

**AUGUST 1998**

## **PROVINCIAL EXAMINATION**

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**MINISTRY OF EDUCATION, SKILLS AND TRAINING**

# **MATHEMATICS 12**

### **GENERAL INSTRUCTIONS**

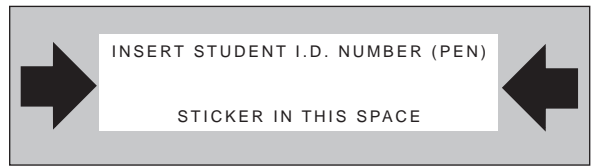
1. Insert the stickers with your Student I.D. Number (PEN) in the allotted spaces above.  
**Under no circumstance is your name or identification, other than your Student I.D. Number, to appear on this paper.**
2. Take the separate Answer Sheet and follow the directions on its front page.
3. Be sure you have an **HB pencil** and an eraser for completing your Answer Sheet.  
Follow the directions on the Answer Sheet when answering multiple-choice questions.
4. For each of the written-response questions, write your answer in the space provided.
5. When instructed to open this booklet, **check the numbering of the pages** to ensure that they are numbered in sequence from page one to the last page, which is identified by

**END OF EXAMINATION**.

6. At the end of the examination, place your Answer Sheet inside the front cover of this booklet and return the booklet and your Answer Sheet to the supervisor.

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**FOR OFFICE USE ONLY**



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**MATHEMATICS 12 AUGUST 1998 PROVINCIAL**

**Course Code = MA      Examination Type = P**

1.  $\frac{\quad}{(2)}$

2.  $\frac{\quad}{(2)}$

3.  $\frac{\quad}{(1)}$

4.  $\frac{\quad}{(3)}$

5.  $\frac{\quad}{(3)}$

6.  $\frac{\quad}{(3)}$

7.  $\frac{\quad}{(2)}$

8.  $\frac{\quad}{(4)}$

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## MATHEMATICS 12 PROVINCIAL EXAMINATION

	Value	Suggested Time
1. This examination consists of <b>two</b> parts:		
PART A: 50 multiple-choice questions	50	75
PART B: 7 written-response questions 2 questions worth <b>two</b> marks each, 4 questions worth <b>three</b> marks each, and 1 question worth <b>four</b> marks.	20	45
	<b>Total: 70 marks</b>	<b>120 minutes</b>
2. The last <b>three</b> pages inside the back cover contain <b>A Summary of Basic Identities and Formulae</b> , <b>Rough Work for Graphing</b> , and <b>Rough Work for Multiple-Choice</b> . These pages may be detached for convenient reference prior to writing this examination.		
3. You will not be provided with any additional paper since rough-work space for the written-response questions has been incorporated into the space allowed for answering each question. You may not need all of the space provided to answer each question.		
4. An approved scientific calculator is essential for the examination. The calculator must be a hand-held device designed <b>only</b> for mathematical computations such as logarithmic and trigonometric functions. It <b>can be</b> programmable, but <b>must not</b> contain any graphing capabilities. You <b>must not</b> bring into the examination room any devices to support calculators such as manuals, printed or electronic cards, printers, memory expansion chips or cards, or keyboards.		
5. You are permitted to use rulers, compasses, and protractors.		
6. You have <b>two hours</b> to complete this examination.		

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**PART A: MULTIPLE CHOICE**

**Value: 50 marks**

**Suggested Time: 75 minutes**

**INSTRUCTIONS:** For each question, select the **best** answer and record your choice on the Answer Sheet provided. Using an HB pencil, completely fill in the circle that has the letter corresponding to your answer.

- Which of the following represents a parabola?
  - $xy = 1$
  - $x + y^2 = 1$
  - $x^2 + y^2 = 1$
  - $x^2 - y^2 = 1$
  
- Find the midpoint of the line segment with endpoints  $A(-4, -2)$  and  $B(6, -8)$ .
  - $(1, -5)$
  - $(1, -3)$
  - $(-1, 5)$
  - $(-5, 3)$
  
- Determine the radius of the circle  $9x^2 + 9y^2 = 25$ .
  - $\frac{5}{3}$
  - $\frac{25}{9}$
  - 5
  - 25
  
- What are the equations of the asymptotes for the hyperbola  $x^2 - y^2 = -7$ ?
  - $y = \pm \sqrt{7}x$
  - $x = \pm \sqrt{7}y$
  - $y = \pm x$
  - $y = 0, x = 0$

5. Solve:  $|x + 4| > 6$



6. A point  $P(x, y)$  moves such that it is twice as far from  $R(5, -2)$  as it is from  $S(-1, -2)$ . Which equation represents this locus?

