

JUNE 1997

PROVINCIAL EXAMINATION

MINISTRY OF EDUCATION, SKILLS AND TRAINING

MATHEMATICS 12

GENERAL INSTRUCTIONS

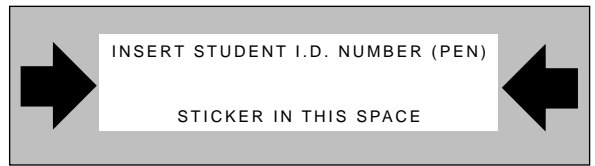
1. Insert the stickers with your Student I.D. Number (PEN) in the allotted spaces above.
Under no circumstance is your name or identification, other than your Student I.D. Number, to appear on this paper.
2. Take the separate Answer Sheet and follow the directions on its front page.
3. Be sure you have an **HB pencil** and an eraser for completing your Answer Sheet.
Follow the directions on the Answer Sheet when answering multiple-choice questions.
4. For each of the written-response questions, write your answer in the space provided.
5. When instructed to open this booklet, **check the numbering of the pages** to ensure that they are numbered in sequence from page one to the last page, which is identified by

END OF EXAMINATION .

6. At the end of the examination, place your Answer Sheet inside the front cover of this booklet and return the booklet and your Answer Sheet to the supervisor.

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FOR OFFICE USE ONLY



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MATHEMATICS 12 JUNE 1997 PROVINCIAL

Course Code = MA Examination Type = P

1. $\frac{\quad}{(2)}$

2. $\frac{\quad}{(3)}$

3. $\frac{\quad}{(2)}$

4. $\frac{\quad}{(1)}$

5. $\frac{\quad}{(3)}$

6. $\frac{\quad}{(3)}$

7. $\frac{\quad}{(2)}$

8. $\frac{\quad}{(4)}$

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MATHEMATICS 12 PROVINCIAL EXAMINATION

	Value	Suggested Time
1. This examination consists of two parts:		
PART A: 50 multiple-choice questions	50	75
PART B: 7 written-response questions 2 questions worth two marks each, 4 questions worth three marks each, and 1 question worth four marks.	20	45
	Total: 70 marks	120 minutes
2. The last three pages inside the back cover contain A Summary of Basic Identities and Formulae , Rough Work for Graphing , and Rough Work for Multiple-Choice . These pages may be detached for convenient reference prior to writing this examination.		
3. You will not be provided with any additional paper since rough-work space for the written-response questions has been incorporated into the space allowed for answering each question. You may not need all of the space provided to answer each question.		
4. An approved scientific calculator is essential for the examination. The calculator must be a hand-held device designed only for mathematical computations such as logarithmic and trigonometric functions. It can be programmable, but must not contain any graphing capabilities. You must not bring into the examination room any devices to support calculators such as manuals, printed or electronic cards, printers, memory expansion chips or cards, or keyboards.		
5. You are permitted to use rulers, compasses, and protractors.		
6. You have two hours to complete this examination.		

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PART A: MULTIPLE CHOICE

Value: 50 marks

Suggested Time: 75 minutes

INSTRUCTIONS: For each question, select the **best** answer and record your choice on the Answer Sheet provided. Using an HB pencil, completely fill in the circle that has the letter corresponding to your answer.

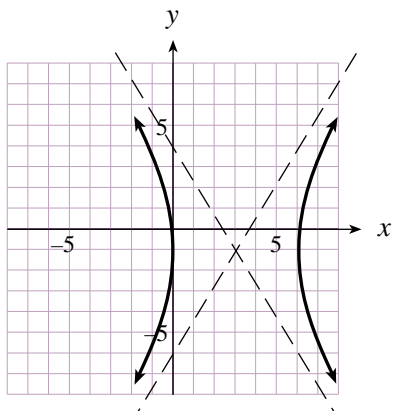
1. Which conic is represented by the equation $4x^2 - 2y^2 - x + y - 7 = 0$?
 - A. circle
 - B. ellipse
 - C. parabola
 - D. hyperbola

2. Determine the midpoint of the line segment joining A(-11, 10) and B(13, -6).
 - A. (-12, 8)
 - B. (1, 2)
 - C. $\left(\frac{3}{2}, 3\right)$
 - D. (2, 4)

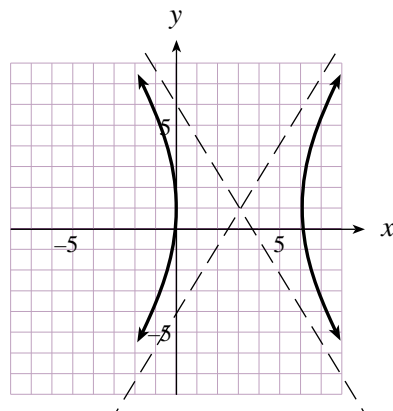
3. What is the vertex of the parabola $f(x) = -2(x+1)^2 + 3$?
 - A. (1, 3)
 - B. (-1, 3)
 - C. (1, -3)
 - D. (-1, -3)

4. Which graph best illustrates $\frac{(x-3)^2}{9} - \frac{(y+1)^2}{25} = 1$?

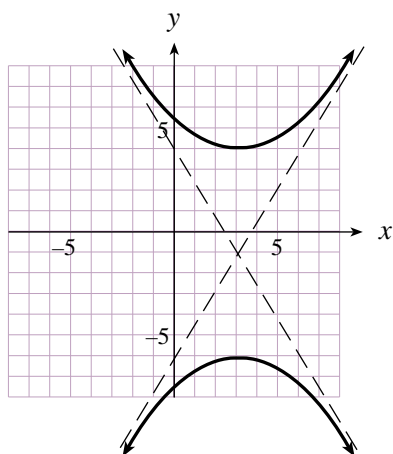
A.



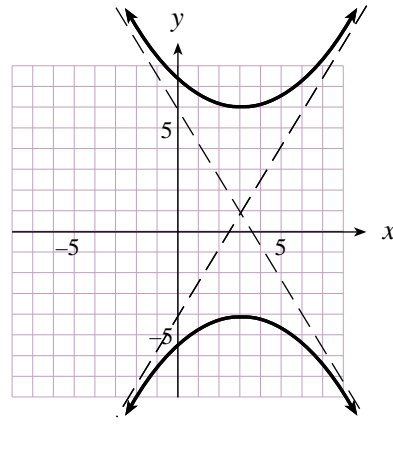
B.



