INSERT STUDENT I.D. NUMBER (PEN)

STICKER IN THIS SPACE

JANUARY 1997

PROVINCIAL EXAMINATION

MINISTRY OF EDUCATION, SKILLS AND TRAINING

MATHEMATICS 12

GENERAL INSTRUCTIONS

1. Insert the stickers with your Student I.D. Number (PEN) in the allotted spaces above. **Under no circumstance is your name or identification, other than your Student I.D. Number, to appear on this paper.**

2. Take the separate Answer Sheet and follow the directions on its front page.

3. Be sure you have an **HB pencil** and an eraser for completing your Answer Sheet. Follow the directions on the Answer Sheet when answering multiple-choice questions.

4. For each of the written-response questions, write your answer in the space provided.

5. When instructed to open this booklet, **check the numbering of the pages** to ensure that they are numbered in sequence from page one to the last page, which is identified by

   END OF EXAMINATION .

6. At the end of the examination, place your Answer Sheet inside the front cover of this booklet and return the booklet and your Answer Sheet to the supervisor.

THIS PAGE INTENTIONALLY BLANK

FOR OFFICE USE ONLY

INSERT STUDENT I.D. NUMBER (PEN)

STICKER IN THIS SPACE

_____ – _____

MATHEMATICS 12 JANUARY 1997 PROVINCIAL

Course Code = MA Examination Type = P

1. _____
(2)

2. _____
(3)

3. _____
(3)

4. _____
(3)

5. _____
(2)

6. _____
(4)

7. _____
(3)

THIS PAGE INTENTIONALLY BLANK

MATHEMATICS 12 PROVINCIAL EXAMINATION

		Value	Suggested Time
1.	This examination consists of **two** parts:		
PART A:	50 multiple-choice questions	50	75
PART B:	7 written-response questions 2 questions worth **two** marks each, 4 questions worth **three** marks each, and 1 question worth **four** marks.	20	45
	Total:	**70 marks**	**120 minutes**

2. The last **three** pages inside the back cover contain **A Summary of Basic Identities and Formulae**, **Rough Work for Graphing**, and **Rough Work for Multiple-Choice**. These pages may be detached for convenient reference prior to writing this examination.

3. You will not be provided with any additional paper since rough-work space for the written-response questions has been incorporated into the space allowed for answering each question. You may not need all of the space provided to answer each question.

4. An approved scientific calculator is essential for the examination. The calculator must be a hand-held device designed **only** for mathematical computations such as logarithmic and trigonometric functions. It **can be** programmable, but **must not** contain any graphing capabilities. You **must not** bring into the examination room any devices to support calculators such as manuals, printed or electronic cards, printers, memory expansion chips or cards, or keyboards.

5. You are permitted to use rulers, compasses, and protractors.

6. You have **two hours** to complete this examination.

THIS PAGE INTENTIONALLY BLANK

PART A: MULTIPLE CHOICE

Value: 50 marks **Suggested Time: 75 minutes**

INSTRUCTIONS: For each question, select the **best** answer and record your choice on the Answer Sheet provided. Using an HB pencil, completely fill in the circle that has the letter corresponding to your answer.

1. Determine an equation of the circle with centre $(3, -2)$ and radius 4.

   A. $(x-3)^2+(y+2)^2=4$
   B. $(x+3)^2+(y-2)^2=4$
   C. $(x+3)^2+(y-2)^2=16$
   D. $(x-3)^2+(y+2)^2=16$

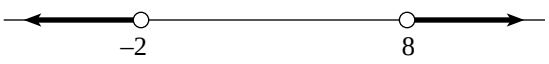
2. Find the midpoint of the line segment joining $P(-8, 4)$ and $Q(12, -20)$.