A.  $\sqrt{(x-5)^2 + (y+2)^2} = 2\sqrt{(x+1)^2 + (y+2)^2}$

B.  $2\sqrt{(x-5)^2 + (y+2)^2} = \sqrt{(x+1)^2 + (y+2)^2}$

C.  $\sqrt{(x+5)^2 + (y-2)^2} = 2\sqrt{(x-1)^2 + (y-2)^2}$

D.  $2\sqrt{(x+5)^2 + (y-2)^2} = \sqrt{(x-1)^2 + (y-2)^2}$

7. Change to standard form:  $3x^2 - y^2 + 18x + 21 = 0$

A.  $\frac{(x+3)^2}{16} - \frac{y^2}{48} = -1$

B.  $\frac{(x+3)^2}{10} - \frac{y^2}{30} = 1$

C.  $\frac{(x+3)^2}{4} - \frac{y^2}{12} = -1$

D.  $\frac{(x+3)^2}{2} - \frac{y^2}{6} = 1$



8. Which of the following describes a parabola with axis of symmetry  $x = 4$  ( $a \neq 0$ ) ?
- A.  $x = a(y + 4)^2 + b$
  - B.  $x = a(y - 4)^2 + b$
  - C.  $y = a(x + 4)^2 + b$
  - D.  $y = a(x - 4)^2 + b$
9. If the graphs defined by  $y = k$  and  $y = -x^2 + 8x - 14$  intersect at two different points, determine all values of  $k$ .
- A.  $k < 2$
  - B.  $k > 2$
  - C.  $k < 4$
  - D.  $k > 4$
10. The length of the major axis of an ellipse is four times the length of the minor axis, and  $P(2, 4)$  is a point on the ellipse. Determine the equation of the ellipse if its centre is  $(0, 0)$  and its major axis is on the  $y$ -axis.
- A.  $x^2 + 16y^2 = 80$
  - B.  $16x^2 + y^2 = 80$
  - C.  $x^2 + 16y^2 = 260$
  - D.  $16x^2 + y^2 = 260$
11. Convert  $225^\circ$  to radians.
- A.  $\frac{\pi}{225}$
  - B.  $\frac{4\pi}{5}$
  - C.  $\frac{5\pi}{4}$
  - D.  $225\pi$

12. Evaluate:  $\sec 3.4$  (Accurate to 2 decimal places.)
- A.  $-3.91$
  - B.  $-1.03$
  - C.  $0.29$
  - D.  $0.96$
13. Which of the following is an equation of an asymptote of the graph of  $y = \tan x$  ?
- A.  $x = 0$
  - B.  $x = 1$
  - C.  $x = \frac{\pi}{2}$
  - D.  $x = \pi$
14. Determine the range of the function  $y = 3 \sin x - 1$ .
- A.  $-2 \leq y \leq 0$
  - B.  $-2 \leq y \leq 4$
  - C.  $-3 \leq y \leq 3$
  - D.  $-4 \leq y \leq 2$
15. Simplify:  $\frac{1 - \sin^2 \theta}{\sin 2\theta}$
- A.  $\frac{1}{2} \cot \theta$
  - B.  $2 \cot \theta$
  - C.  $\frac{1}{2} \tan \theta$
  - D.  $2 \tan \theta$

16. How many solutions are there in the interval  $0 \leq x < 2\pi$  for the equation  $-5 \cos 2x = 3$  ?

- A. 0
- B. 2
- C. 3
- D. 4

17. Determine **all** values of  $x$  for which the following expression is undefined.

$$\frac{\sin x}{1 - \sec^2 x}, \text{ where } 0 \leq x < 2\pi$$

- A.  $\frac{\pi}{2}, \frac{3\pi}{2}$
- B.  $0, \pi$
- C.  $0, \frac{\pi}{2}$
- D.  $0, \pi, \frac{\pi}{2}, \frac{3\pi}{2}$

18. Which of the following is equivalent to  $\cos 5\theta \cos \theta - \sin 5\theta \sin \theta$  ?

- A.  $\cos 4\theta$
- B.  $\cos 6\theta$
- C.  $\cos 4\theta - \sin 4\theta$
- D.  $\cos 6\theta - \sin 6\theta$

19. Determine the exponential form of  $\log_p r = m$ .

- A.  $m = p^r$
- B.  $m = r^p$
- C.  $r = m^p$
- D.  $r = p^m$

20. Evaluate:  $3 \log_7 2$  (Accurate to 2 decimal places.)

- A. 0.90
- B. 0.92
- C. 1.07
- D. 1.13

21. Solve for  $x$ :  $\log x - \log 3 = 2$

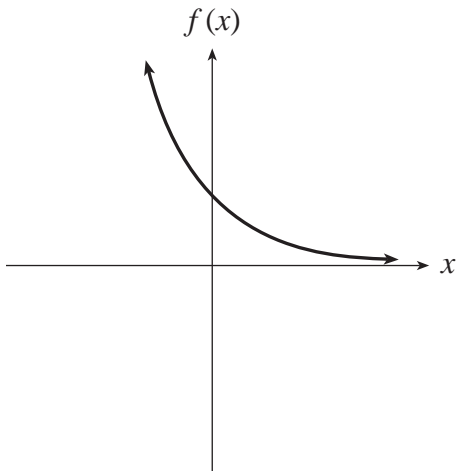
- A.  $0.\bar{6}$
- B. 6
- C.  $33.\bar{3}$
- D. 300

22. If  $\log a = x$  and  $\log b = y$ , express  $\log \left( \frac{a^2}{\sqrt{b}} \right)$  in terms of  $x$  and  $y$ .