C.



D.



5. Solve for x : $|3-4x|=4$

A. -2, 4

B. $-\frac{1}{4}, \frac{7}{4}$

C. $\frac{1}{4}, \frac{7}{4}$

D. 2, 4

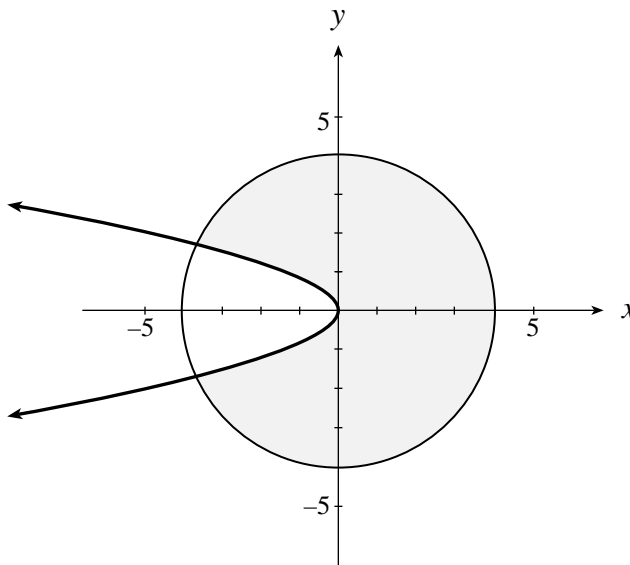
6. Solve the following system for x only.

$$2x^2 - y^2 = 1$$

$$2x^2 + 3y^2 = 13$$

- A. $\pm\sqrt{2}$
- B. $\pm\frac{\sqrt{10}}{2}$
- C. $\pm\sqrt{3}$
- D. $\pm\sqrt{7}$

7. Which system describes the shaded region in the diagram below?



- | | |
|--|--|
| A. $x^2 + y^2 \leq 16$
$x \geq -2y^2$ | B. $x^2 + y^2 \geq 16$
$x \geq -2y^2$ |
| C. $x^2 + y^2 \geq 16$
$x \leq -2y^2$ | D. $x^2 + y^2 \leq 16$
$x \leq -2y^2$ |

8. Determine the range of $\frac{(x-3)^2}{16} + \frac{(y+2)^2}{25} = 1$.

- A. $-7 \leq y \leq 3$
- B. $-3 \leq y \leq 7$
- C. $-7 \leq x \leq 1$
- D. $-1 \leq x \leq 7$

9. Determine an equation of the line tangent to the circle $(x-5)^2 + (y+12)^2 = 169$ at the point $(0, 0)$.

- A. $y = \frac{5}{12}x$
- B. $y = \frac{12}{5}x$
- C. $y = -\frac{5}{12}x$
- D. $y = -\frac{12}{5}x$

10. For what values of m does the following system have no real solutions?

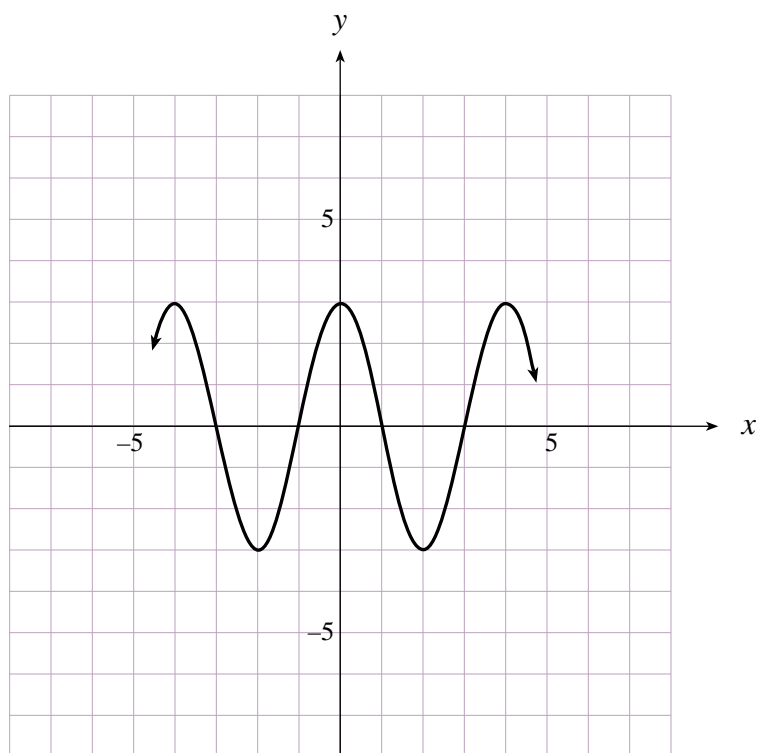
$$\frac{x^2}{4} - \frac{y^2}{16} = 1$$
$$y = mx$$

- A. $-\frac{1}{2} \leq m \leq \frac{1}{2}$
- B. $m \leq -\frac{1}{2}$ or $m \geq \frac{1}{2}$
- C. $-2 \leq m \leq 2$
- D. $m \leq -2$ or $m \geq 2$

11. Convert $\frac{5\pi}{6}$ radians to degrees.

- A. 108°
- B. 150°
- C. 216°
- D. 300°

12. Determine the period of the trigonometric function graphed below.



- A. 2
- B. 3
- C. 4
- D. 6

13. Evaluate $\sec 0.156$ to 3 decimal places.

- A. 0.992
- B. 1.012
- C. 1.414
- D. 6.436

14. Simplify: $\frac{2 \tan x}{\cos^2 x + \sin^2 x + \tan^2 x}$

- A. $2 \sin x$
- B. $\sin 2x$
- C. $\tan 2x$
- D. $2 \cot x$

15. Solve: $2 \cos^2 x - 5 \cos x + 2 = 0$, $0 \leq x < 2\pi$ (Accurate to 2 decimal places.)

- A. 0.00
- B. 1.05, 5.24
- C. 2.09, 4.19
- D. 1.05, 2.09, 4.19, 5.24

16. How many solutions does $\cos 3x = -1$ have over the interval $0 \leq x < 2\pi$?

- A. 1
- B. 2
- C. 3
- D. 6

17. Which expression is equivalent to $4 \sin 6\theta \cos 6\theta$?

- A. $\sin 6\theta$
- B. $\sin 12\theta$
- C. $2 \sin 3\theta$
- D. $2 \sin 12\theta$

18. Given two functions, $f(x) = \sin\left(x - \frac{\pi}{4}\right)$ and $g(x) = \cos(x - a)$, determine the smallest positive value for a so that the graphs are identical.

