

**JANUARY 1998**

## **PROVINCIAL EXAMINATION**

**MINISTRY OF EDUCATION, SKILLS AND TRAINING**

# **MATHEMATICS 12**

### **GENERAL INSTRUCTIONS**

1. Insert the stickers with your Student I.D. Number (PEN) in the allotted spaces above and on the **back** cover of this booklet. **Under no circumstance is your name or identification, other than your Student I.D. Number, to appear on this booklet.**
2. Ensure that in addition to this examination booklet, you have an **Examination Response Form**. Follow the directions on the front of the Response Form.
3. **Disqualification** from the examination will result if you bring books, paper, notes or unauthorized electronic devices into the examination room.
4. All multiple-choice answers must be entered on the Response Form using an **HB pencil**. Multiple-choice answers entered in this examination booklet will **not** be marked.
5. For each of the written-response questions, write your answer in the space provided in this booklet.
6. When instructed to open this booklet, **check the numbering of the pages** to ensure that they are numbered in sequence from page one to the last page, which is identified by

**END OF EXAMINATION**.

7. At the end of the examination, place your Response Form inside the front cover of this booklet and return the booklet and your Response Form to the supervisor.

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## MATHEMATICS 12 PROVINCIAL EXAMINATION

- |   | Value                  | Suggested Time     |
|---|------------------------|--------------------|
| 1. This examination consists of <b>two</b> parts:   |                        |                    |
| PART A: 50 multiple-choice questions  | 50                     | 75                 |
| PART B: 7 written-response questions<br>2 questions worth <b>two</b> marks each,<br>4 questions worth <b>three</b> marks each, and<br>1 question worth <b>four</b> marks. | 20                     | 45                 |
|   | <b>Total: 70 marks</b> | <b>120 minutes</b> |
- Aside from an approved calculator, electronic devices, including dictionaries and pagers, are **not** permitted in the examination room.
  - The last **three** pages inside the back cover contain **A Summary of Basic Identities and Formulae, Rough Work for Graphing, and Rough Work for Multiple-Choice**. These pages may be detached for convenient reference prior to writing this examination.
  - You will not be provided with any additional paper since rough-work space for the written-response questions has been incorporated into the space allowed for answering each question. You may not need all of the space provided to answer each question.
  - An approved scientific calculator is essential for the examination. The calculator must be a hand-held device designed **only** for mathematical computations such as logarithmic and trigonometric functions. It **can be** programmable, but **must not** contain any graphing capabilities. You **must not** bring into the examination room any devices to support calculators such as manuals, printed or electronic cards, printers, memory expansion chips or cards, or keyboards.
  - You are permitted to use rulers, compasses and protractors.
  - The time allotted for this examination is **two hours**.

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**PART A: MULTIPLE CHOICE**

**Value: 50 marks**

**Suggested Time: 75 minutes**

**INSTRUCTIONS:** For each question, select the **best** answer and record your choice on the Response Form provided. Using an HB pencil, completely fill in the circle that has the letter corresponding to your answer.

1. Find the radius of the circle  $(x + 5)^2 + (y - 3)^2 = 16$ .

- A. 2
- B. 4
- C. 8
- D. 16

2. What is the length of the major axis of the ellipse  $\frac{x^2}{36} + \frac{y^2}{64} = 1$  ?

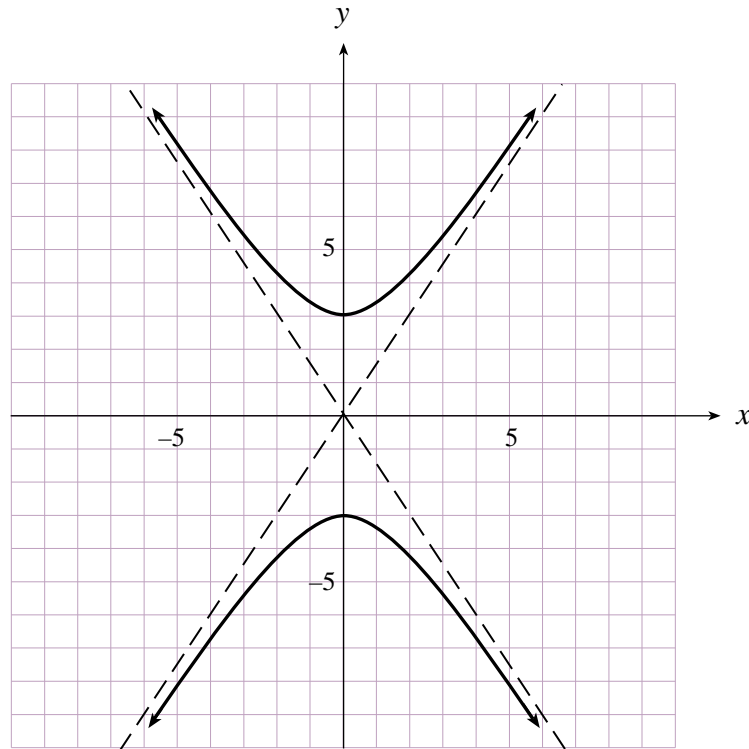
- A. 6
- B. 8
- C. 12
- D. 16

3. Which absolute value inequality describes the solution shown?



- A.  $|x - 6| < 14$
- B.  $|x - 14| < 6$
- C.  $|x - 6| > 14$
- D.  $|x - 14| > 6$

4. Determine an equation of the hyperbola graphed below.



- A.  $\frac{x^2}{4} - \frac{y^2}{9} = 1$
- B.  $\frac{x^2}{9} - \frac{y^2}{4} = 1$
- C.  $\frac{x^2}{4} - \frac{y^2}{9} = -1$
- D.  $\frac{x^2}{9} - \frac{y^2}{4} = -1$