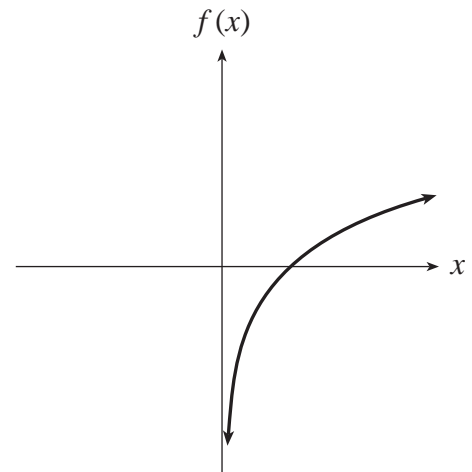
- A.  $2x - \frac{y}{2}$
- B.  $2y - \frac{x}{2}$
- C.  $x^2 - \sqrt{y}$
- D.  $\frac{x^2}{\sqrt{y}}$

23. Which graph **best** represents the function  $f(x) = 3^{-x}$  ?

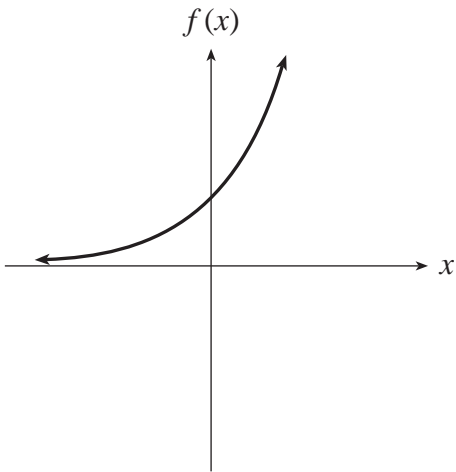
A.



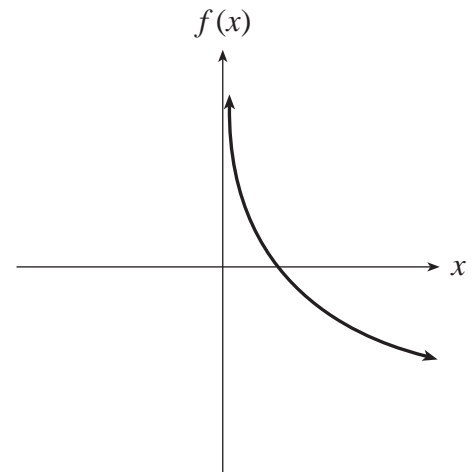
B.



C.



D.



24. A bacteria population halves in size every 5 days. Determine an expression for the number of bacteria remaining after  $t$  days, if the beginning population is  $k$ .

A.  $k(5)^{\frac{t}{5}}$

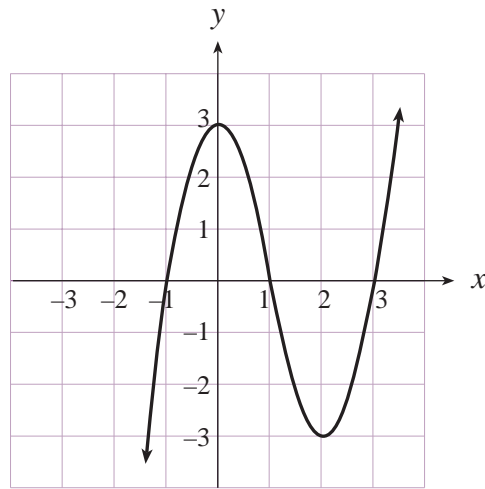
B.  $k(5)^{-2t}$

C.  $k\left(\frac{1}{2}\right)^{5t}$

D.  $k\left(\frac{1}{2}\right)^{\frac{t}{5}}$

25. If  $f(x) = ax$ , determine **all** values of  $a$  such that  $f^{-1}(x) = f(x)$  for every  $x$ , where  $f^{-1}(x)$  is the inverse of  $f(x)$ .
- A.  $-1$   
 B.  $1$   
 C.  $\pm 1$   
 D. any non-zero real number

26. Determine the real zeros of the function graphed below.



- A.  $0, 2$   
 B.  $-1, 1, 3$   
 C.  $-1, 1, -3$   
 D.  $-3, 3$
27. If  $x - 3$  is a factor of the polynomial  $f(x)$ , then which of the following **must** be true?
- A.  $f(3) = 0$   
 B.  $f(-3) = 0$   
 C.  $f(0) = -3$   
 D.  $f(0) = 3$
28. Determine the remainder when  $x^{42} + 3x^{31} - 2$  is divided by  $x + 1$ .
- A.  $-6$   
 B.  $-4$   
 C.  $-2$   
 D.  $2$

29. Solve:  $x^3 - x^2 - 5x + 5 = 0$

- A.  $-1, \pm\sqrt{5}$
- B.  $-1, \pm 5$
- C.  $1, \pm\sqrt{5}$
- D.  $1, \pm 5$

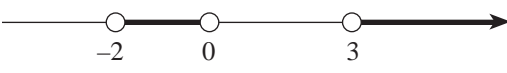

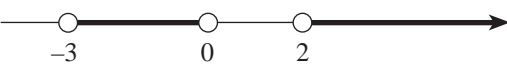

30. Determine the quotient when  $x^3 - x^2 - 18$  is divided by  $x^2 + 2x + 6$ .

- A.  $x - 3$
- B.  $x + 3$
- C.  $x - 1$
- D.  $x + 1$

31. Determine a polynomial function of degree 3 which has zeros at  $-2, 1, 4$  and a  $y$ -intercept of  $-16$ .

- A.  $f(x) = -(x + 2)(x - 1)(x - 4)$
- B.  $f(x) = (x + 2)(x - 1)(x - 4) - 16$
- C.  $f(x) = 2(x + 2)(x - 1)(x - 4)$
- D.  $f(x) = -2(x + 2)(x - 1)(x - 4)$

32. Solve:  $x(x - 3)^2(x + 2) > 0$

- A. 
- B. 
- C. 
- D. 