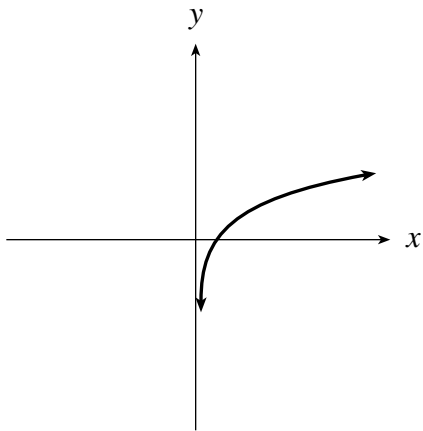
- A. $\frac{\pi}{4}$
- B. $\frac{\pi}{2}$
- C. $\frac{3\pi}{4}$
- D. $\frac{5\pi}{4}$

19. Change $\log_a b = c$ to exponential form.

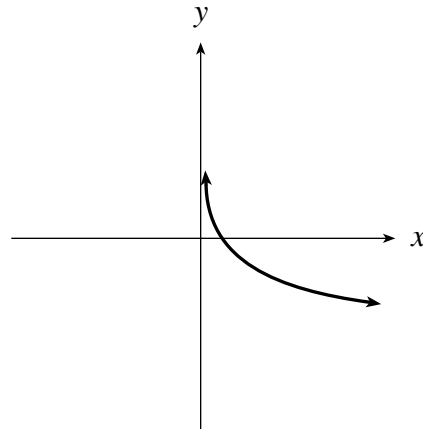
- A. $b = a^c$
- B. $b = c^a$
- C. $c = a^b$
- D. $c = b^a$

20. Select the graph of $y = \left(\frac{1}{2}\right)^x$.

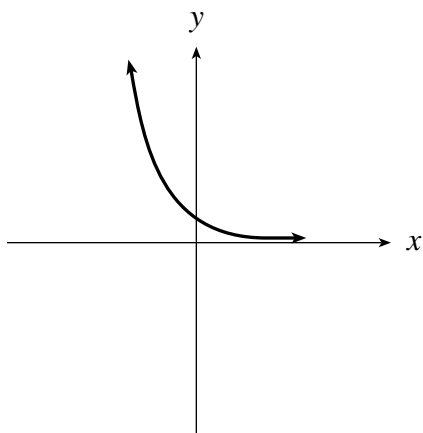
A.



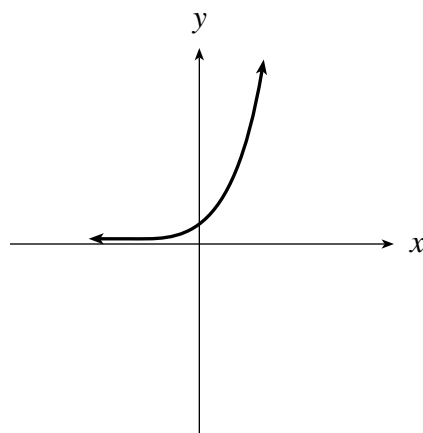
B.



C.



D.



21. Evaluate: $\log_8 16$

A. $\frac{1}{2}$

B. $\frac{3}{4}$

C. $\frac{4}{3}$

D. 2

22. Determine the inverse of $f(x) = x - 2$.

A. $f^{-1}(x) = x + 2$

B. $f^{-1}(x) = \frac{1}{x} - \frac{1}{2}$

C. $f^{-1}(x) = -\frac{x}{2}$

D. $f^{-1}(x) = \frac{1}{x-2}$

23. Solve: $2^{\log x} = \frac{1}{4}$

A. -2

B. $\frac{1}{100}$

C. 2

D. 100

24. If $\log_a b = 0.5$, evaluate $\log_a \sqrt{b} + \log_a \left(\frac{a}{b}\right)$.

A. 0.5

B. 0.75

C. 1

D. 1.5

25. Simplify: $\frac{1}{\log_a x} + \frac{1}{\log_b x}$

A. $-\log_{ab} x$

B. $-\log_x ab$

C. $\log_{ab} x$

D. $\log_x ab$

26. According to the Rational Root Theorem, list all possible rational roots of $2x^8 - 5x^3 + 6x^2 - 4 = 0$.

A. $\pm 1, \pm 2, \pm 4$

B. $\pm \frac{1}{2}, \pm 1, \pm 2, \pm 4$

C. $\pm \frac{1}{4}, \pm \frac{1}{2}, \pm 1, \pm 2$

D. $\pm \frac{1}{2}, \pm 1, \pm 2, \pm 4, \pm 8$

27. Determine the remainder when $6x^3 - 11x^2 + 14x - 5$ is divided by $2x^2 - 7x + 3$.

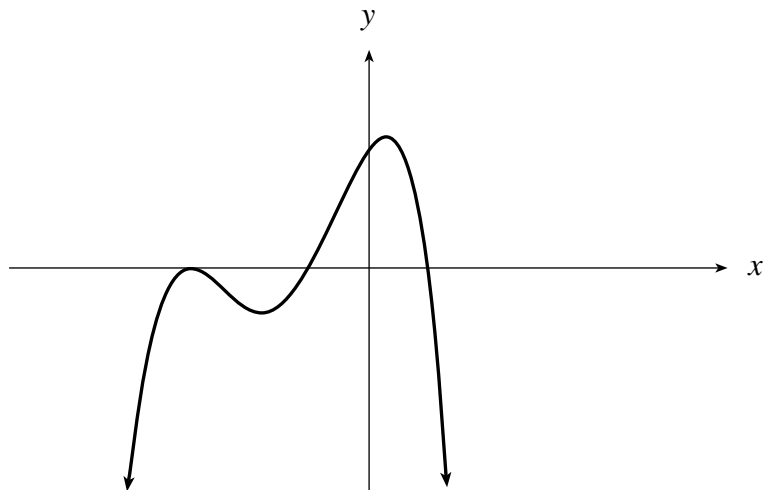
A. $-107x - 53$

B. $-107x + 43$

C. $40x - 20$

D. 20

28. The following graph represents the polynomial function $y = Ax^4 + Bx^3 + Cx^2 + Dx + E$.
What conditions must be satisfied by A and E ?