5. Find all real solutions of the following system.

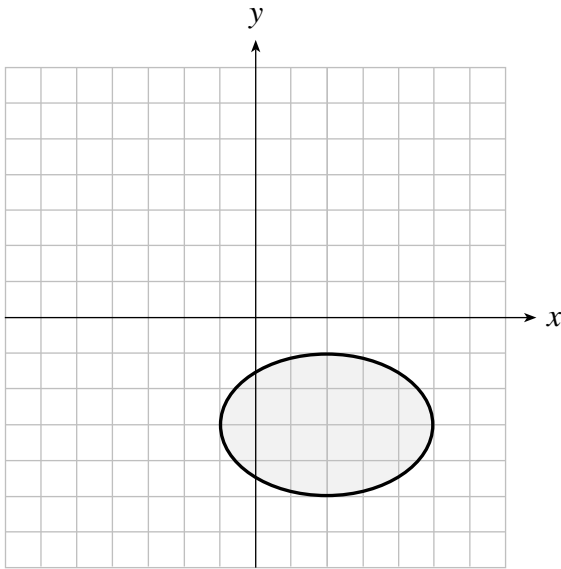
$$2x^2 + y^2 = 13$$

$$x^2 - y^2 = 5$$

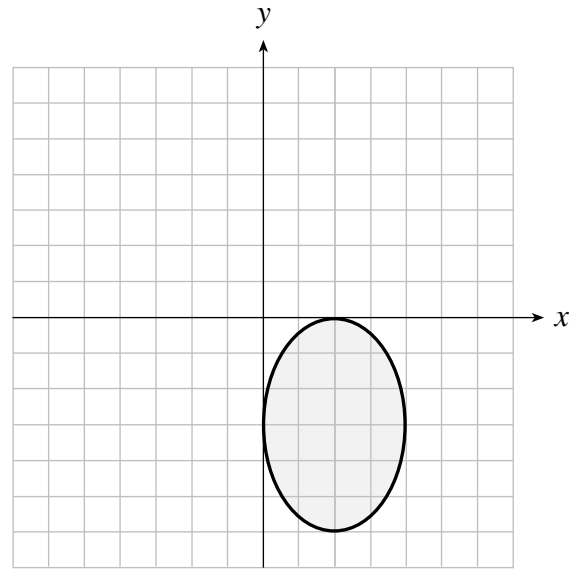
- A.  $(2\sqrt{2}, \sqrt{3}), (-2\sqrt{2}, \sqrt{3})$
- B.  $(\sqrt{6}, 1), (-\sqrt{6}, 1)$
- C.  $(2\sqrt{2}, \sqrt{3}), (-2\sqrt{2}, \sqrt{3}), (2\sqrt{2}, -\sqrt{3}), (-2\sqrt{2}, -\sqrt{3})$
- D.  $(\sqrt{6}, 1), (-\sqrt{6}, 1), (\sqrt{6}, -1), (-\sqrt{6}, -1)$

6. Which graph represents the solution of the inequality  $4(x - 2)^2 + 9(y + 3)^2 \leq 36$  ?

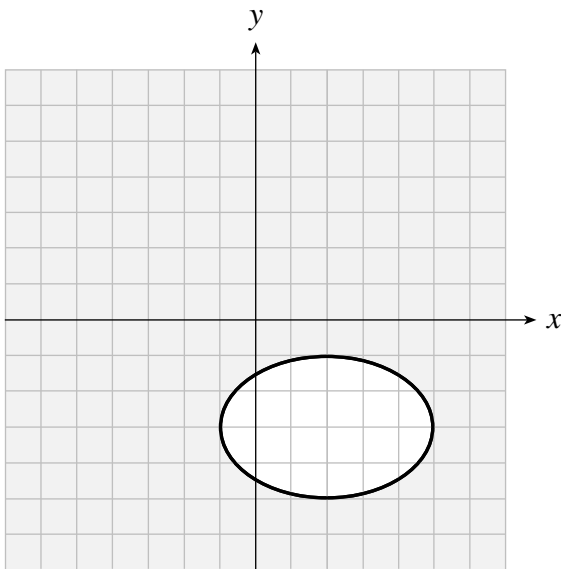
A.



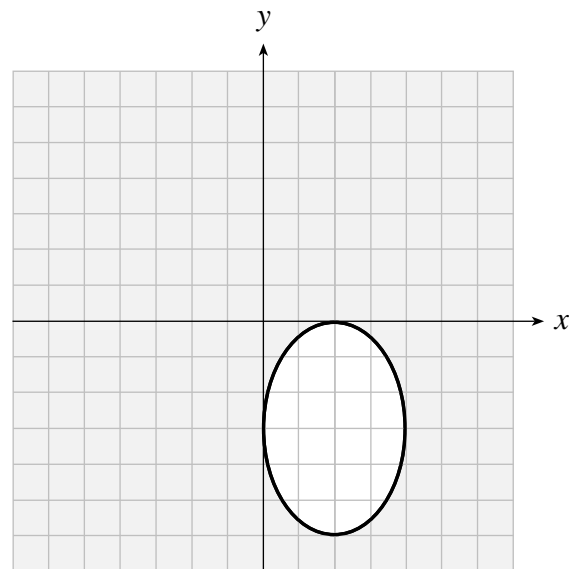
B.



C.



D.



7. Change the following equation to standard form.

$$y = 2x^2 - 24x + 75$$

- A.  $y = 2(x - 6)^2 + 39$
- B.  $y = -2(x - 6)^2 - 39$
- C.  $y = 2(x - 6)^2 + 3$
- D.  $y = -2(x - 6)^2 - 3$

8. A parabola with vertex  $(-3, 2)$  has a horizontal axis of symmetry and passes through the point  $(5, 4)$ . Determine an equation of the parabola.
- A.  $x = \frac{1}{2}(y-2)^2 - 3$   
 B.  $x = 2(y-2)^2 - 3$   
 C.  $x = \frac{1}{2}(y+2)^2 + 3$   
 D.  $x = 2(y+2)^2 + 3$
9. A point  $P(x, y)$  moves such that the slope of the line through  $P$  and  $A(-6, 0)$  is always 3 greater than the slope of the line through  $P$  and  $B(6, 0)$ . Determine an equation of this locus.
- A.  $\frac{y}{x+6} + 3 = \frac{y}{x-6}$   
 B.  $\frac{y}{x+6} = \frac{y}{x-6} + 3$   
 C.  $\sqrt{(x+6)^2 + y^2} = \sqrt{(x-6)^2 + y^2} + 3$   
 D.  $\sqrt{(x+6)^2 + y^2} + 3 = \sqrt{(x-6)^2 + y^2}$
10. Determine all values of the constant  $h$  such that the following system has exactly 3 different real solutions.

