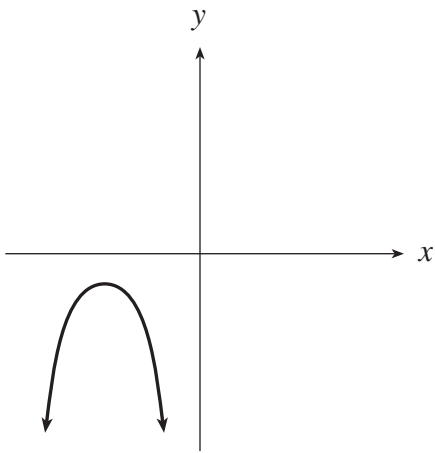
15. Express $2 \log a - \log 3b$ as a single logarithm.

- A. $\log\left(\frac{a^2}{3b}\right)$
- B. $\log\left(\frac{a^2}{b^3}\right)$
- C. $2 \log\left(\frac{a}{3b}\right)$
- D. $\log(a^2 - 3b)$

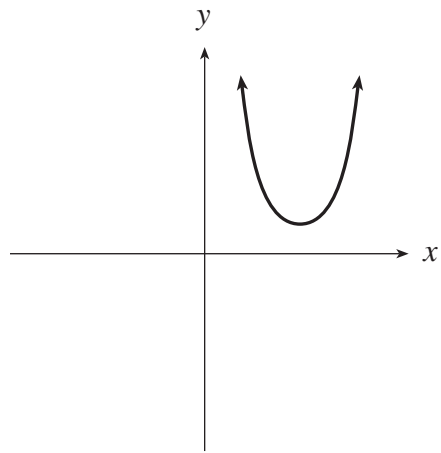
16. Which graph best represents the inverse relation of the graph shown below?



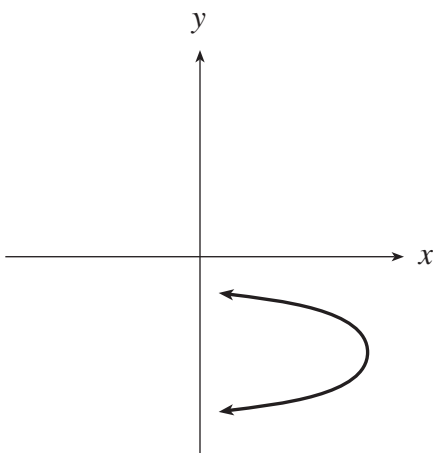
A.



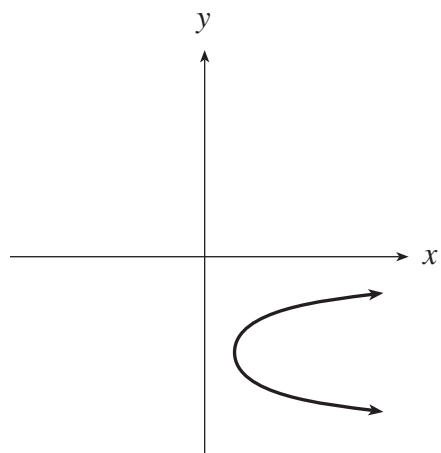
B.



C.



D.



17. Determine an equation of the asymptote of the graph of $y = 5^{x+2} - 3$.
- A. $y = -3$
 - B. $y = 3$
 - C. $x = -2$
 - D. $x = 2$
18. The population of a town grows at a constant annual rate of 1.3%. In how many years will the present population of 1 250 grow to 10 000? (Answer accurate to the nearest year.)
- A. 8
 - B. 17
 - C. 161
 - D. 214
19. Solve: $\log_x 6 - \log_x(x+1) = 1$
- A. 2
 - B. 3
 - C. 5
 - D. -3, 2
20. Solve for x : $\log_{\frac{1}{2}} \frac{1}{9} = \log_2 x^2$
- A. ± 3
 - B. $\pm \frac{1}{3}$
 - C. $\frac{1}{3}$
 - D. 3

21. If $t_n = \frac{6n+2}{5n+4}$, find t_5 .

A. $\frac{13}{14}$

B. $\frac{31}{27}$

C. $\frac{32}{29}$

D. $\frac{67}{59}$

22. Determine t_{35} for the arithmetic sequence 16, 12, 8, ...

A. -116

B. -120

C. -124

D. -128

23. Determine the arithmetic mean between 4 and 16.

A. 8

B. 9

C. 10

D. 12

24. What is the 3rd term of the sequence given by the following recursive definition?

$$t_1 = 3$$

$$t_n = 3t_{n-1} + n, \quad n > 1$$

A. 33

B. 35

C. 36

D. 39

25. The sum of an infinite geometric series is 15. If the 1st term is 6, find the common ratio r .

A. $-\frac{3}{5}$

B. $-\frac{2}{5}$

C. $\frac{2}{5}$

D. $\frac{3}{5}$

26. In a geometric sequence, $t_3 = 256$ and $t_8 = 781.25$. Find the common ratio r .

A. 1.20

B. 1.25

C. 1.32

D. 3.50

27. An arithmetic series has a first term of a and a common difference of d . If the sum of the first 8 terms is 20, which of the following must be true?