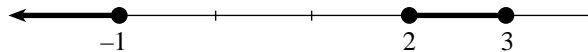
A. $A < 0$ and $E < 0$

B. $A < 0$ and $E > 0$

C. $A > 0$ and $E < 0$

D. $A > 0$ and $E > 0$

29. Which polynomial inequality describes the solution shown?



- A. $(x-1)(x+2)(x+3) \geq 0$
- B. $(x-1)(x+2)(x+3) \leq 0$
- C. $(x+1)(x-2)(x-3) \geq 0$
- D. $(x+1)(x-2)(x-3) \leq 0$

30. For the polynomial function $p(x) = ax^3 + bx - 3$, $p(-1) = 4$. Determine the value of $p(1)$.

- A. -10
- B. -4
- C. 4
- D. 10

31. Which of the following is an arithmetic sequence?

- A. -1, 2, 5, 8
- B. 1, 0, -1, 0
- C. -1, -2, -4, -8
- D. $1, \sqrt{2}, 2, 2\sqrt{2}$

32. Determine a single geometric mean between 1 and 4.

- A. 1.5
- B. 2
- C. 2.5
- D. 3

33. What are the first 3 terms of the sequence defined by the formula below if $n > 1$ and $t_1 = 5$?

$$t_n = 5t_{n-1} - 2$$

- A. 5, 23, 113
- B. 5, 3, 1
- C. 5, 15, 35
- D. 5, 35, 245

34. Find the sum of the infinite geometric series $2 - 1 + \frac{1}{2} - \frac{1}{4} + \dots$

- A. 0
- B. $\frac{5}{4}$
- C. $\frac{4}{3}$
- D. 4

35. In an arithmetic sequence, $t_4 = 41$ and $t_{12} = -7$. Determine the value of t_1 .

- A. 23
- B. 53
- C. 53.75
- D. 59

36. Determine the 20th term of the geometric sequence 2, 2.5, ... (Accurate to 2 decimal places.)

- A. 11.50
- B. 111.02
- C. 138.78
- D. 173.47

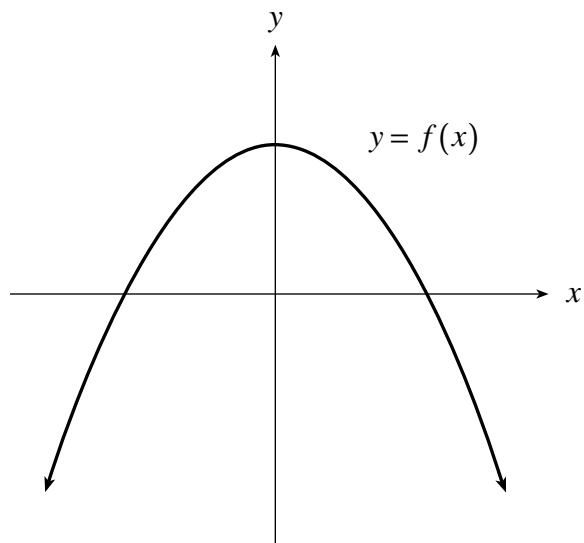
37. Evaluate: $\sum_{k=4}^{15} (3k - 15)$

- A. 148.5
- B. 162
- C. 177
- D. 202.5

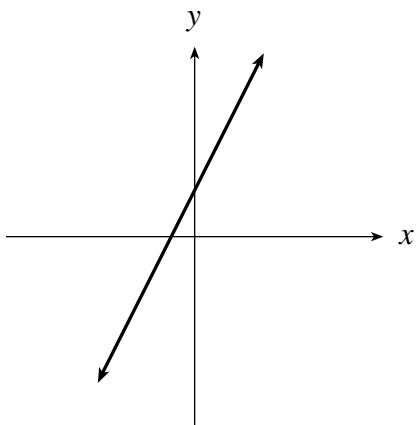
38. Determine the sum of the first 12 terms of the series $\log_b 1 + \log_b 10 + \log_b 100 + \dots$
- A. $\frac{66}{\log b}$
 - B. $\frac{72}{\log b}$
 - C. 66
 - D. 72
39. If $y = 2x^3 + 3x$, find $\frac{dy}{dx}$.
- A. $5x^2 + 3$
 - B. $6x^2 + 3$
 - C. $5x^2 + 3x$
 - D. $6x^2 + 3x$
40. Evaluate: $\lim_{n \rightarrow \infty} \frac{n-1}{n}$
- A. -1
 - B. 0
 - C. 1
 - D. limit does not exist (no finite limit)
41. Evaluate: $\lim_{x \rightarrow 4} \frac{x^2 - 2x - 8}{x^2 - 6x + 8}$
- A. 0
 - B. 1
 - C. 3
 - D. limit does not exist (no finite limit)

42. Given the function $f(x) = x^2 - 4x$, determine the slope of the secant line intersecting the graph of f at the points where $x = 2$ and $x = 5$.
- A. 0
 - B. 2
 - C. 3
 - D. 6
43. Determine the minimum value of the function $g(x) = 2x^2 - 12x + 25$.
- A. 0
 - B. 3
 - C. 7
 - D. 25
44. Find an equation of the line tangent to the graph of $y = x^3 - 3x^2 + 3x + 2$ at $(0, 2)$.
- A. $y = -3x + 2$
 - B. $y = -2x + 2$
 - C. $y = 2x + 2$
 - D. $y = 3x + 2$

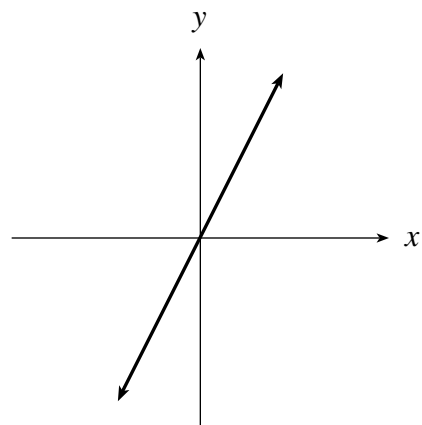
45. Given the graph of $y = f(x)$ below, which of the following **best** represents the graph of $y = f'(x)$?