33. Determine the number of **rational** roots for the equation  $x^5 - 2x - 1 = 0$ .

- A. 1
- B. 2
- C. 3
- D. 5

34. The  $n^{\text{th}}$  term of a sequence is given by  $t_n = \frac{2n}{n^2 + 1}$ . Determine the 5<sup>th</sup> term.

- A.  $\frac{12}{37}$
- B.  $\frac{5}{13}$
- C.  $\frac{8}{17}$
- D. 1

35. Find the 25<sup>th</sup> term of the arithmetic sequence 37, 33, 29, 25, ...

- A. -67
- B. -63
- C. -59
- D. 133

36. Determine the common difference in the terms of the series  $\sum_{k=1}^5 (3 + 2k)$ .

- A. 1
- B. 2
- C. 3
- D. 5



37. Find the sum of the infinite geometric series  $5 - \frac{5}{3} + \frac{5}{9} - \dots$

A.  $\frac{15}{8}$

B.  $\frac{15}{4}$

C.  $\frac{15}{2}$

D.  $\frac{35}{9}$

38. The middle term of an arithmetic series is  $k$ . If the sum of the series is  $p$ , determine an expression for the number of terms in the series.

A.  $\frac{k}{p}$

B.  $\frac{p}{k}$

C.  $\frac{2p}{k}$

D.  $\frac{k}{2p}$

39. Evaluate:  $\lim_{x \rightarrow 3} (x^2 - 2x + 2)$

A.  $-8$

B.  $4$

C.  $5$

D.  $17$

40. Evaluate:  $\lim_{x \rightarrow \infty} \frac{2x^2 + 3x + 4}{2 - 5x^3}$

A.  $-\frac{2}{5}$

B.  $0$

C.  $2$

D. limit does not exist (no finite limit)

**OVER**

41. Which of the following limits represents the derivative of the function  $f(x) = x^2 - 3x + 1$  ?

A.  $\lim_{h \rightarrow 0} \frac{2(x+h) - 3}{h}$

B.  $\lim_{x \rightarrow 0} \frac{2(x+h) - 3}{h}$

C.  $\lim_{h \rightarrow 0} \frac{(x+h)^2 - 3(x+h) + 1 - (x^2 - 3x + 1)}{h}$

D.  $\lim_{x \rightarrow 0} \frac{(x+h)^2 - 3(x+h) + 1 - (x^2 - 3x + 1)}{h}$

42. If  $f(x) = \frac{3}{x}$ , determine  $f'(x)$ .

A.  $-\frac{3}{x^2}$

B.  $\frac{3}{x}$

C.  $-3x$

D.  $3x$

43. Given the function  $f(x) = 3x^2 - 4x + 3$ , for what value(s) of  $x$  is the slope of the tangent line equal to 2 ?

A.  $\frac{2}{3}$

B. 1

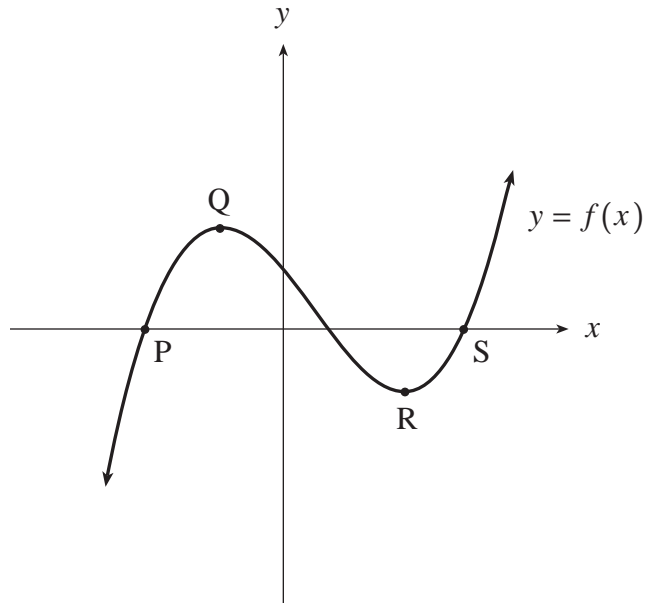
C. 8

D. 1,  $\frac{1}{3}$

44. Determine the minimum value of the function  $f(x) = 3x^2 - 12x + 13$ .

- A. 0
- B. 1
- C. 2
- D. 13

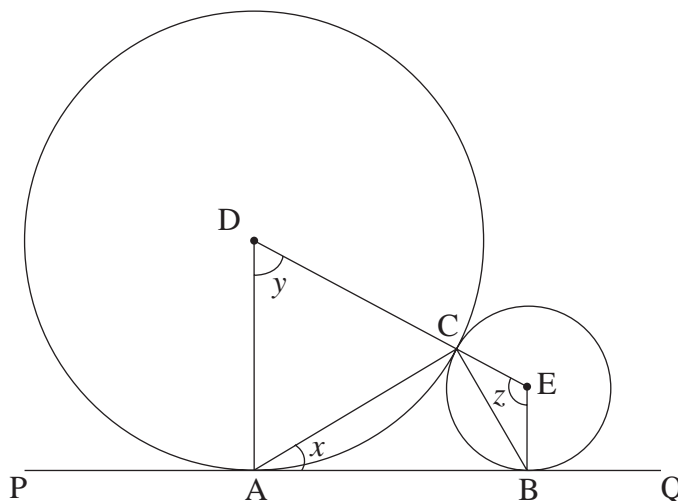
45. The graph of the function  $y = f(x)$  is given below. At which of the four labelled points is  $f'(x) = 0$  and  $x > f(x)$ ?