A. $a + d = 5$

B. $a + 7d = 20$

C. $2a + 7d = 10$

D. $2a + 7d = 5$

28. Evaluate: $\sum_{k=2}^5 \log_k k^k$

A. 8

B. 14

C. 28

D. 54

29. Convert 215° to radians.

- A. 1.19
- B. 3.58
- C. 3.75
- D. 7.51

30. Determine the amplitude of the graph of $y = -3 \cos 4x$.

- A. -3
- B. 3
- C. 4
- D. $\frac{\pi}{2}$

31. Evaluate: $\csc \frac{\pi}{5}$

- A. 0.70
- B. 1.00
- C. 1.24
- D. 1.70

32. Solve: $\tan x = -2.3$, $0 \leq x < 2\pi$

- A. 1.12, 4.26
- B. 1.16, 1.98
- C. 1.16, 4.30
- D. 1.98, 5.12

33. The point $(-2, 3)$ lies on the terminal arm of position angle θ . Find $\cot \theta$.

- A. $-\frac{2}{\sqrt{13}}$
- B. $-\frac{2}{3}$
- C. $-\frac{3}{2}$
- D. $\frac{3}{\sqrt{13}}$

34. If the period of the function $y = \sin Bx$ is $\frac{3\pi}{5}$, find B .

A. $\frac{2}{15}$

B. $\frac{3}{10}$

C. $\frac{10}{3}$

D. $\frac{15}{2}$

35. Simplify: $\frac{\cos 2\theta - 1}{2 \sin \theta}$

A. $-\sin \theta$

B. $\sin \theta$

C. $\cot \theta - 1$

D. $\frac{\cos^2 \theta}{\sin \theta}$

36. Simplify: $\frac{\cot \theta}{\sec \theta} + \sin \theta$

A. 1

B. $\csc \theta$

C. $\cos^2 \theta$

D. $2 \sin \theta$

37. Express $\sin 4x \cos 4x$ as a single trigonometric function.

A. $\frac{1}{2} \sin 8x$

B. $\frac{1}{2} \sin 2x$

C. $2 \sin 8x$

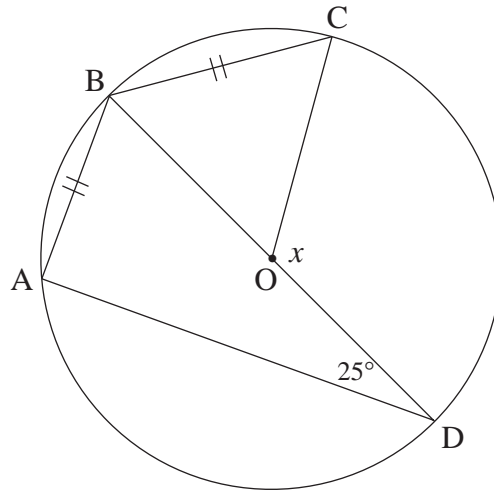
D. $2 \sin 2x$

38. Solve: $\sin x + 1 = \sec x$, $0 \leq x < 2\pi$

- A. 0, 1.00
- B. 0, 1.84
- C. 1.00, 1.84
- D. 0, 1.00, 1.57, 4.71

For questions 39 to 41, diagrams are not drawn to scale.

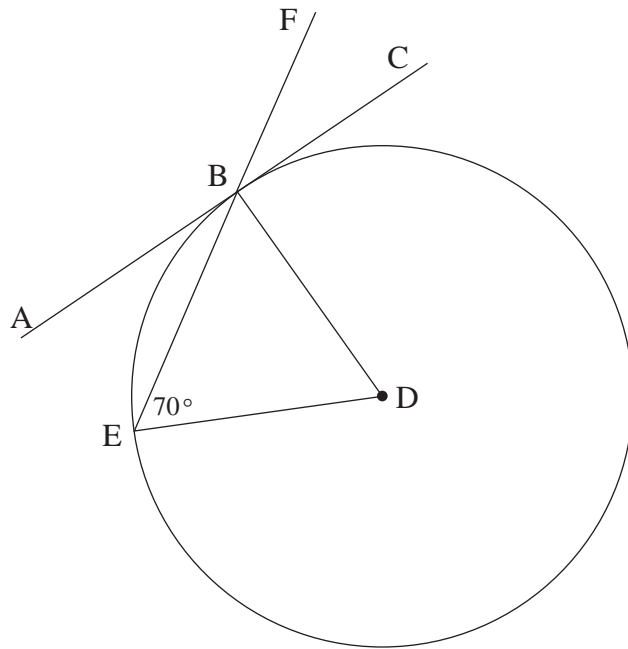
39. In the diagram below, O is the centre of the circle, BD is a diameter, and $AB = BC$. Determine the measure of $\angle x$.



- A. 120°
- B. 130°
- C. 145°
- D. 155°

Use the following diagram to answer question 40.

Given: Circle with centre D
AC is tangent to the circle at B
E, B, F are collinear

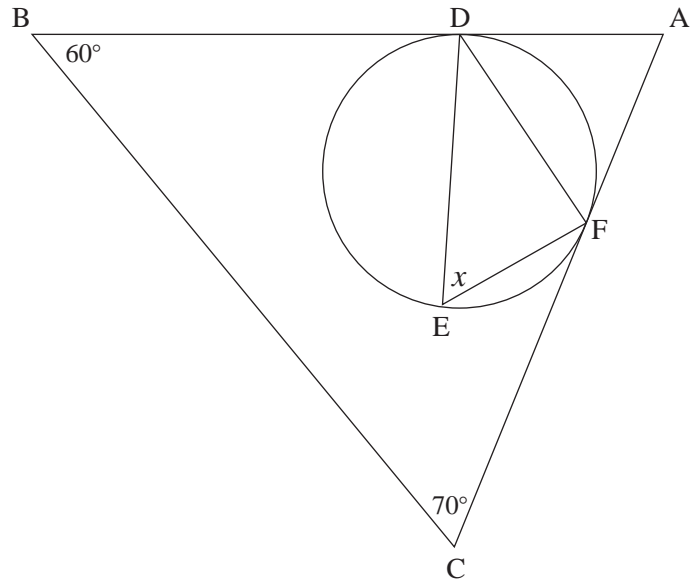


40. Determine the measure of $\angle CBF$.

- A. 20°
- B. 25°
- C. 35°
- D. 40°

Use the following diagram to answer question 41.