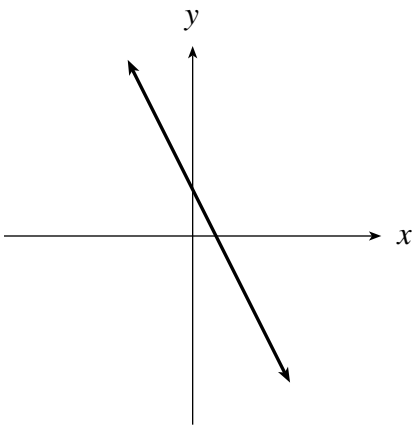
A.



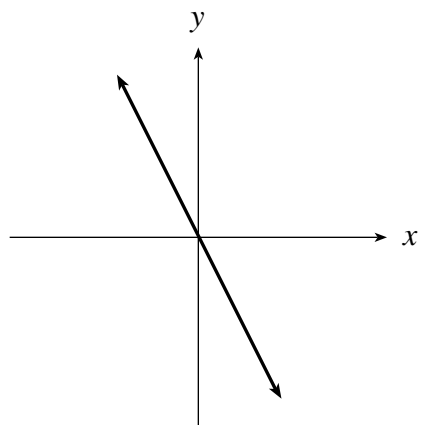
B.



C.



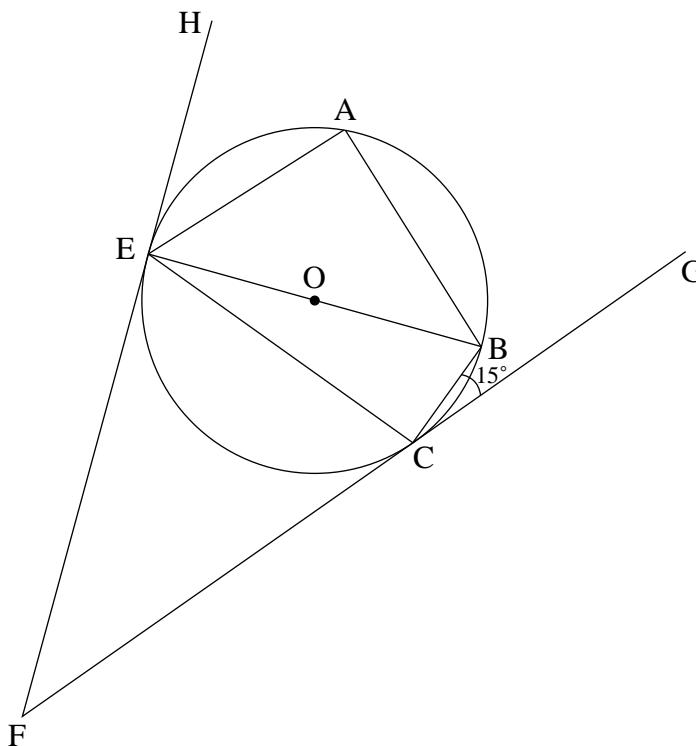
D.



Use the following diagram to answer questions 46 and 47.

Diagram is not drawn to scale.

Given: Circle with centre O
 FG is tangent at C
 FH is tangent at E
 $\angle BCG = 15^\circ$
 $AE = AB$



46. Determine the measure of $\angle ABC$.

- A. 60°
- B. 105°
- C. 120°
- D. 165°

47. Determine the measure of $\angle CFE$.

- A. 15°
- B. 30°
- C. 45°
- D. 60°

48. A basketball court is made up of square tiles, laid side by side to form a rectangle 40 tiles by 30 tiles. If a straight line is drawn diagonally from one corner of the floor to the opposite corner, how many tiles would the diagonal intersect?
- A. 45
 - B. 50
 - C. 55
 - D. 60
49. To the nearest power of 10, determine the difference between 10^{600} and 10^{200} .
- A. 10^3
 - B. 10^{200}
 - C. 10^{400}
 - D. 10^{600}
50. Given $\log 6 = x$ and $\log 8 = y$, determine an expression for $\log 3$ in terms of x and y .
- A. $\frac{x}{3} - y$
 - B. $x - \frac{y}{3}$
 - C. $\frac{y}{3} - x$
 - D. $y - \frac{x}{3}$

**This is the end of the multiple-choice section.
Answer the remaining questions directly in this examination booklet.**

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PART B: WRITTEN RESPONSE

Value: 20 marks

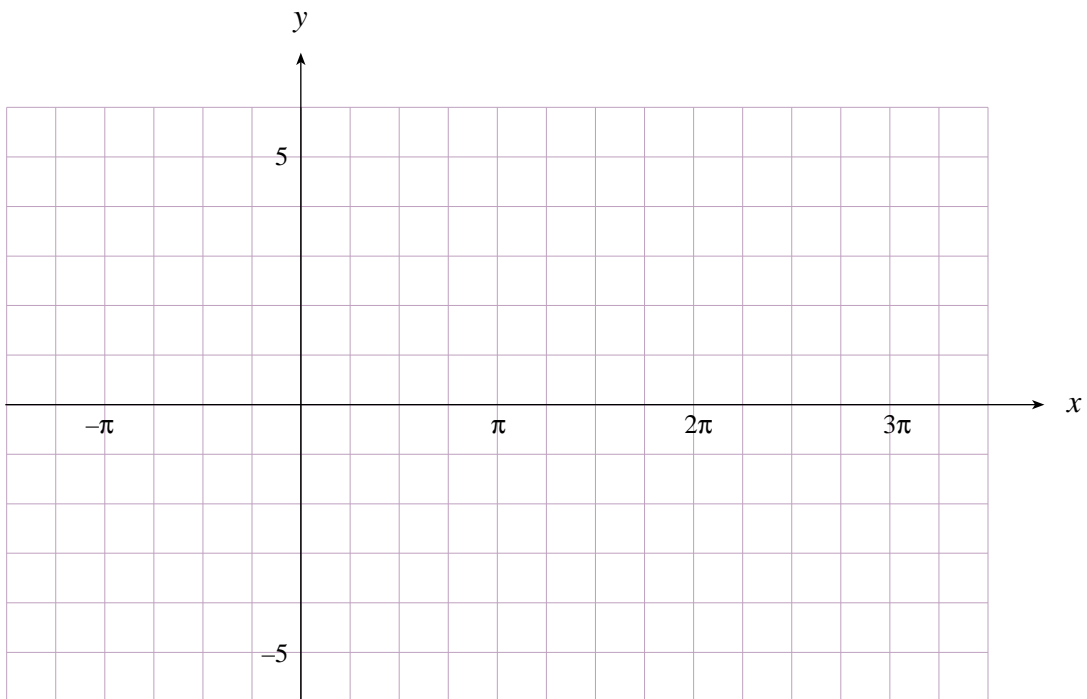
Suggested Time: 45 minutes

INSTRUCTIONS: Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

Full marks will NOT be given for the final answer only.