- A. P
- B. Q
- C. R
- D. S

Use the following diagram to answer questions 46 and 47.

- Given:
- Circles with centres D and E.
  - The circles are tangent to each other at C and to line PQ at A and B respectively.



46. What relationship exists between  $x$  and  $y$  ?

- A.  $y = x$
- B.  $y = 2x$
- C.  $y = \frac{1}{2}x$
- D.  $y = 180^\circ - x$

47. If  $x = 40^\circ$ , determine the measure of  $z$ . (Diagram is not drawn to scale.)

- A.  $80^\circ$
- B.  $90^\circ$
- C.  $100^\circ$
- D.  $110^\circ$

48. To the nearest power of 10, what is the difference between  $10^{640}$  and  $10^{643}$ ?
- A.  $10^3$
  - B.  $10^{\frac{643}{640}}$
  - C.  $10^{640}$
  - D.  $10^{643}$
49. What percent annual increase will allow a population of 2 million fish to increase to a population of 3 million fish over 10 years? (Accurate to 2 decimal places.)
- A. 0.96%
  - B. 1.04%
  - C. 4.14%
  - D. 5.68%
50. If  $A = (7, 10)$  and  $B = (2, 1)$ , find the y-coordinate of the point P which divides the line segment AB in a ratio of 1:2, that is,  $\frac{AP}{PB} = \frac{1}{2}$ .
- A. 6
  - B. 7
  - C.  $7\frac{1}{3}$
  - D.  $8\frac{1}{3}$

**This is the end of the multiple-choice section.  
Answer the remaining questions directly in this examination booklet.**

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**PART B: WRITTEN RESPONSE**

**Value: 20 marks**

**Suggested Time: 45 minutes**

**INSTRUCTIONS:** Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

**Full marks will NOT be given for the final answer only.**

1. Find four geometric means between 96 and 729.

**(2 marks)**



ANSWER:

Score for  
Question 1:

1.           
(2)

**OVER**

2. A particle moves along the  $x$ -axis so that its position at time  $t$  is given by  $x(t) = 4t^3 - 33t^2 + 30t + 12$ , where  $t$  is measured in seconds and  $x$  is measured in metres.

a) Determine the velocity, in m/s, of the particle at time  $t = 2$  seconds. **(2 marks)**

ANSWER:	Score for Question 2a:  2. <u>        </u> (2)
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b) Determine the time(s), in seconds, when the particle is stationary.

**(1 mark)**

ANSWER:

Score for  
Question 2b:

3.           
(1)

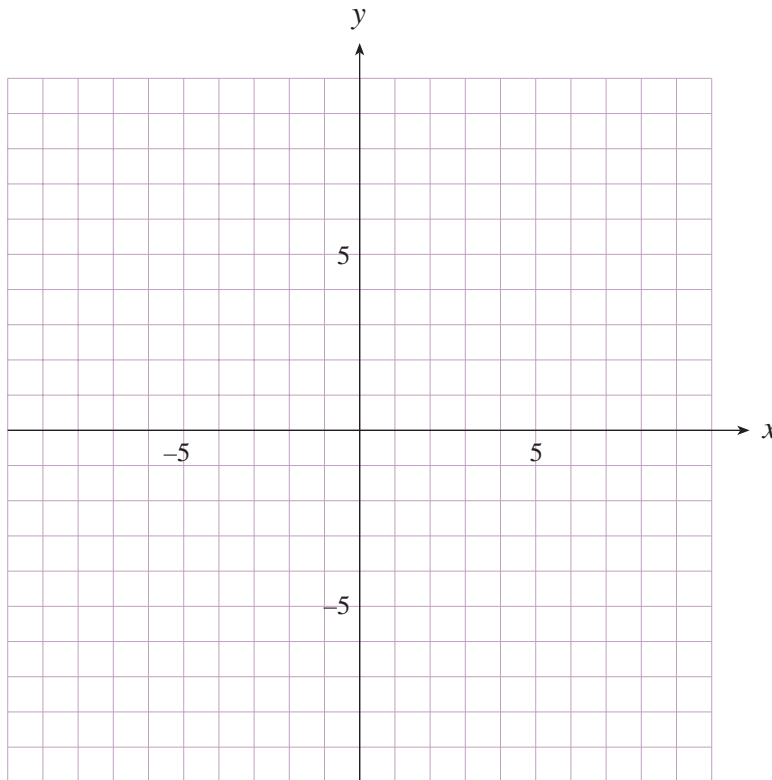
**OVER**

3. Graph the solution of the following system:

**(3 marks)**

$$\frac{x^2}{25} + \frac{y^2}{9} < 1$$

$$x^2 + 4x + y^2 \geq 12$$



Score for  
Question 3:

4.           
(3)

**OVER**

4. Solve for  $x$ :  $500 = 2.5(7.04)^{\frac{x}{3}}$  (Accurate to at least 2 decimal places.)

**(3 marks)**

ANSWER:

Score for  
Question 4:

5.           
(3)

**OVER**

5. Solve for  $x$ ,  $0 \leq x < 2\pi$ .

$$2 \sin x = 3 \sin x \cos x$$

(Accurate to at least 2 decimal places.)