   A. $(-10, 12)$
   B. $(-2, 8)$
   C. $(2, -8)$
   D. $(10, -12)$

3. Which conic is represented by the equation $4x^2-4y^2+8x-24y-9=0$?

   A. circle
   B. ellipse
   C. parabola
   D. hyperbola

4. Which absolute value inequality describes the solution shown?



   A. $|x-5|<3$
   B. $|x-3|<5$
   C. $|x-5|>3$
   D. $|x-3|>5$

OVER

5. Determine an equation of a rectangular hyperbola with centre at $(-2,\ 0)$ and one vertex at $(4,\ 0)$.

A. $(x-2)^2 - y^2 = 16$

B. $(x+2)^2 - y^2 = 16$

C. $(x-2)^2 - y^2 = 36$

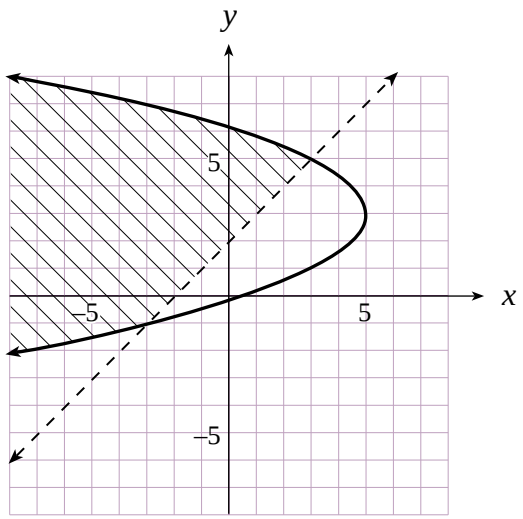
D. $(x+2)^2 - y^2 = 36$

6. Which graph represents the solution of the following system of inequalities?

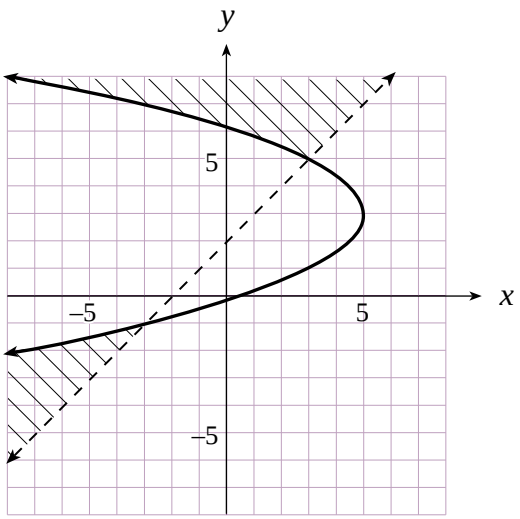
$$x \geq -\frac{1}{2}(y-3)^2 + 5$$

$$y > x + 2$$

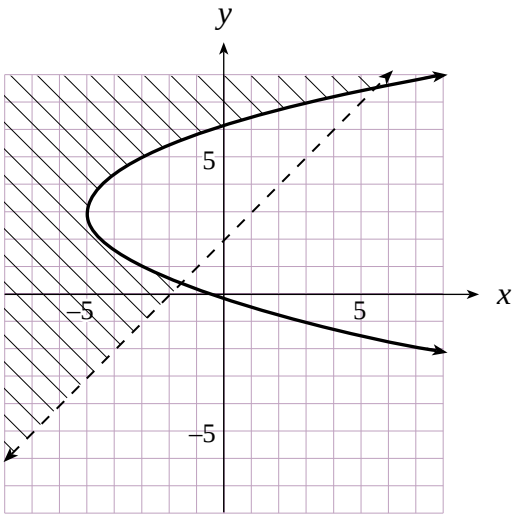
A.



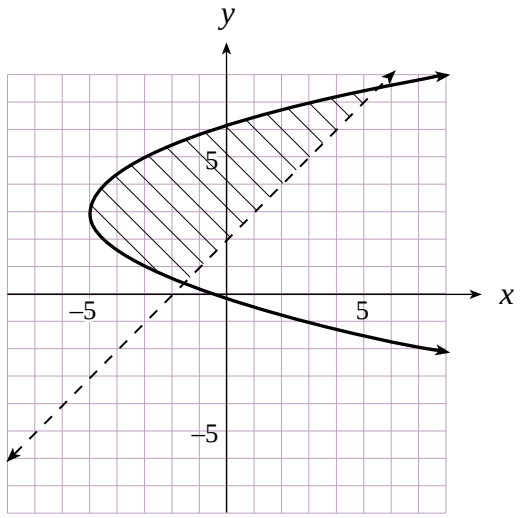
B.



C.



D.