1. Graph at least one period of $y = -3 \sin\left(x - \frac{3\pi}{4}\right)$.

(2 marks)



Score for
Question 1:

1. $\frac{\quad}{(2)}$

OVER

2. Solve for x : $2 \log(4 - x) - \log 3 = \log(10 - x)$

(3 marks)

ANSWER:

Score for
Question 2:

2.
(3)

OVER

3. A particle moves along the x -axis so that its position at time t is $x = 4t^3 - 21t^2 + 30t$, where t is measured in seconds, and x is measured in metres.

a) Determine the time(s) when the particle is stopped.

(2 marks)

ANSWER:	Score for Question 3a: 3. <u> </u> (2)
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b) Determine when the particle is moving to the left.

(1 mark)

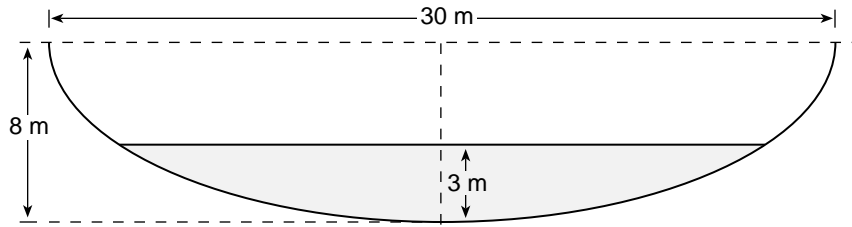
ANSWER:

Score for
Question 3b:

4.
(1)

OVER

4. The cross section of a drainage canal has the shape of a semi-ellipse and measures 30 m across and 8 m deep at its deepest point. Presently the maximum depth of water in the canal is 3 m. Calculate the width of the water surface. (Accurate to at least 2 decimal places.) **(3 marks)**



ANSWER:

Score for
Question 4:

5.
(3)

OVER

5. If -2 is a root of $2x^3 + kx^2 - 11x + 6 = 0$, determine the other two roots.

(3 marks)

ANSWER:

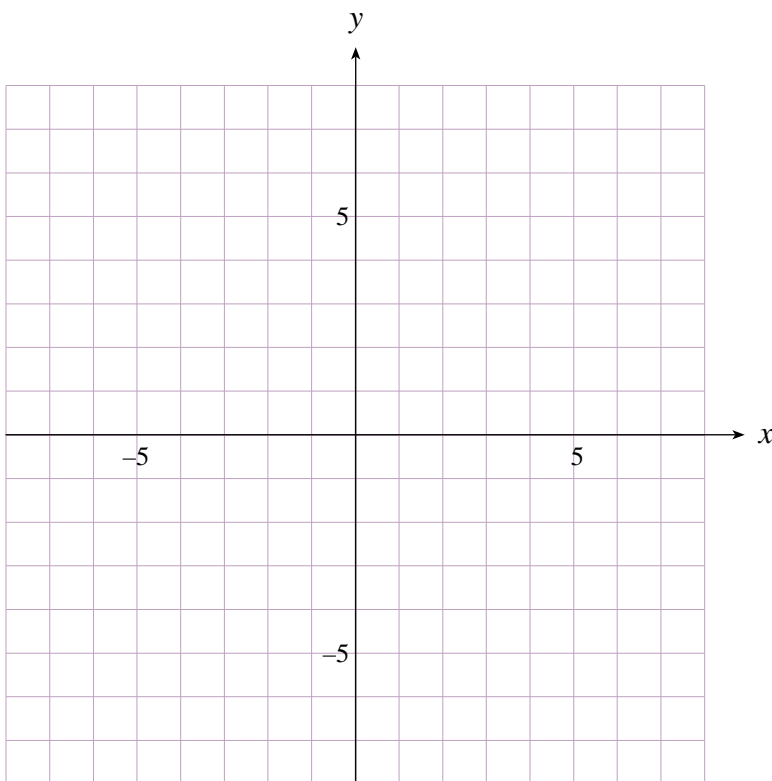
Score for
Question 5:

6.
(3)

OVER

6. Two functions are defined by the equations $f(t) = t^2 - 4t - 6$ and $g(t) = t^2 + 2t - 5$. Graph the region defined by the following inequality. **(2 marks)**

$$f(x) + g(y) \leq 0$$



Score for
Question 6:

7.
(2)

OVER

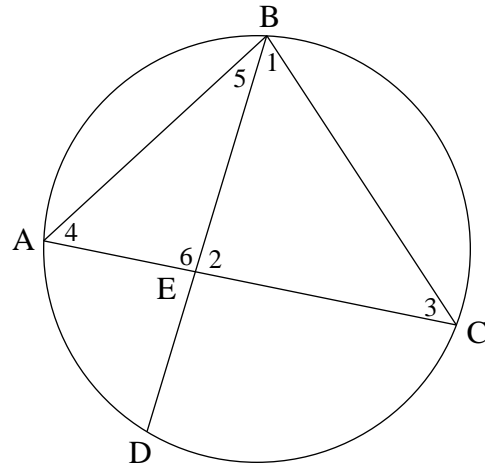
7. Complete the proof.

(4 marks)

Given: $BE = EC$

Prove: $AE = DE$

Note: Students are encouraged to number angles.