Given: $\triangle DEF$ is inscribed in the circle
BA is tangent to the circle at D
AC is tangent to the circle at F

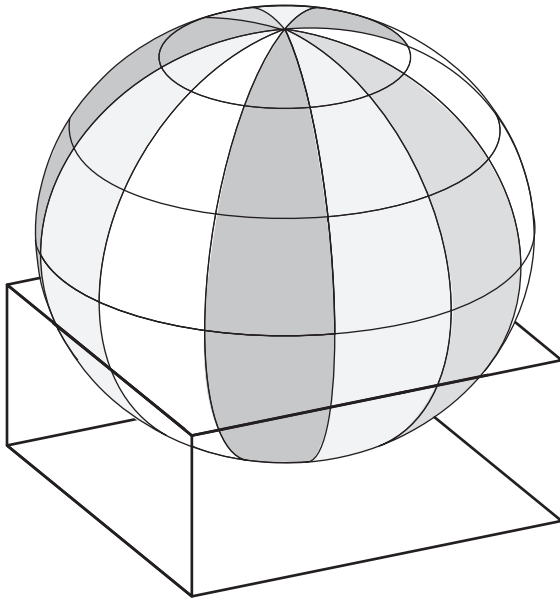


41. Find the measure of $\angle x$.

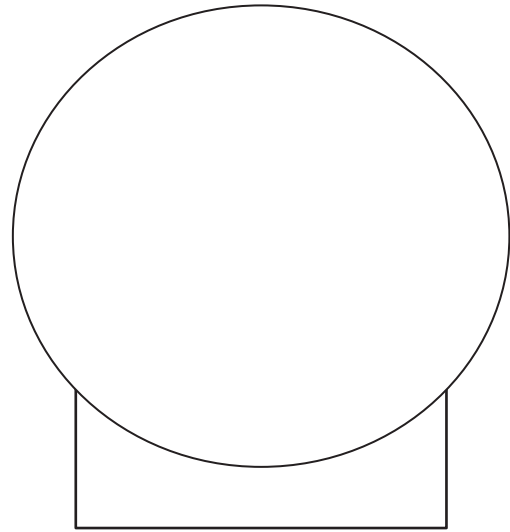
- A. 50°
- B. 55°
- C. 60°
- D. 65°

42. A ball with diameter 20 cm rests on an open top glass box such that part of the ball lies within the box, as shown in the two diagrams below. The bottom of the box is a $12\text{ cm} \times 12\text{ cm}$ square and the height of the box is 5 cm. How far is the bottom of the ball above the bottom of the box?

Three-dimensional view



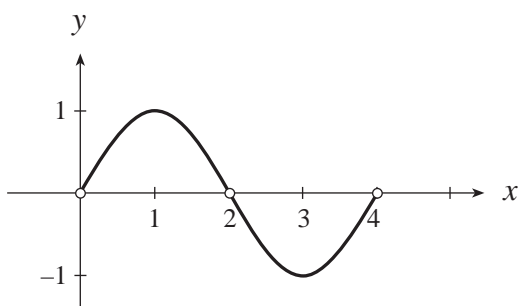
Cross-sectional view



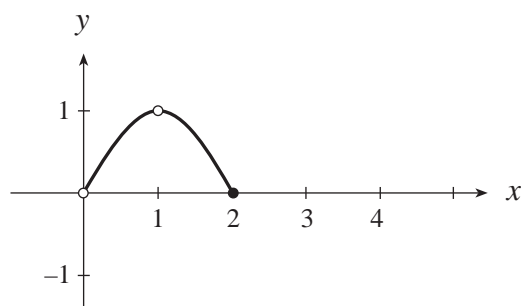
- A. 1 cm
- B. 2 cm
- C. 3 cm
- D. 4 cm

43. Identify the graph of $\log_x y = \log_x \sin \frac{\pi}{2} x$, $0 < x < 4$.

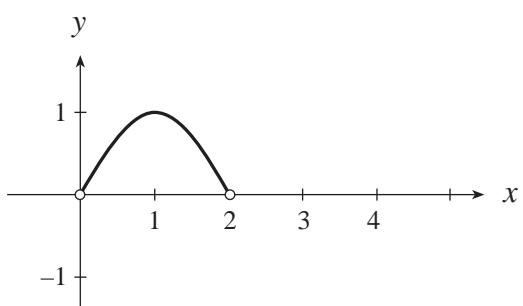
A.



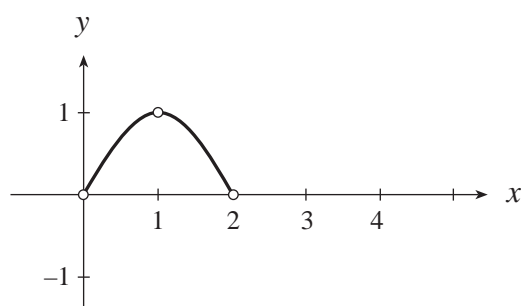
B.