**(3 marks)**



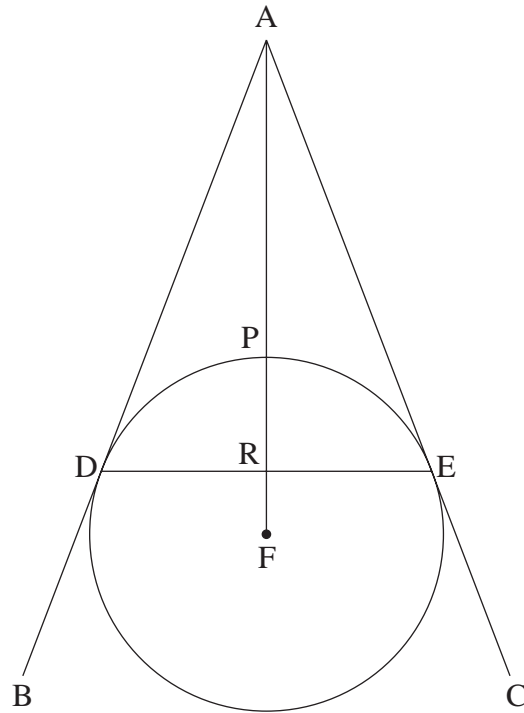
ANSWER:

Score for  
Question 5:

6.           
(3)

**OVER**

6. The given circle has centre  $F$  and tangents  $AB$  and  $AC$  at  $D$  and  $E$  respectively.  $A, P, R, F$  are collinear and chord  $DE$  is perpendicular to  $AF$ . If the radius is  $6$  and  $FR$  is  $2$ , determine the length of  $AR$ . (2 marks)



ANSWER:

Score for  
Question 6:

7.           
(2)

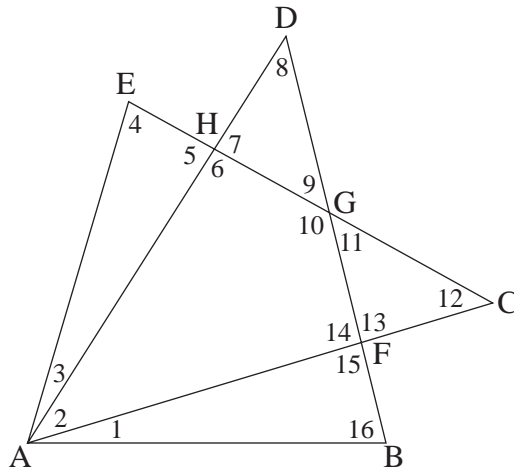
**OVER**

7. Complete the proof.

(4 marks)

Given: •  $\angle 8 = \angle 12$   
 •  $\angle 4 = \angle 16$

Prove: •  $\angle 3 = \angle 1$



Proof	
Statement	Reason

Score for  
Question 7:

8. \_\_\_\_\_  
(4)

**END OF EXAMINATION**

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## A SUMMARY OF BASIC IDENTITIES AND FORMULAE

### Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

### Reciprocal and Quotient Identities

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

### Addition Identities

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

### Double-Angle Identities

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - 2 \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

### Formulae

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t_n = a + (n-1)d$$

$$t_n = ar^{n-1}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$S_n = \frac{n}{2}(a + \ell)$$