Proof	
Statement	Reason

Score for
Question 7:

8.
(4)

END OF EXAMINATION

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A SUMMARY OF BASIC IDENTITIES AND FORMULAE

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Reciprocal and Quotient Identities

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Addition Identities

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

Double-Angle Identities

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - 2 \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Formulae

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t_n = a + (n-1)d$$

$$t_n = ar^{n-1}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$S_n = \frac{n}{2}(a+1)$$

$$S_n = \frac{a-1r}{1-r}$$

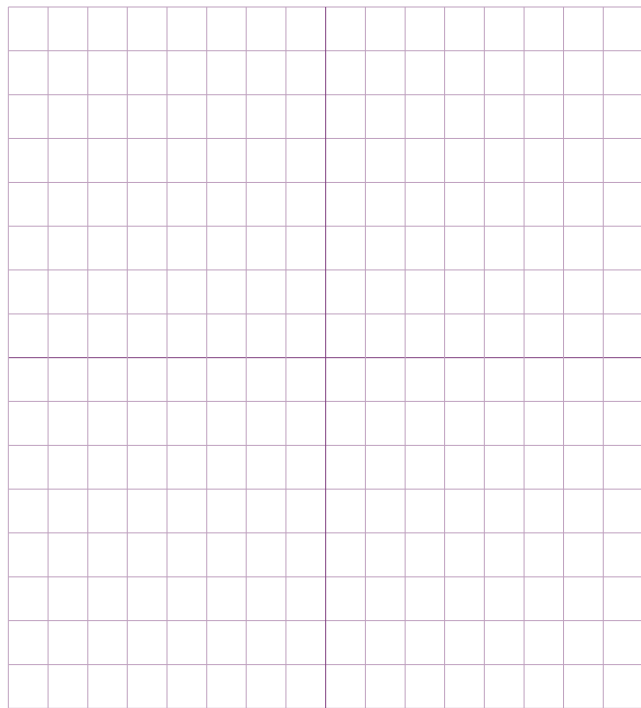
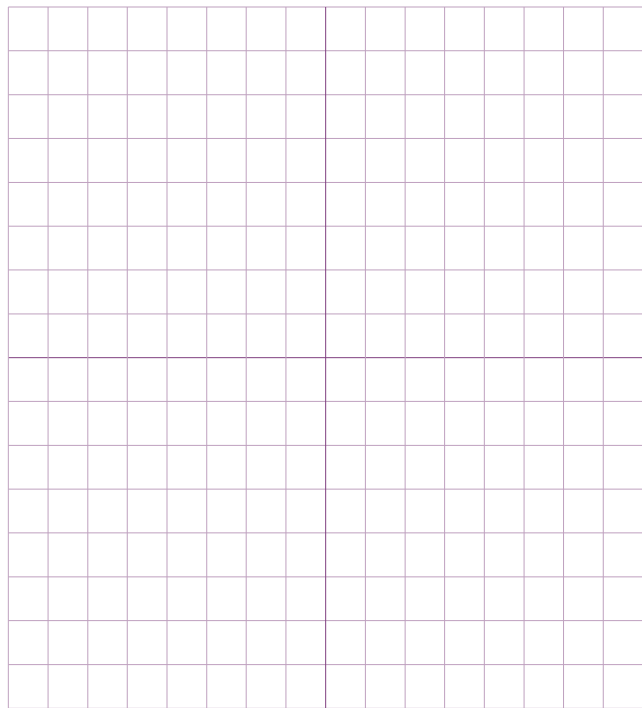
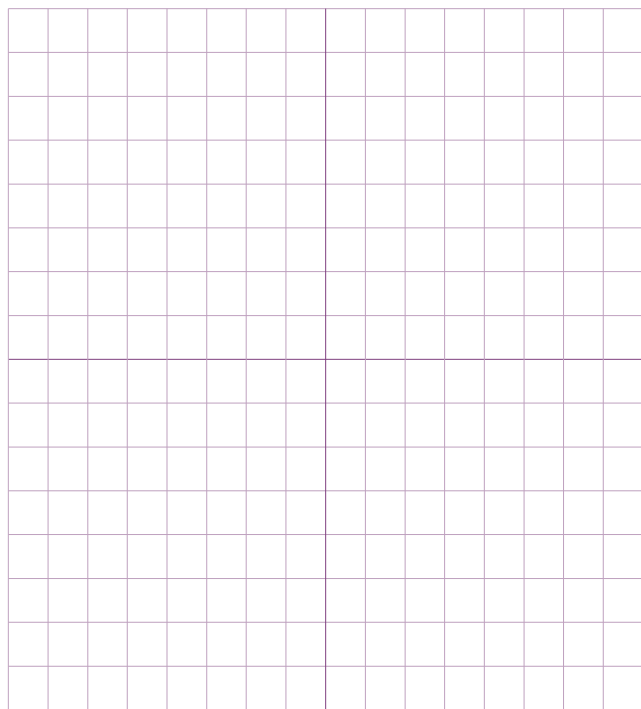
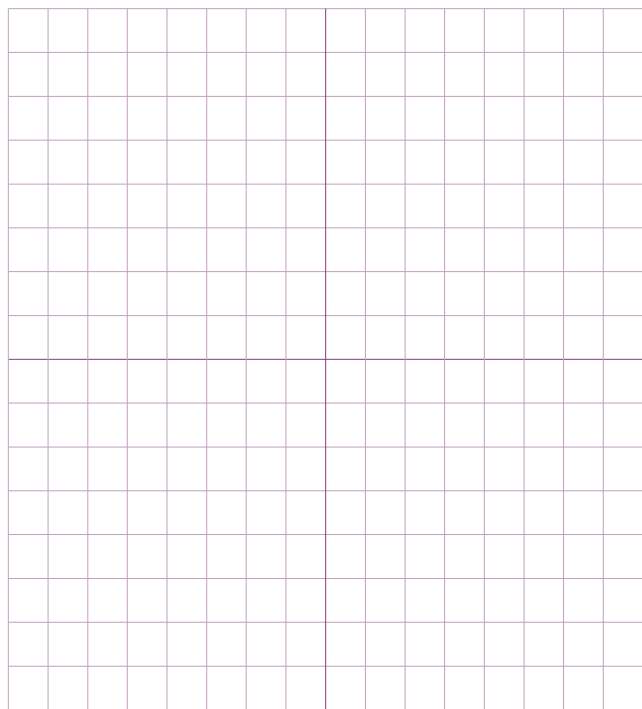
$$S = \frac{a}{1-r}$$