$$\frac{x^2}{36} - \frac{y^2}{25} = 1$$

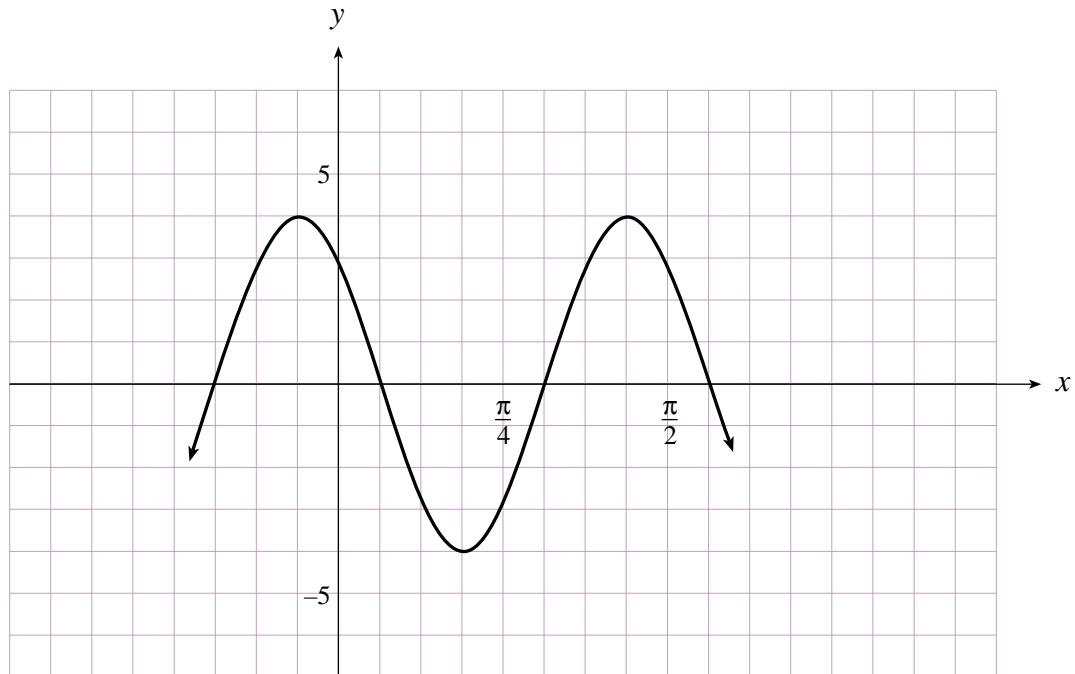
$$(x-h)^2 + y^2 = 64$$

- A.  $h = -14$  or  $14$   
 B.  $h = -2$  or  $2$   
 C.  $-14 \leq h \leq 14$   
 D.  $-2 \leq h \leq 2$



11. Convert  $162^\circ$  to radian measure.
- A. 0.90
  - B. 2.83
  - C. 508.94
  - D. 9 281.92
12. Determine the amplitude of the graph of  $y = -2 \cos 3x$ .
- A. -2
  - B. 2
  - C. 3
  - D.  $\frac{2\pi}{3}$
13. Evaluate:  $\csc 1.2$  (Accurate to 2 decimal places.)
- A. 0.67
  - B. 0.74
  - C. 1.07
  - D. 2.76
14. If  $\sin x = \frac{3}{4}$ , determine the smallest positive angle  $x$ , in radians.  
(Accurate to 2 decimal places.)
- A. 0.68
  - B. 0.72
  - C. 0.85
  - D. 1.47

15. Which equation describes the following graph?



- A.  $y = -4 \sin 4\left(x - \frac{\pi}{16}\right)$
- B.  $y = 4 \sin 4\left(x - \frac{\pi}{16}\right)$
- C.  $y = 4 \sin 4\left(x - \frac{3\pi}{16}\right)$
- D.  $y = -4 \sin 4\left(x + \frac{3\pi}{16}\right)$

16. Simplify:  $2 \cot x \sin^2 x$

- A.  $\frac{\sin 2x}{2}$
- B.  $\sin 2x$
- C.  $\frac{2 \sin^3 x}{\cos x}$
- D.  $\cos 2x \sin x$

17. Solve for  $x$ , where  $0 \leq x < 2\pi$ . (Accurate to 2 decimal places.)

$$2 \sec^2 x + 5 \sec x - 3 = 0$$

- A. 1.23 , 5.05
- B. 1.91 , 4.37
- C. 3.48 , 5.94
- D. 1.05 , 1.91 , 4.37 , 5.23

18. If  $\sin \theta = a$  and  $0 < \theta < \frac{\pi}{2}$ , determine an expression for  $\cos(\pi + \theta)$ .

- A.  $1 - a$
- B.  $a - 1$
- C.  $\sqrt{1 - a^2}$
- D.  $-\sqrt{1 - a^2}$

19. Evaluate:  $\log_3 3^{-15}$

- A. -15
- B. -5
- C.  $\frac{1}{15}$
- D.  $\frac{1}{5}$

20. Which expression is equivalent to  $2 \log \frac{3}{x}$  ?

- A.  $\log 9 - 2 \log x$
- B.  $\log 9 - \log x$
- C.  $\log 6 - \log 2x$
- D.  $(\log 3 - \log x)^2$

21. Give the domain of the function  $y = \log_2(x - 4)$ .

- A.  $x > -4$
- B.  $x > 0$
- C.  $x > 2$
- D.  $x > 4$

**OVER**

22. Solve:  $\log_6 x + \log_6(x - 5) = 2$

- A. 3.5
- B. 8.5
- C. 9
- D. 20.5

23. Given  $f(x) = \log_3 x$ , determine its inverse,  $f^{-1}(x)$ .

- A.  $f^{-1}(x) = x^3$
- B.  $f^{-1}(x) = 3^x$
- C.  $f^{-1}(x) = \log_x 3$
- D.  $f^{-1}(x) = \log_3 \frac{1}{x}$

24. Solve for  $x$ :  $(\log_2 3)(\log_x 5)(\log_5 2) = 9$

- A.  $3^{\frac{1}{9}}$
- B.  $9^{\frac{1}{3}}$
- C. 2
- D. 3

25. Consider the graph of  $y = -\log_2 x$ . Which value for  $x$  in the interval  $a \leq x \leq b$ ,  $a > 0$ , will give the largest value for  $y$ ?