$$S_n = \frac{a - \ell r}{1-r}$$

$$S = \frac{a}{1-r}$$

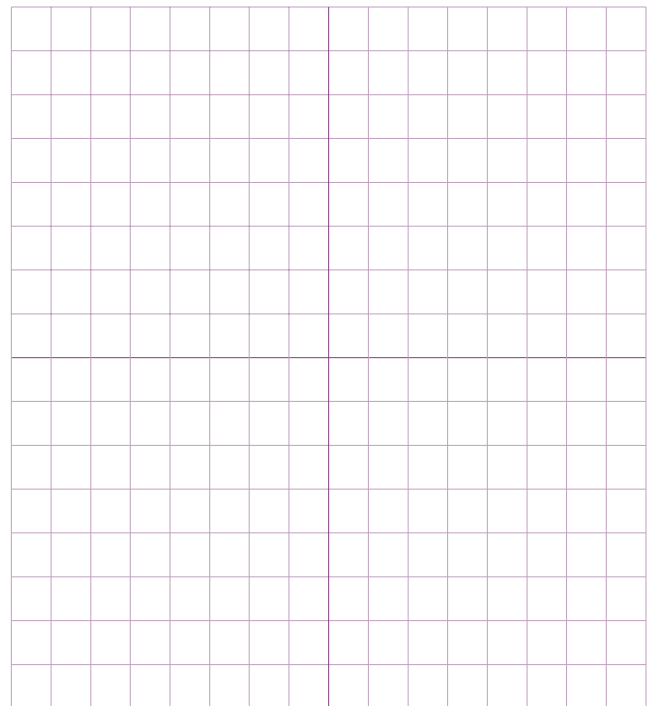
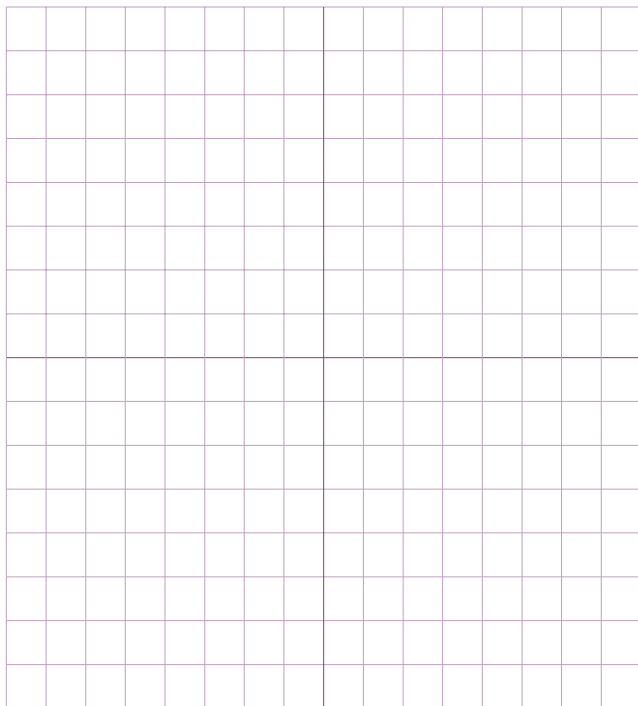
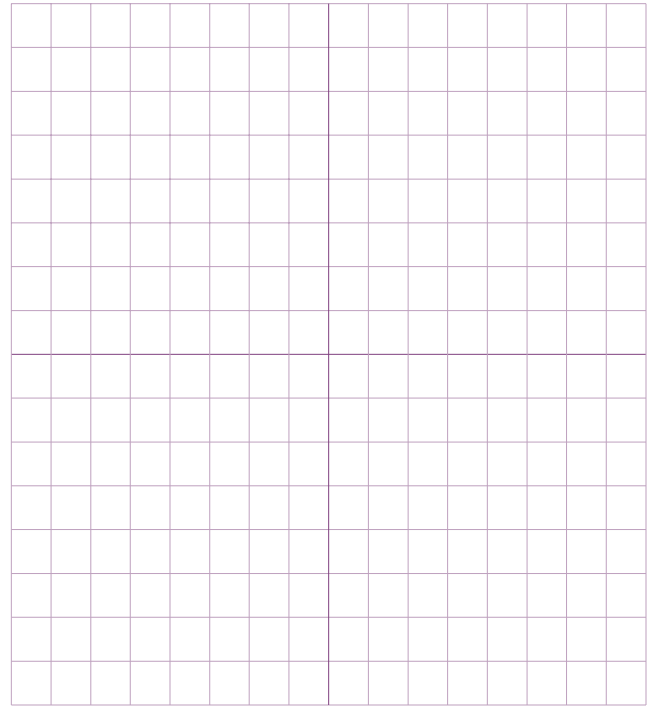
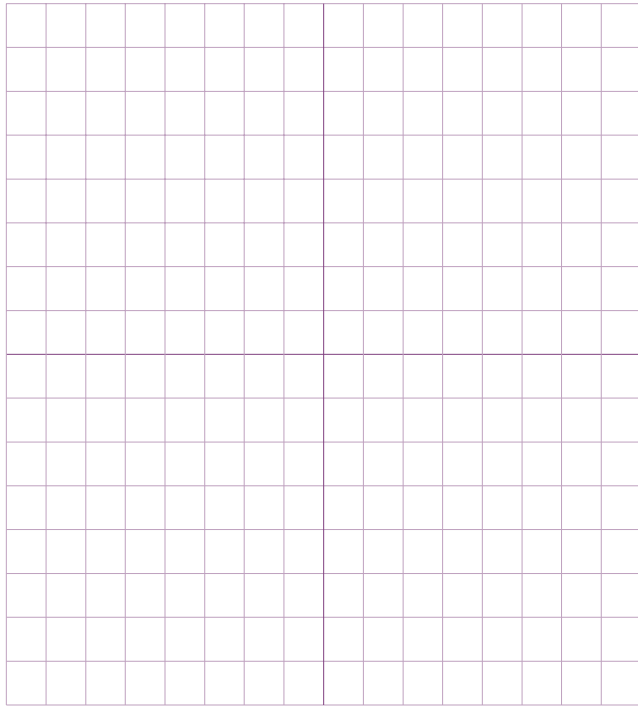
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**ROUGH WORK FOR GRAPHING**