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ROUGH WORK FOR GRAPHING

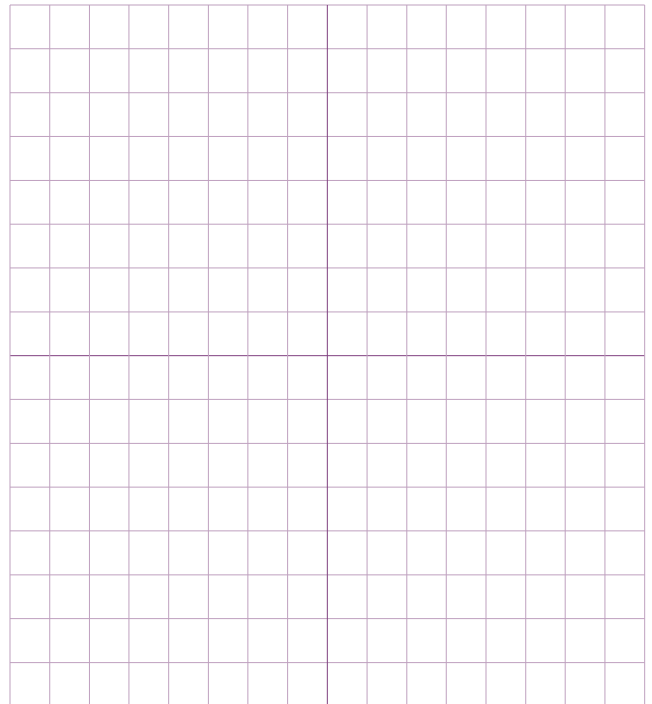
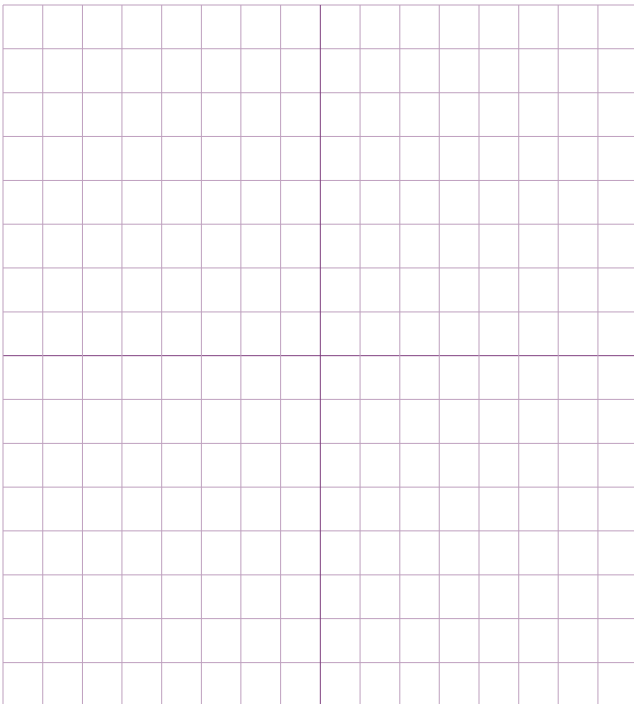
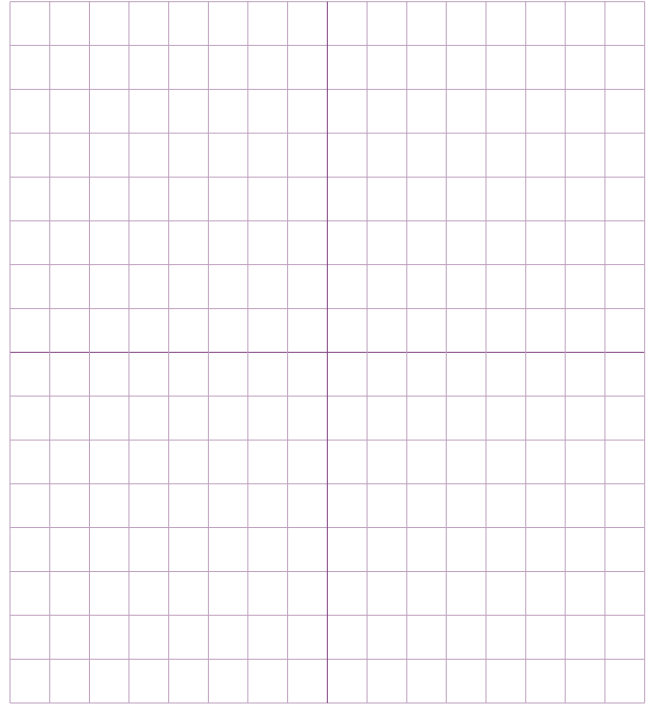
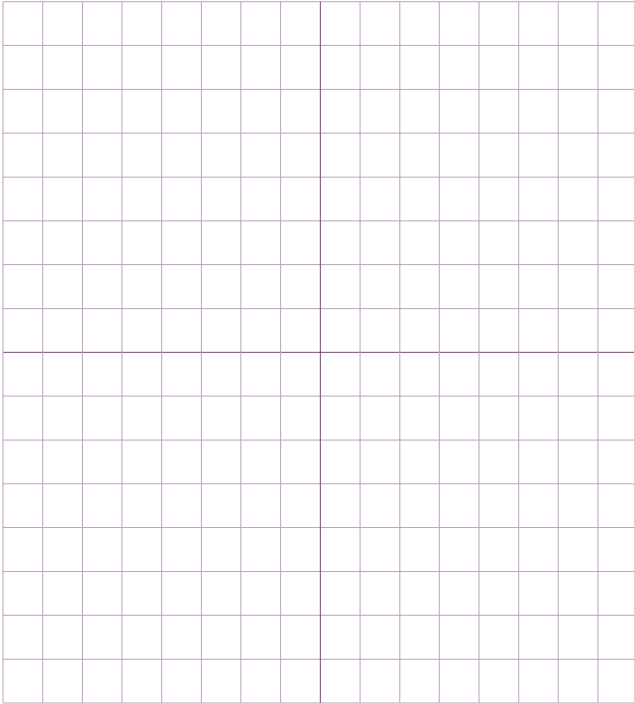
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ROUGH WORK FOR GRAPHING

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ROUGH WORK FOR MULTIPLE-CHOICE

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ROUGH WORK FOR MULTIPLE-CHOICE