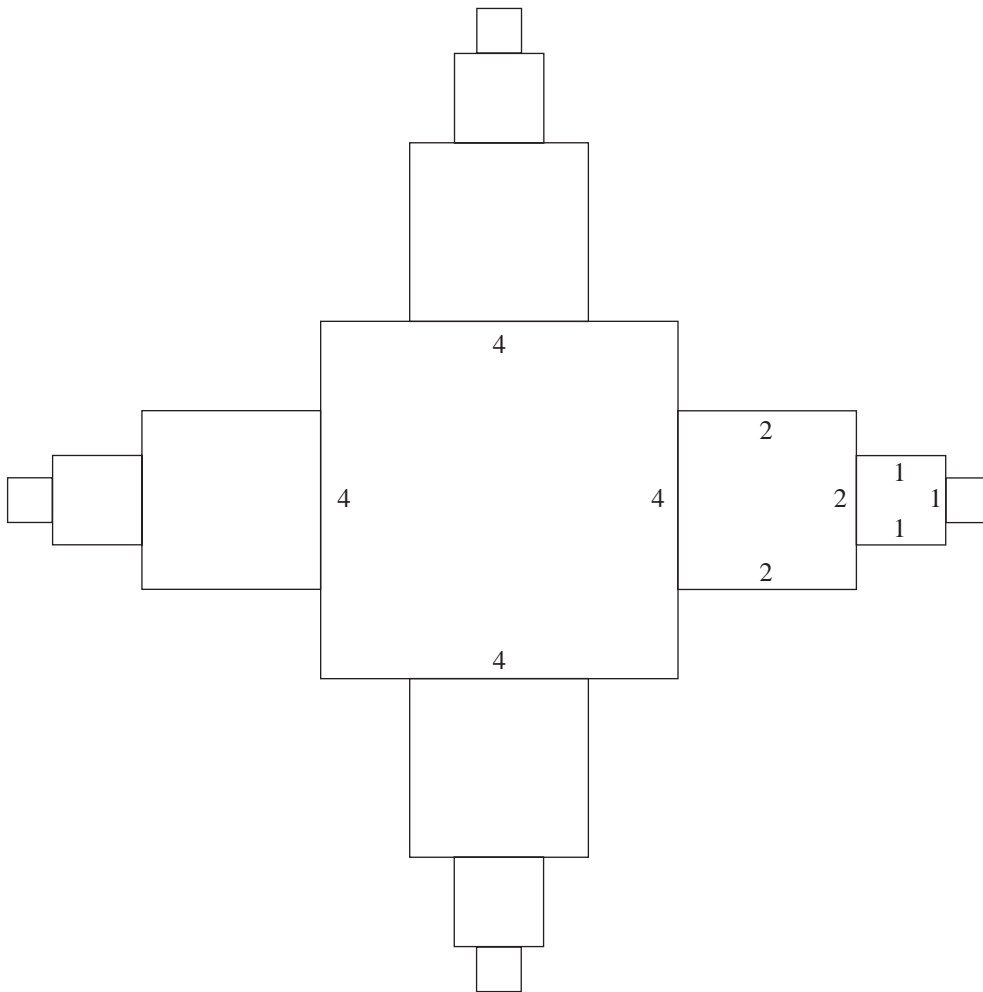
C.



D.

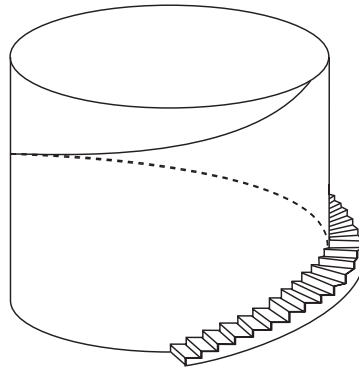


44. A “tower” of squares is constructed off each side of an original $4\text{ cm} \times 4\text{ cm}$ square as shown in the diagram below. Each successive square has sides that are $\frac{1}{2}$ the length of the sides of the previous square. If this process is continued without end, determine the sum of the areas of all the squares formed including the first 4×4 square.



- A. $21\frac{1}{3}$
 B. $37\frac{1}{3}$
 C. 48
 D. 64

45. A cylindrical storage tank has a radius of 6 metres and a height of 18 metres. A staircase allowing access from the ground to the top of the tank rises at an angle of elevation of 20° . Determine the length of the inner edge of this staircase.



- A. 19.16 m
- B. 41.78 m
- C. 49.45 m
- D. 52.63 m

**This is the end of the multiple-choice section.
Answer the remaining questions directly in this examination booklet.**

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PART B: WRITTEN RESPONSE

Value: 25 marks

Suggested Time: 45 minutes

INSTRUCTIONS: Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

If, in a justification, you refer to information produced by the calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem, it is important to sketch the graph, showing its general shape and indicating the appropriate window dimensions.

When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.

Full marks will NOT be given for the final answer only.

1. Solve the following system algebraically. Express all solutions as ordered pairs. **(3 marks)**

$$x^2 + y^2 = 25$$

$$x = y^2 - 5$$

ANSWER:



2. A polynomial function of degree 3 has a zero of -1 and a double zero of 4 . Determine this function if it passes through the point $(1, 10)$. Answer may be left in factored form. **(3 marks)**

ANSWER:



3. A point $P(x, y)$ moves such that it is always equidistant from the point $A(2, 3)$ and the line $y = -1$. Determine the equation of this locus, in standard form. **(3 marks)**