7. Change the following equation to standard form.

$$2x^2 + y^2 - 12x - 10 = 0$$

A. $\frac{(x+3)^2}{4} + \frac{y^2}{8} = 1$

B. $\frac{(x-3)^2}{4} + \frac{y^2}{8} = 1$

C. $\frac{(x+3)^2}{14} + \frac{y^2}{28} = 1$

D. $\frac{(x-3)^2}{14} + \frac{y^2}{28} = 1$

8. Find all real solutions of the following system.

$$x - 2y = 0$$

$$x^2 - y^2 = 48$$

A. $(8,\ 4)$

B. $(8,\ 4), (-8,\ -4)$

C. $(8,\ 4), (-8,\ 4)$

D. $(8,\ 4), (-8,\ 4), (8,\ -4), (-8,\ -4)$

9. Determine the value of k $(k > 0)$ so that the conjugate axis of the hyperbola $x^2 - \frac{y^2}{k} = 1$ is 2 units longer than the minor axis of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

A. 8

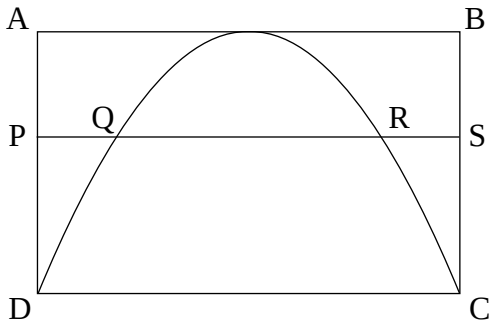
B. 10

C. 16

D. 25

OVER

10. A parabola is drawn within rectangle $ABCD$ with its vertex at the midpoint of AB. PS is parallel to AB. If $AB = 80$, $BC = 60$ and $AP = 20$, determine the length of QR. (Accurate to 1 decimal place.)



A. 45.6
B. 46.2
C. 48.3
D. 49.7

11. Convert $256°$ to radians. (Accurate to 2 decimal places.)

A. 2.23
B. 3.39
C. 4.47
D. 8.93

12. Determine the period of the function $f(x) = 3\sin 4x + 1$.

A. $\frac{\pi}{2}$
B. $\frac{2\pi}{3}$
C. 6π
D. 8π

13. If the point $(-4,\ 2)$ lies on the terminal arm of an angle θ in standard position, determine the exact value of $\csc\theta$.