- A.  $a$
- B.  $b$
- C.  $\frac{a+b}{2}$
- D.  $\sqrt{ab}$

26. Given a polynomial  $p(x)$ , what condition must be true for  $x - 2$  to be a factor of  $p(x)$  ?
- A.  $p(2) = 0$
  - B.  $p(-2) = 0$
  - C.  $p(x) = 2$
  - D.  $p(x) = -2$
27. According to the Rational Root Theorem, give all possible rational roots of  $2x^3 - 5x^2 + 12x - 6 = 0$  .
- A.  $\pm 1, \pm 2, \pm 3, \pm 6$
  - B.  $\pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{1}{6}$
  - C.  $\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$
  - D.  $\pm 1, \pm 2, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{1}{6}$
28. What is the quotient when  $5x^3 - 6x^2 + 64$  is divided by  $x + 2$  ?
- A.  $5x^2 + 4x + 8$
  - B.  $5x^2 - 16x + 32$
  - C.  $5x^2 + 4x + 72$
  - D.  $5x^2 - 16x + 96$
29. Find the remainder when  $3x^{45} + 4x^8 - 5x^3 + 2$  is divided by  $x + 1$  .
- A.  $-10$
  - B.  $-2$
  - C.  $4$
  - D.  $8$
30. What is the least number of real zeros that a polynomial function can have if its degree is 5 ?
- A. 0
  - B. 1
  - C. 3
  - D. 5

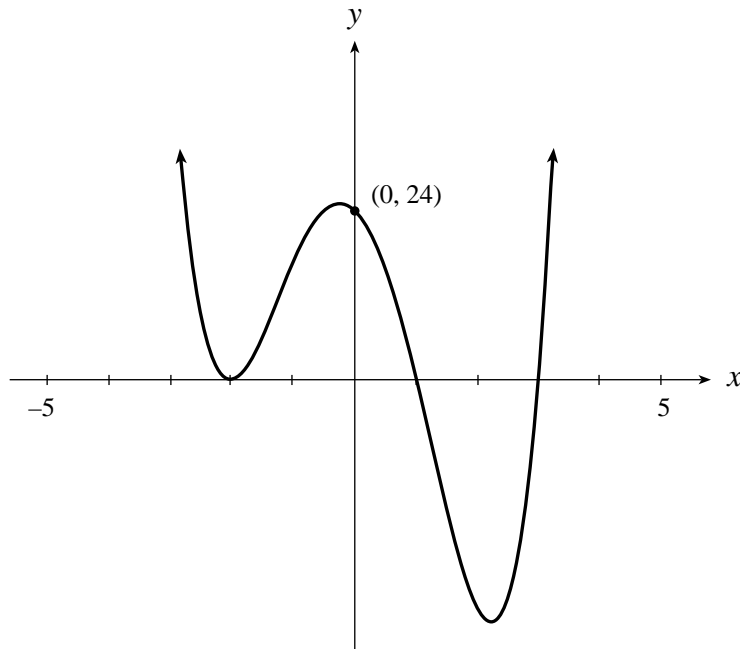
31. Determine the real roots:  $x^3 + 3x^2 - 6x - 8 = 0$

- A. -4, -1, 2
- B. -4, 1, 2
- C. -2, 1, 4
- D. -1, 2, 4

32. Solve the inequality:  $(x + 5)(x - 2)(6 - x) > 0$

- A.  $x < -5$  or  $x > 6$
- B.  $x < -5$  or  $x > 2$
- C.  $-5 < x < 2$  or  $x > 6$
- D.  $x < -5$  or  $2 < x < 6$

33. Determine an equation of the polynomial function  $y = f(x)$  graphed below.



- A.  $f(x) = 2(x + 2)^2(x - 1)(x - 3)$
- B.  $f(x) = 4(x + 2)^2(x - 1)(x - 3)$
- C.  $f(x) = 2(x - 2)^2(x + 1)(x + 3)$
- D.  $f(x) = 4(x + 2)(x - 1)(x - 3)$

34. Which of the following is a geometric sequence?
- A. 10, 20, 30, 40
  - B. 2, 8, 12, 16
  - C. 12, 36, 72, 144
  - D. 8, 16, 32, 64
35. Determine the number of terms in the arithmetic sequence 5, 1, -3, ... , -111.
- A. 27
  - B. 28
  - C. 29
  - D. 30
36. Evaluate:  $\sum_{k=1}^{120} (4k - 1)$
- A. 28 441
  - B. 28 920
  - C. 29 161
  - D. 29 403
37. If the sum of an infinite geometric series is 90 and the common ratio is  $-\frac{1}{5}$ , determine the value of the first term.
- A. 72
  - B. 75
  - C. 108
  - D. 112.5
38. Given that  $t_n = 2t_{n-1} + n$ ,  $n > 1$  and  $t_3 = 17$ , find  $t_1$ .
- A. 2
  - B. 2.25
  - C. 2.5
  - D. 2.75

39. Find the derivative of  $f(x) = 4x^2 + 7x - 5$ .

- A.  $4x + 7$
- B.  $6x + 7$
- C.  $8x + 7$
- D.  $8x^2 + 7x$

40. Evaluate:  $\lim_{x \rightarrow \infty} \frac{2x - 7}{4x + 3}$

- A.  $-\frac{7}{3}$
- B.  $-\frac{3}{7}$
- C.  $\frac{1}{2}$
- D. 2

41. Evaluate:  $\lim_{x \rightarrow 10} \frac{x^2 - 9x - 10}{x^2 - 100}$

- A.  $\frac{1}{10}$
- B.  $\frac{11}{20}$
- C. 1
- D. limit does not exist (no finite limit)

42. Which expression represents the derivative of  $f(x) = x^4$  ?

- A.  $\lim_{x \rightarrow 0} \frac{4(x+h)^3 - 4x^3}{x}$
- B.  $\lim_{h \rightarrow 0} \frac{4(x+h)^3 - 4x^3}{h}$
- C.  $\lim_{x \rightarrow 0} \frac{(x+h)^4 - x^4}{x}$
- D.  $\lim_{h \rightarrow 0} \frac{(x+h)^4 - x^4}{h}$



43. For the function  $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 6x$ , find the  $x$ -coordinate of the critical point where the local minimum point occurs.