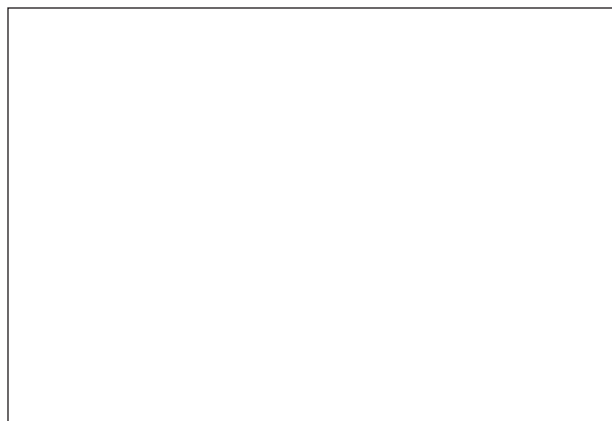
ANSWER:



4. Use a graphing calculator to solve the following equation for x where $0 \leq x < 2\pi$. (3 marks)

$$2 \cos x = -\sin 3x$$

Sketch the graph in the viewing window below and indicate appropriate window dimensions. State the function(s) used in your graph. Ensure that the relative maximum and relative minimum points of the function(s) are visible within the viewing window.



$Y_1 =$

$Y_2 =$

$Y_3 =$

$Y_4 =$

[,] [,]

x
min x
max

y
min y
max

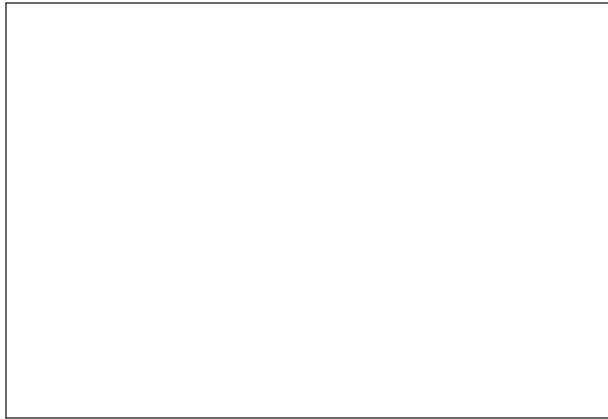
ANSWER:

5. Solve the following system using a graphing calculator. Express all solutions as ordered pairs. **(3 marks)**

$$y = 3\log(x + 2) + 1$$

$$y = 2^x - 1$$

Sketch the graph in the viewing window below. State the function(s) that you entered to obtain your graph and your solution. Indicate the dimensions of the viewing window that will show enough of the graph so that recognizable characteristics of the function(s) and all intersection points are visible.



Y₁ =

Y₂ =

Y₃ =

Y₄ =

[,]

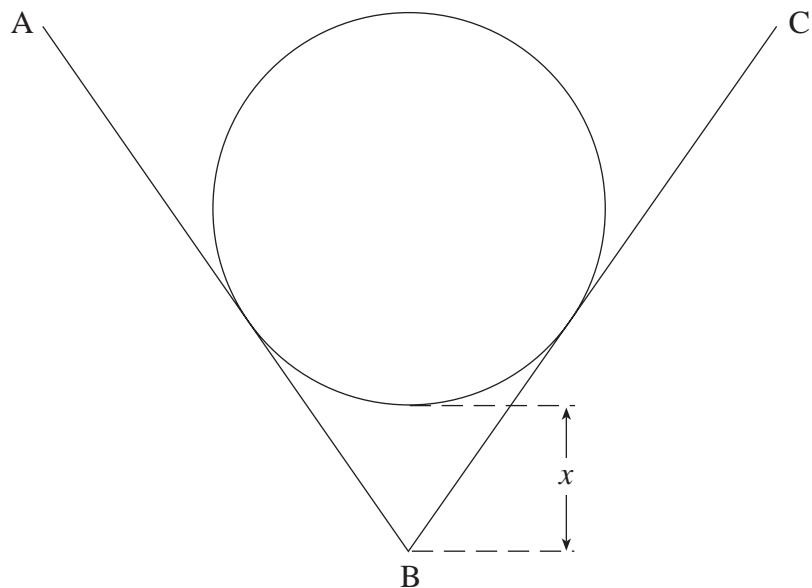
[,]

x
min x
max

y
min y
max

ANSWER:

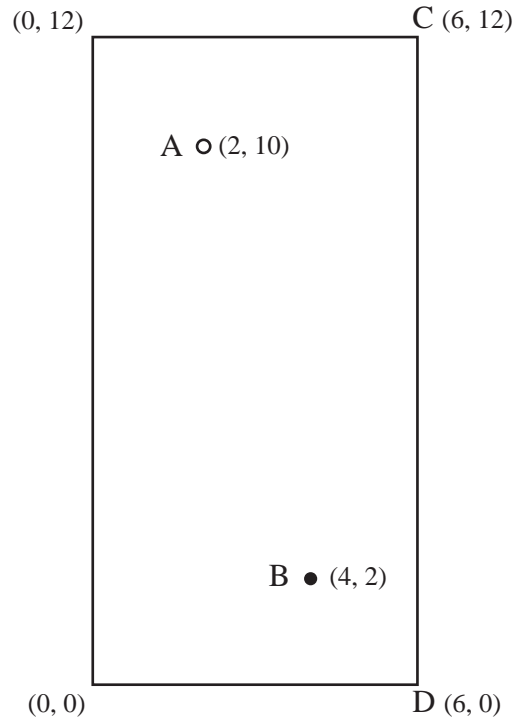
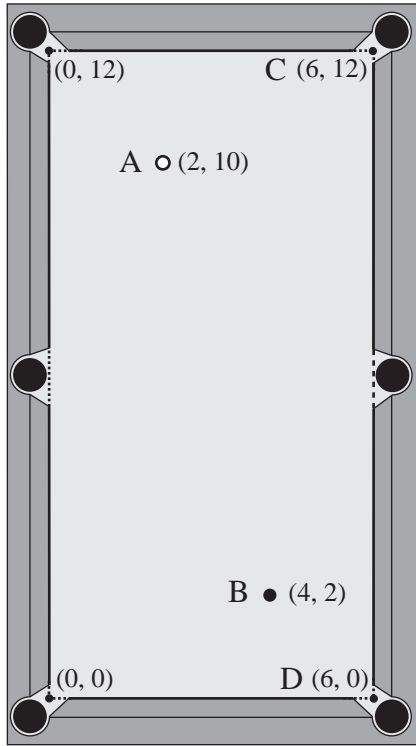
6. In the diagram below, AB and CB are tangents to a circle with radius 10. If $\angle ABC = 80^\circ$, find x , the shortest distance from B to the circle. **(3 marks)**