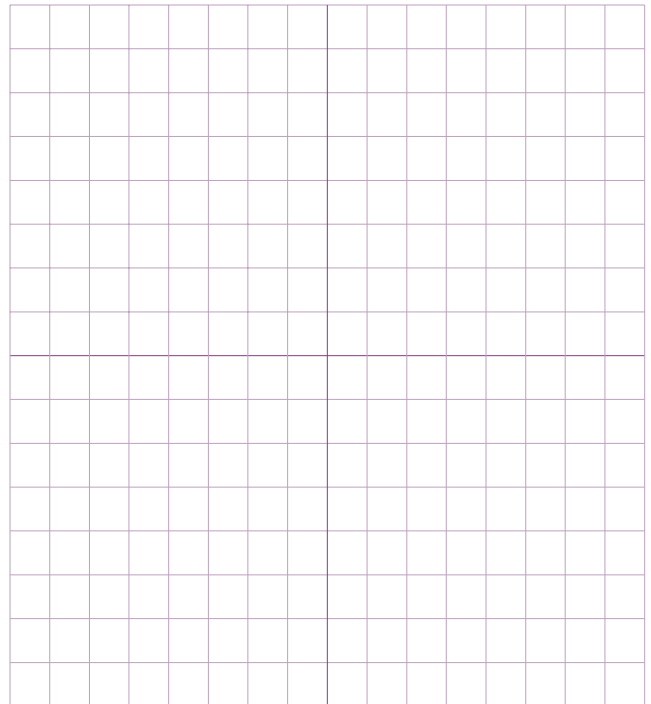
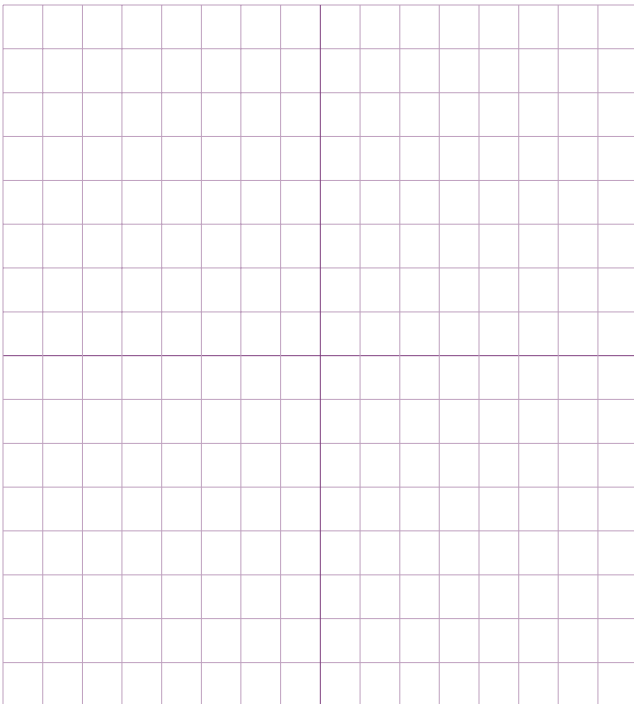
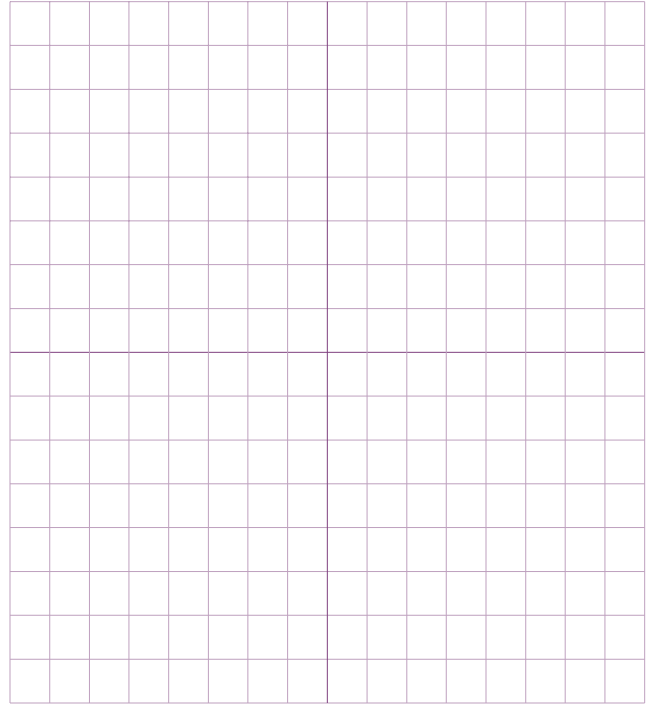
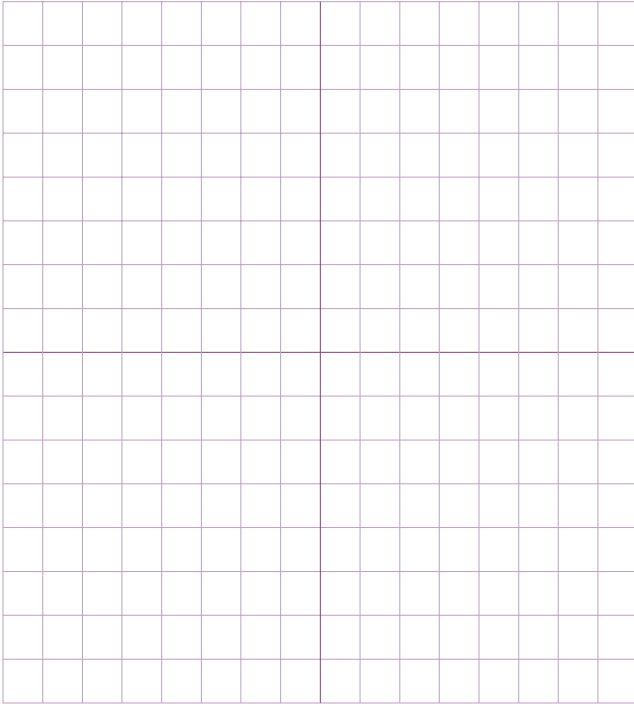
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# ROUGH WORK FOR GRAPHING

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**ROUGH WORK FOR MULTIPLE CHOICE**

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## ROUGH WORK FOR MULTIPLE CHOICE