- A. -3
- B. -2
- C. 2
- D. 3

44. Choose the most appropriate volume function to solve the following problem:

“A rectangular box with no top is to be constructed from a square piece of cardboard 24 cm on each side by cutting equal squares of side  $x$  cm from the four corners and bending up the four resulting flaps to form the sides of the box. What should the length of  $x$  be to give a box that has the largest possible volume?”

- A.  $V = x(24 - x)^2$
- B.  $V = x(24 - 2x)^2$
- C.  $V = 2x(24 - x)^2$
- D.  $V = 4x^2(24 - 2x)^2$

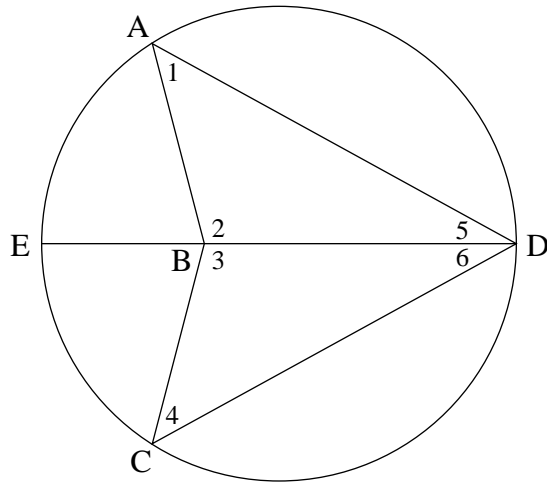
45. What are the coordinates of the point on the graph of  $y = \sqrt{x}$  where the slope of the tangent is  $\frac{1}{8}$ ?

- A.  $\left(\frac{1}{16}, \frac{1}{4}\right)$
- B.  $\left(\frac{1}{16}, 4\right)$
- C.  $\left(16, \frac{1}{8}\right)$
- D.  $(16, 4)$

Use the following diagram and proof to answer questions 46 and 47.

Given:  $\widehat{AE} = \widehat{EC}$   
 $AD = CD$   
 E, B, D are collinear

Prove:  $AB = CB$



PROOF	
Statement	Reason
$\widehat{AE} = \widehat{EC}$	given
(a) $\angle 5 = \angle 6$	inscribed $\angle$ s on = arcs are =
(b) $AD = CD$	given
(c) $BD = BD$	same side
(d) $\angle 1 = \angle 4$	inscribed $\angle$ s on same chord are =
(e) $\triangle ABD \cong \triangle CBD$	_____
$AB = CB$	CPCTC

46. Which line is **incorrect** in the given proof?

- A. (a)
- B. (b)
- C. (c)
- D. (d)

47. Give the reason for line (e).

- A. SAS
- B. ASA
- C. AAS
- D. SSS

48. A basketball court is made up of identical square tiles, laid side by side to form a rectangle 54 tiles wide by 72 tiles long. If a straight line is drawn diagonally from one corner of the floor to the opposite corner, how many tiles would the diagonal intersect?
- A. 90
  - B. 108
  - C. 117
  - D. 126
49. To the nearest power of 10, what is the difference between  $10^{700}$  and  $10^{600}$  ?
- A.  $10^{599}$
  - B.  $10^{600}$
  - C.  $10^{699}$
  - D.  $10^{700}$
50. Solve over the real numbers:  $\cos \frac{\pi}{2} x = 1$
- A.  $2n$  ( $n$  is an integer)
  - B.  $4n$  ( $n$  is an integer)
  - C.  $2 + 2n$  ( $n$  is an integer)
  - D.  $2 + 4n$  ( $n$  is an integer)

**This is the end of the multiple-choice section.  
Answer the remaining questions directly in this examination booklet.**

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**PART B: WRITTEN RESPONSE**

**Value: 20 marks**

**Suggested Time: 45 minutes**

**INSTRUCTIONS:** Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

**Full marks will NOT be given for the final answer only.**

1. Find the sum of the first 30 terms of the geometric series  $10 + 12 + 14.4 + \dots$   
(Accurate to at least 2 decimal places.)

**(3 marks)**

ANSWER:

2. Prove the identity.

(2 marks)