ANSWER:



7. Pam and Donna are playing snooker on a 6×12 snooker table. To establish the position of the balls, consider a coordinate system with $(0, 0)$ at the bottom left corner and the other corners at $(6, 0)$, $(6, 12)$ and $(0, 12)$. The cue ball is at $A(2, 10)$ and the ball that must be hit is at $B(4, 2)$. If the cue ball at A must bounce off side CD and then hit the ball at B , determine the coordinates of the point on side CD that the cue ball must hit. **(3 marks)**



ANSWER:

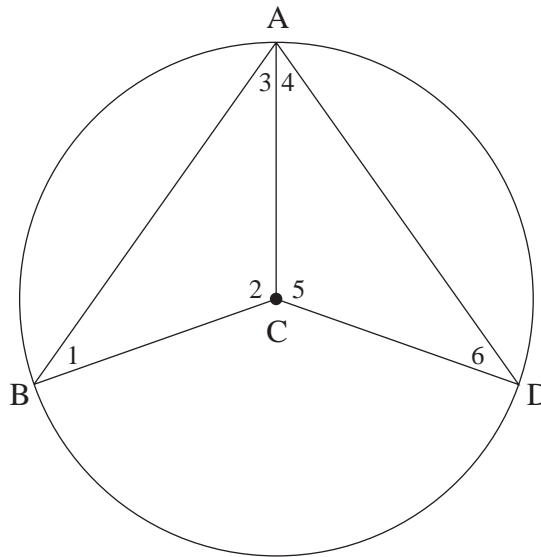


8. Complete the proof.

(4 marks)

Given: Circle with centre C
 $\angle 1 = \angle 6$

Prove: $AB = AD$



PROOF

END OF EXAMINATION

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A SUMMARY OF BASIC IDENTITIES AND FORMULAE

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Reciprocal and Quotient Identities

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Addition Identities

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

Double-Angle Identities

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - 2 \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Formulae

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t_n = a + (n-1)d$$

$$t_n = ar^{n-1}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$S_n = \frac{n}{2}(a + \ell)$$

$$S_n = \frac{a - \ell r}{1 - r}$$

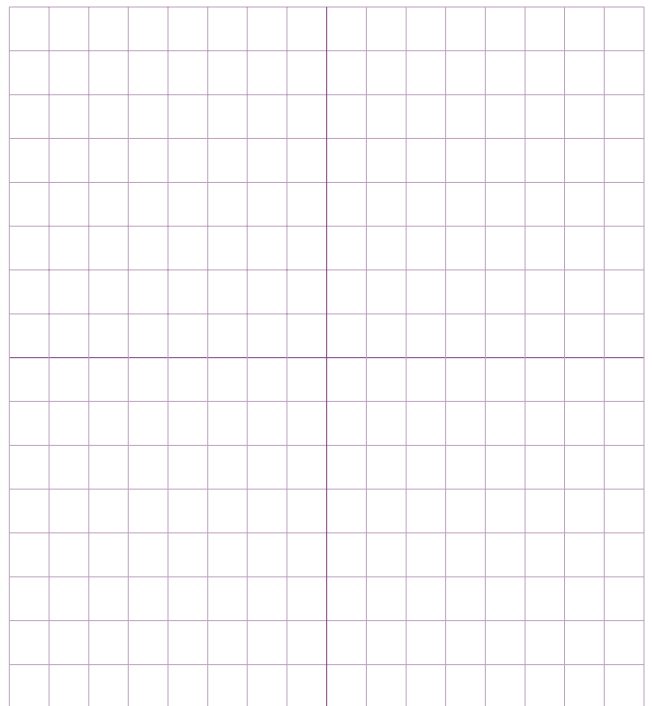
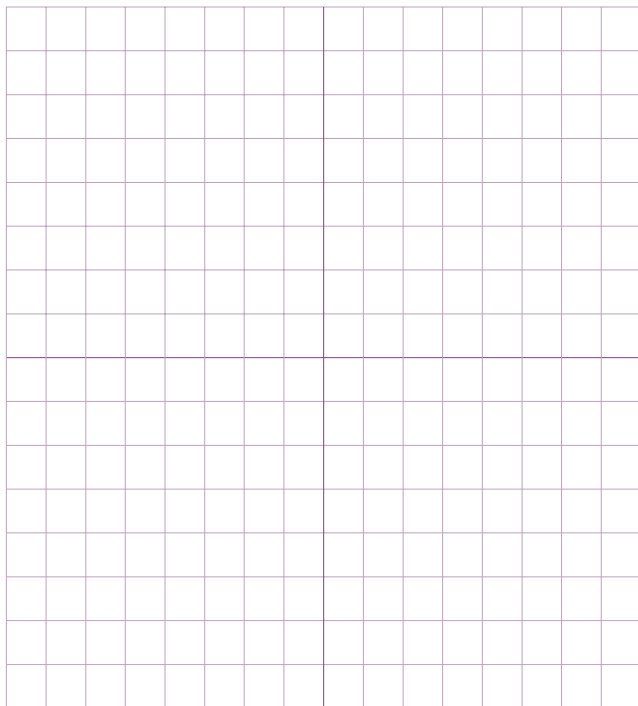
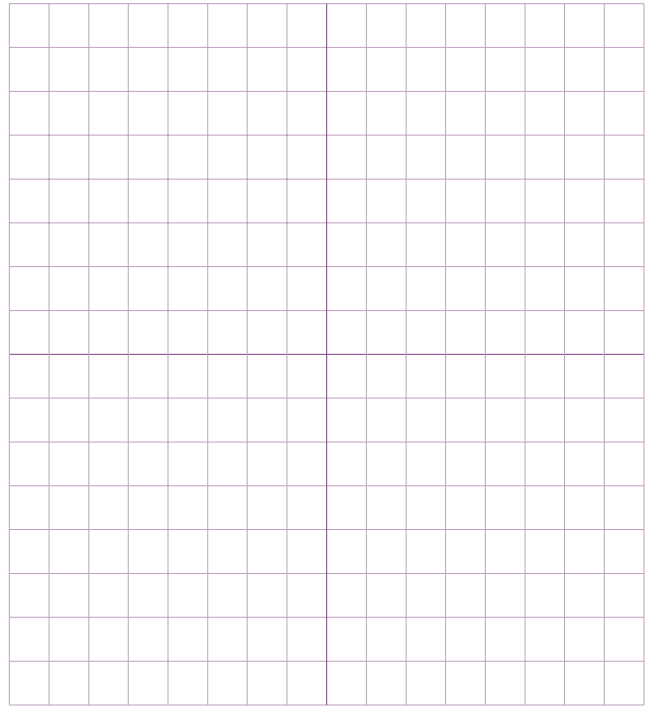
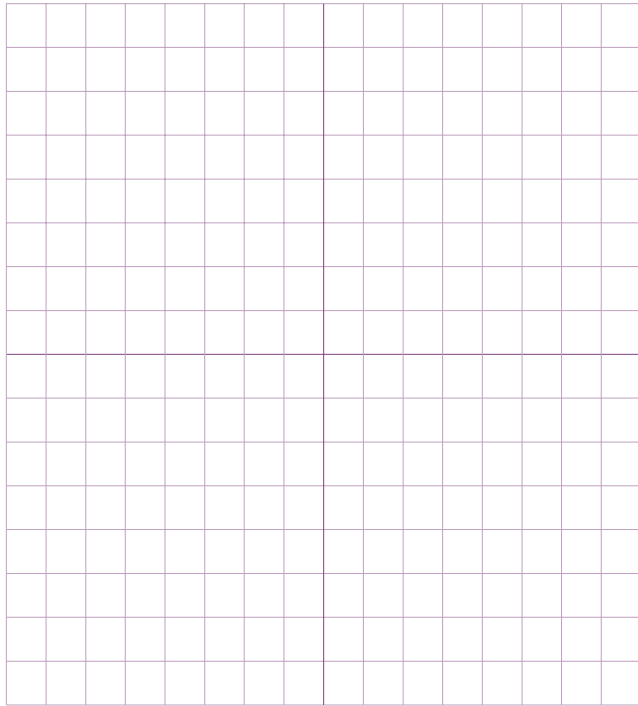
$$S = \frac{a}{1-r}$$