$$\frac{1 - \cos \theta}{\sin^2 \theta} = \frac{1}{1 + \cos \theta}$$

Left Side

Right Side



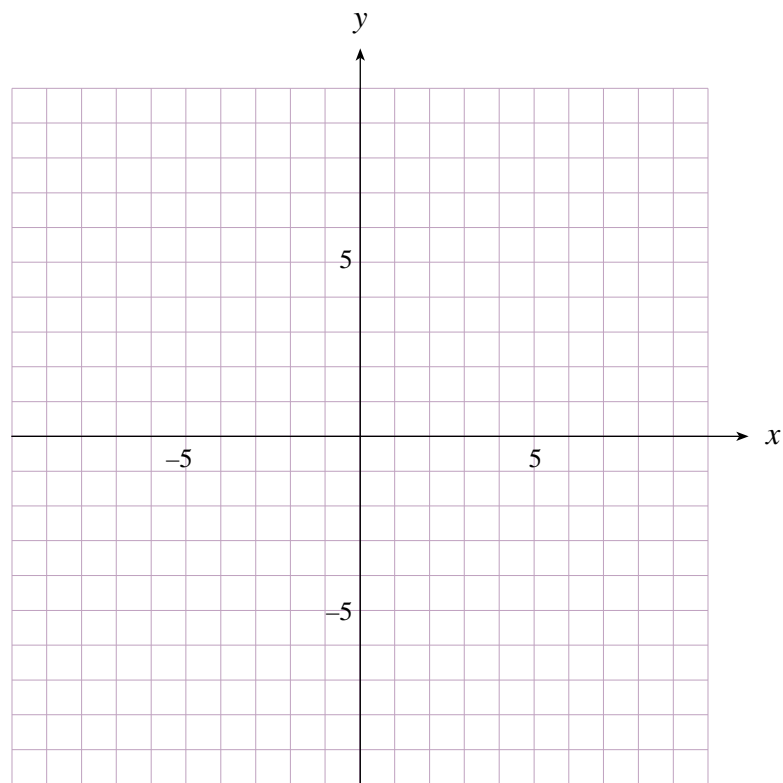


3. A particle moves along the  $x$ -axis such that its distance from the origin is given by  $x(t) = 2t^2 + 60t$  where  $x$  is in centimetres and  $t$  is in seconds. When the particle's velocity is 72 cm/sec, determine its distance  $x(t)$  from the origin. **(3 marks)**

ANSWER:

**The following question has two parts: a) and b)**

4. a) Graph the function  $y = 3^{x+2} - 4$  on the grid provided. Indicate the asymptote with a dotted or broken line and clearly show at least three points on the curve. **(2 marks)**



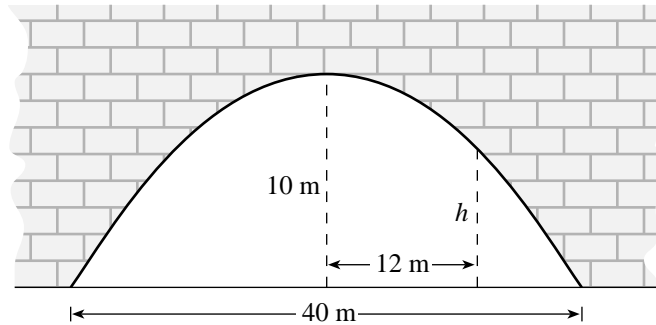
b) Calculate the zero of this function. (Accurate to at least 2 decimal places.)

**(1 mark)**

ANSWER:

**OVER**

5. The arch of a bridge over a river has the shape of a parabola. The base is 40 m wide and the maximum height of the arch is 10 m. Find the height  $h$  of the arch at a distance of 12 m from the centre of the arch. (Accurate to at least 1 decimal place.) **(3 marks)**



ANSWER:

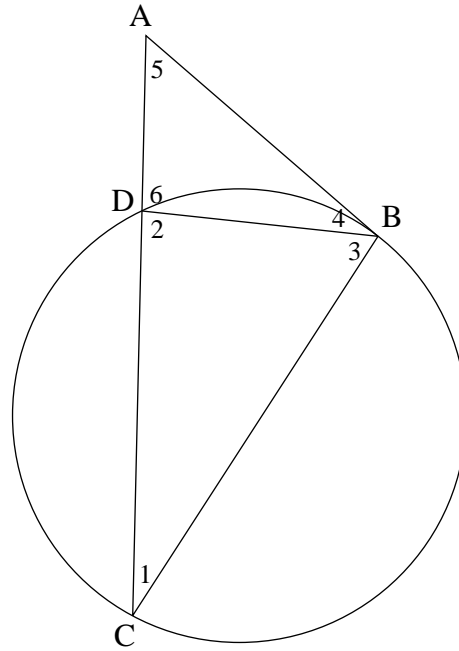
**OVER**

6. Complete the proof.

(4 marks)

Given:  $AB$  is a tangent  
 $\angle 3 = \angle 5$   
 $C, D, A$  are collinear

Prove:  $BC$  is a diameter



PROOF

Statement	Reason





7. A river system has a current population of 4 000 000 fish. In each year, an enhancement program produces a 7% growth in population followed by a 200 000 fish harvest (i.e. 200 000 fish are taken from the population). Determine the number of fish after the second harvest.

**(2 marks)**

ANSWER:

**END OF EXAMINATION**

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## A SUMMARY OF BASIC IDENTITIES AND FORMULAE

### Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

### Reciprocal and Quotient Identities

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

### Addition Identities

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

### Double-Angle Identities

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - 2 \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

### Formulae

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t_n = a + (n-1)d$$

$$t_n = ar^{n-1}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$S_n = \frac{n}{2}(a + \ell)$$