**You may detach this page for convenient reference.
Exercise care when tearing along perforations.**

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ROUGH WORK FOR GRAPHING

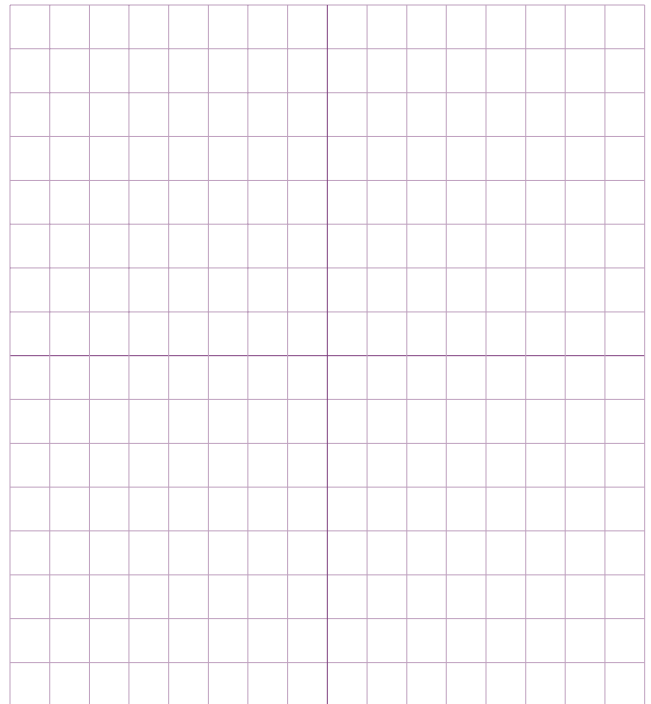
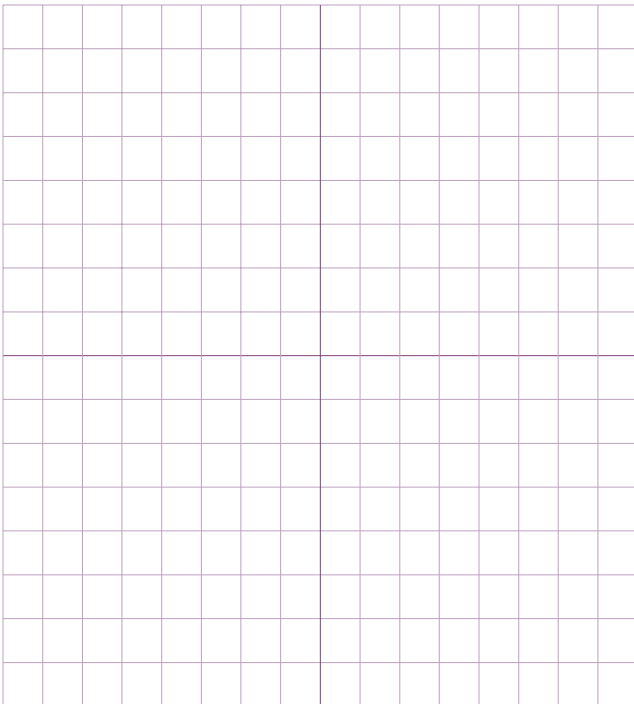
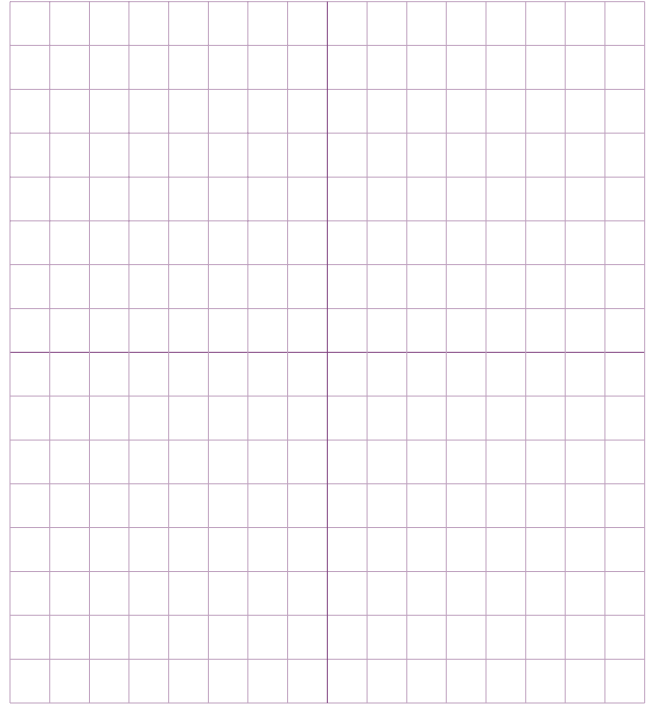
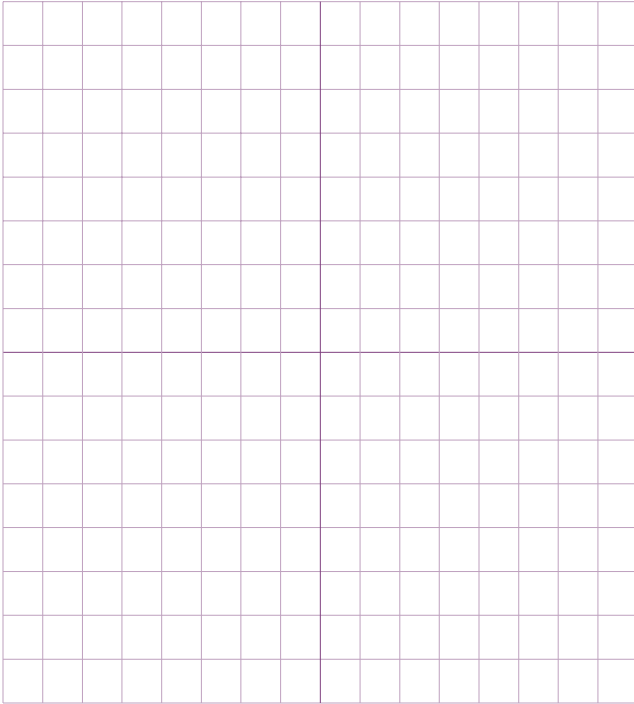
(No marks will be given for work done on this page.)



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ROUGH WORK FOR GRAPHING

(No marks will be given for work done on this page.)

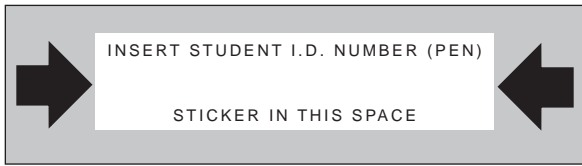


ROUGH WORK FOR MULTIPLE-CHOICE

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ROUGH WORK FOR MULTIPLE-CHOICE





PRINCIPLES OF MATHEMATICS 12

April 1999

Course Code = PMA

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**PRINCIPLES OF
MATHEMATICS 12**

April 1999

Course Code = PMA

Score for
Question 1:

1. $\frac{\quad}{(3)}$

Score for
Question 8:

8. $\frac{\quad}{(4)}$

Score for
Question 2:

2. $\frac{\quad}{(3)}$

Score for
Question 3:

3. $\frac{\quad}{(3)}$

Score for
Question 4:

4. $\frac{\quad}{(3)}$

Score for
Question 5:

5. $\frac{\quad}{(3)}$

Score for
Question 6:

6. $\frac{\quad}{(3)}$

Score for
Question 7:

7. $\frac{\quad}{(3)}$