$$S_n = \frac{a - \ell r}{1-r}$$

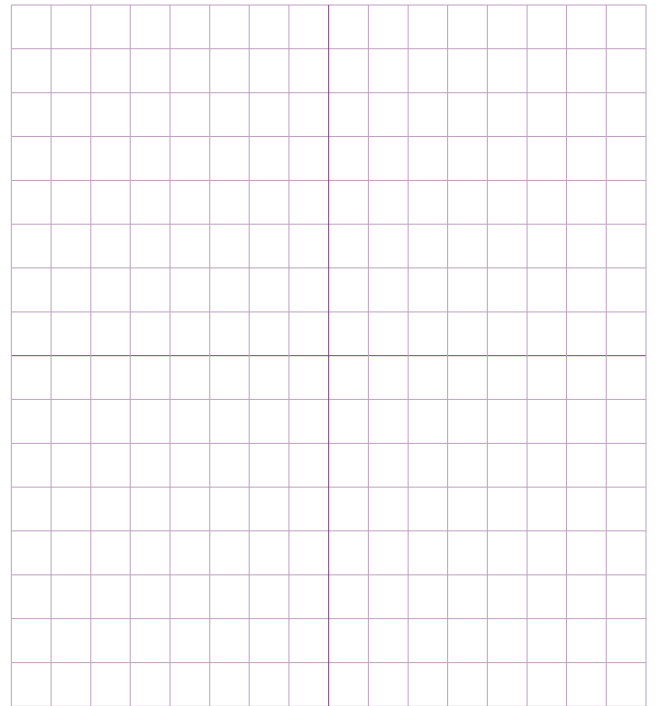
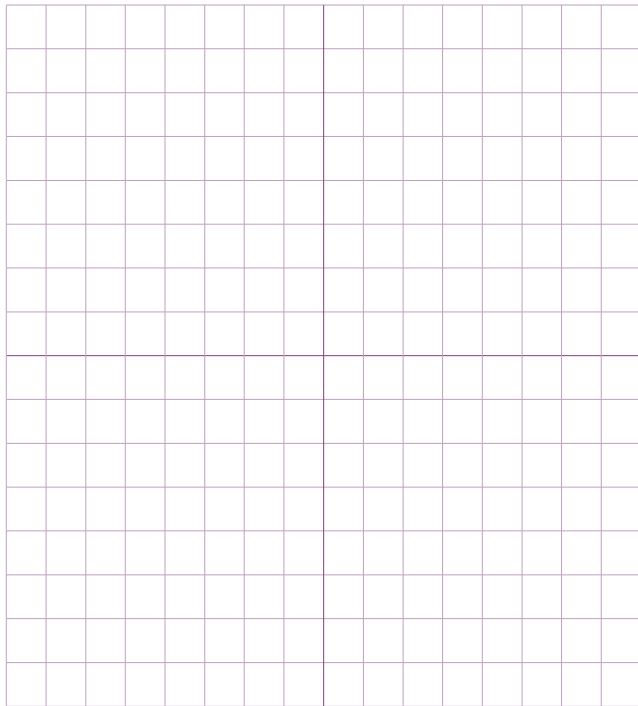
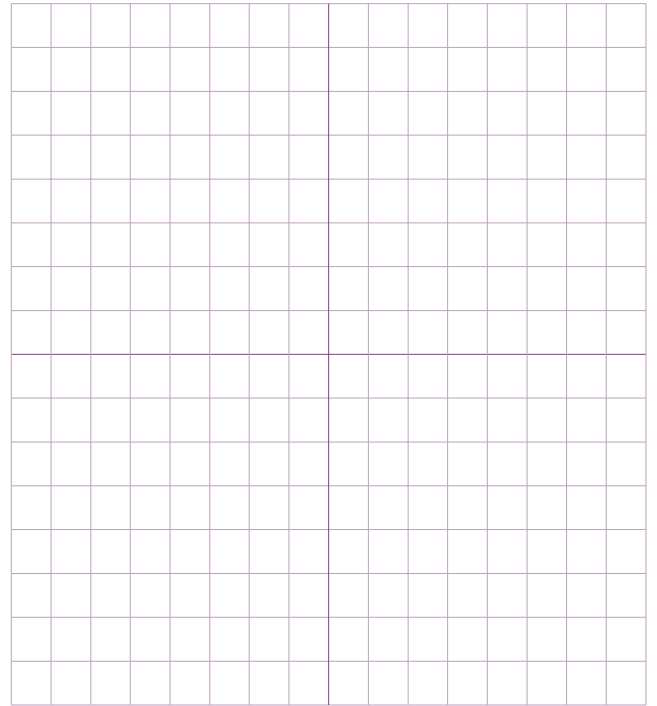
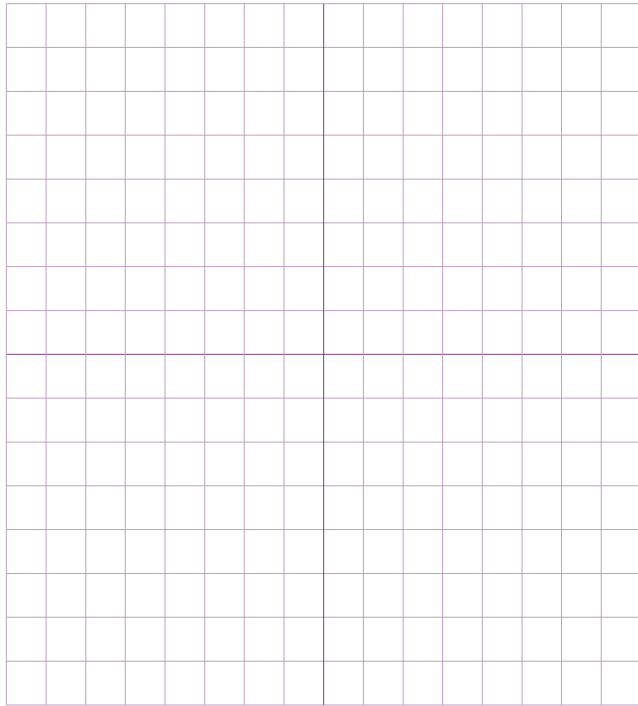
$$S = \frac{a}{1-r}$$

**You may detach this page for convenient reference.  
Exercise care when tearing along perforations.**

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**ROUGH WORK FOR GRAPHING**

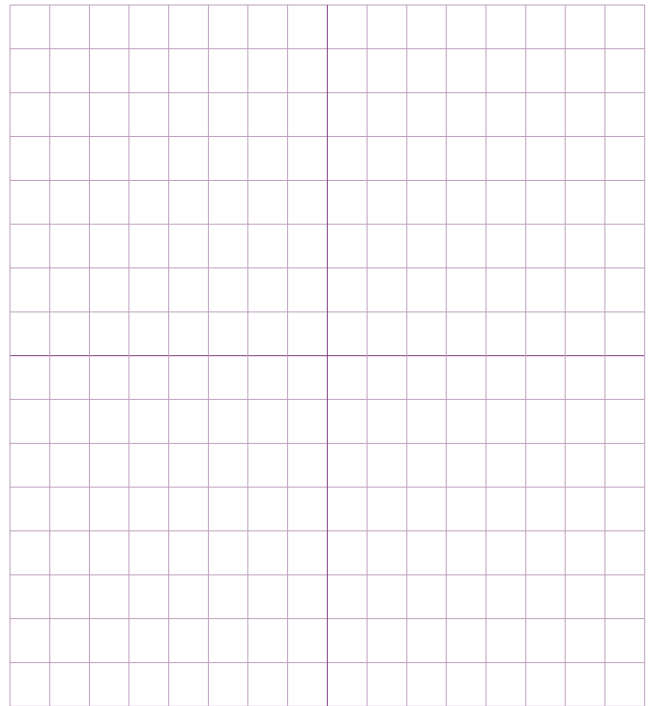
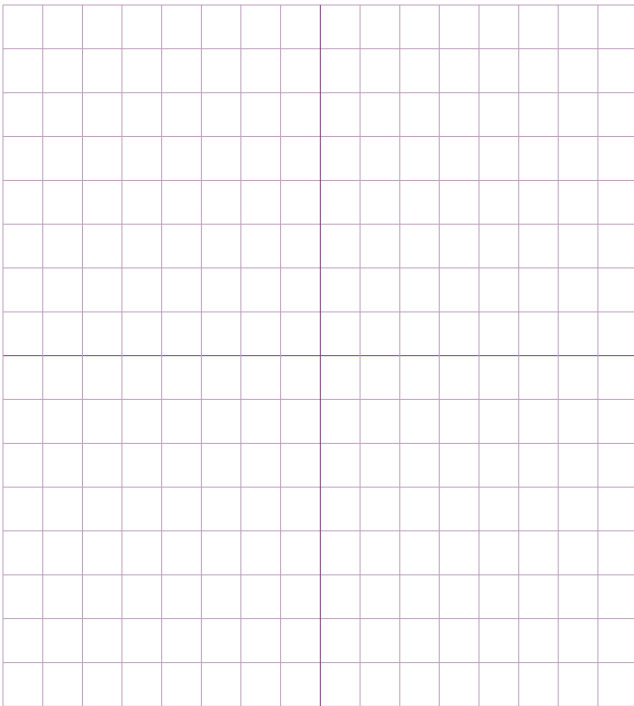
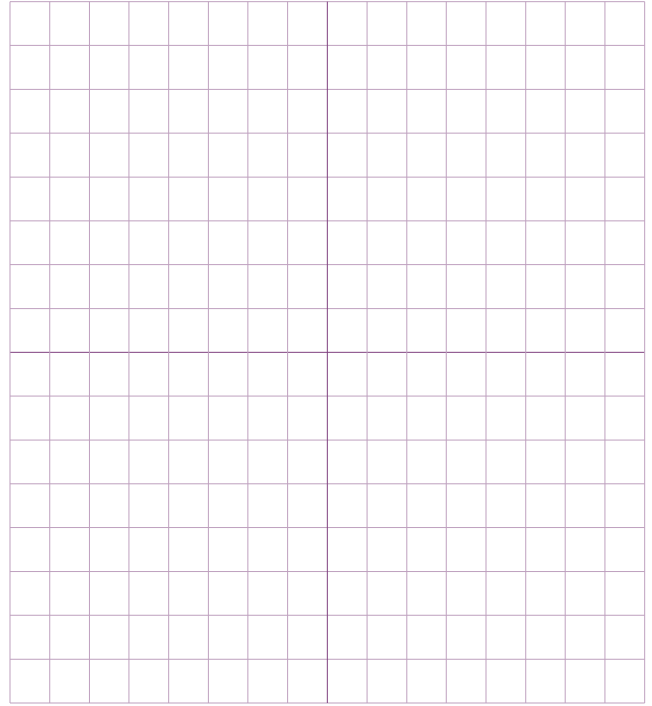
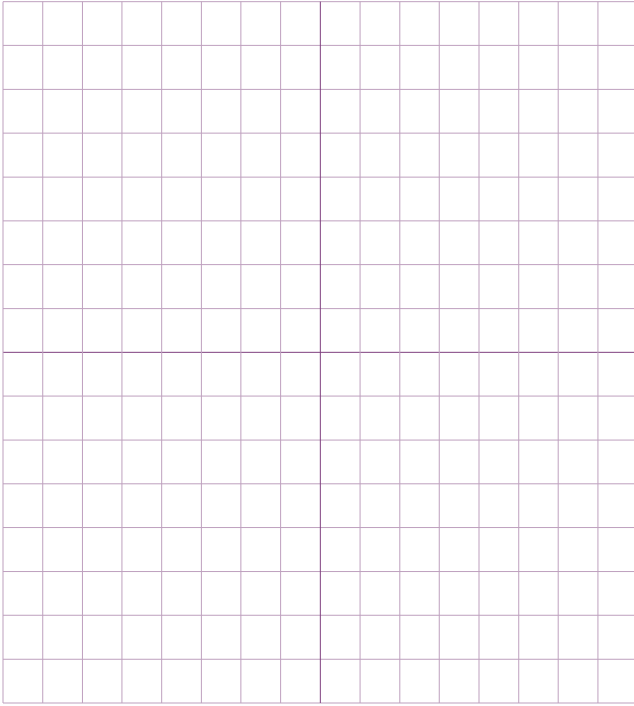
**(No marks will be given for work done on this page.)**



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# ROUGH WORK FOR GRAPHING

(No marks will be given for work done on this page.)



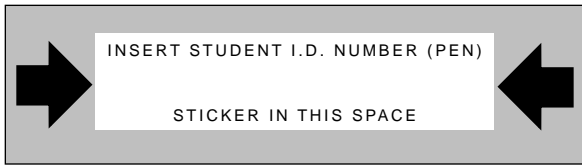


**ROUGH WORK FOR MULTIPLE-CHOICE**

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## ROUGH WORK FOR MULTIPLE-CHOICE





# **MATHEMATICS 12**

**January 1998**

Course Code = MA

FOR OFFICE USE ONLY

# MATHEMATICS 12

January 1998

Course Code = MA

Score for  
Question 1:

1.  $\frac{\quad}{(3)}$

Score for  
Question 7:

8.  $\frac{\quad}{(2)}$

Score for  
Question 2:

2.  $\frac{\quad}{(2)}$

Score for  
Question 3:

3.  $\frac{\quad}{(3)}$

Score for  
Question 4a:

4.  $\frac{\quad}{(2)}$

Score for  
Question 4b:

5.  $\frac{\quad}{(1)}$

Score for  
Question 5:

6.  $\frac{\quad}{(3)}$

Score for  
Question 6:

7.  $\frac{\quad}{(4)}$