A. $-\sqrt{5}$

B. $-\frac{\sqrt{5}}{2}$

C. $\frac{\sqrt{5}}{2}$

D. $\sqrt{5}$

14. Which expression is equivalent to $\cot\theta + \tan\theta$?

A. $\frac{1}{\sin\theta\cos\theta}$

B. $\frac{\cos\theta + \sin\theta}{\sin\theta\cos\theta}$

C. 1

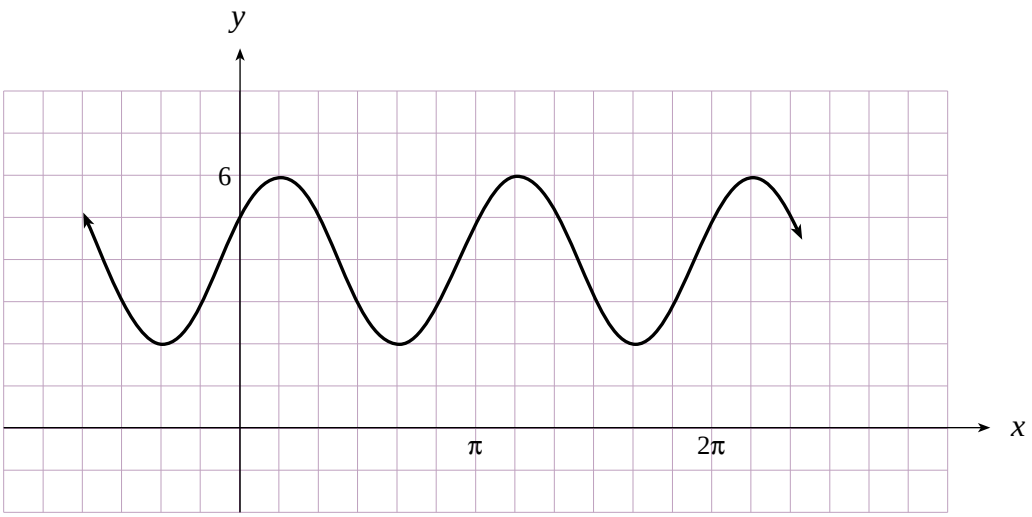
D. 2

15. Solve: $2\cot x + 3 = 0$, where $0 \le x < 2\pi$ (Accurate to 2 decimal places.)

A. 0.59 , 3.73
B. 0.98 , 4.12
C. 2.16 , 5.30
D. 2.55 , 5.70

OVER

16. Which equation describes the following graph?



A. $y = 2\cos 2\left(x - \frac{\pi}{6}\right) + 4$

B. $y = 2\cos 2\left(x + \frac{\pi}{6}\right) + 4$

C. $y = 4\cos\left(x + \frac{\pi}{6}\right) + 2$

D. $y = 4\cos\left(x - \frac{\pi}{6}\right) + 2$

17. Which expression is equivalent to $\frac{1 - \cos 2\theta}{\sin 2\theta}$?

A. $\tan\theta$

B. $\cot\theta$

C. $-\tan\theta$

D. $-\cot\theta$

18. How many solutions does the following equation have over the interval $0° \le \theta < 360°$?

$$(2\sin\theta + 5)(3\cos\theta + 3)\left(\tan^2\theta - 2\right) = 0$$

A. 4

B. 5

C. 6

D. 7

19. Change $r^s = t$ to logarithmic form.

A. $r = \log_s t$
B. $s = \log_t r$
C. $t = \log_r s$
D. $s = \log_r t$

20. Evaluate: $\log_4 135$ (Accurate to 2 decimal places.)

A. 0.53
B. 1.53
C. 2.13
D. 3.54

21. Determine the range of the function $y = 3^x + 2$.

A. $y > -2$
B. $y > 2$
C. $y > 0$
D. all real numbers

22. Which expression is equivalent to $3\log a + \log b - \frac{1}{2}\log c$?

A. $\log\left(\frac{6ab}{c}\right)$
B. $\log\left(\frac{a^3 b}{\sqrt{c}}\right)$
C. $\log\left(3a + b - \frac{1}{2}c\right)$
D. $\log\left(\frac{a^3 + b}{\sqrt{c}}\right)$

OVER

23. Solve: $\log_2 x - \log_2 4 = 3$

A. 2
B. 12
C. 32
D. 36

24. If $\log_9 5 = x$ and $\log_{27} 2 = y$, express $\log_3 100$ in terms of x and y.

A. $2\left(9^x + 27^y\right)$
B. $9x + 27y$
C. $24xy$
D. $4x + 6y$

25. Given the function $f(x) = 7^{\frac{x}{2}} - 3$, determine its inverse, $f^{-1}(x)$.

A. $f^{-1}(x) = 2\log_7 x - 6$
B. $f^{-1}(x) = 2\log_7 (x - 6)$
C. $f^{-1}(x) = 2\log_7 (x + 3)$
D. $f^{-1}(x) = 2\log_7 x + 3$

26. If $x+4$ is a factor of the polynomial $p(x)$, then which of the following must be true?

A. $p(-4)=0$

B. $p(4)=0$

C. $p(0)=-4$

D. $p(0)=4$

27. According to the Rational Root Theorem, which number could **not** be a root of the equation $4x^3+kx^2+3x-3=0$, where k is an integer?

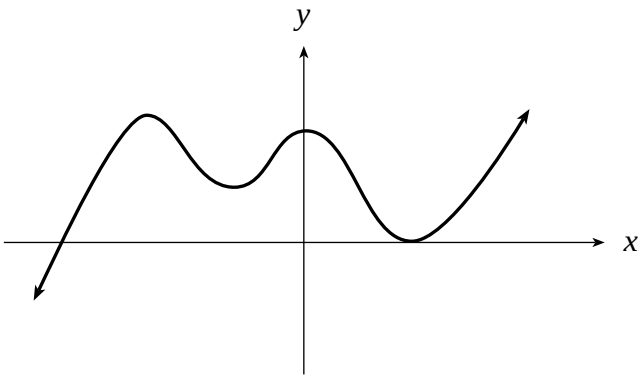
A. –3

B. –1

C. $\frac{4}{3}$

D. $\frac{3}{2}$

28. What is the minimum degree of the polynomial function graphed below?



A. 3
B. 4
C. 5
D. 7

29. When x^3+x^2-kx-5 is divided by $x-2$, the remainder is 1. Find the value of k.

A. 3
B. 3.5
C. 4.5
D. 5

OVER

30. Determine a polynomial equation that has the roots ± 2 , $\pm \sqrt{7}$.

A. $x^4 - 11x^2 + 28 = 0$
B. $x^4 + 11x^2 + 28 = 0$
C. $x^4 - 9x^2 + 14 = 0$
D. $x^4 + 9x^2 + 14 = 0$

31. Find the remainder for the following division.

$$x^2 + 2x - 4 \overline{)x^4 + 2x^3 - 3x^2 + 2x + 6}$$

A. 2
B. 10
C. $4x + 2$
D. $16x - 22$

32. Determine the real root(s): $2x^3 - 3x^2 + 6x - 9 = 0$

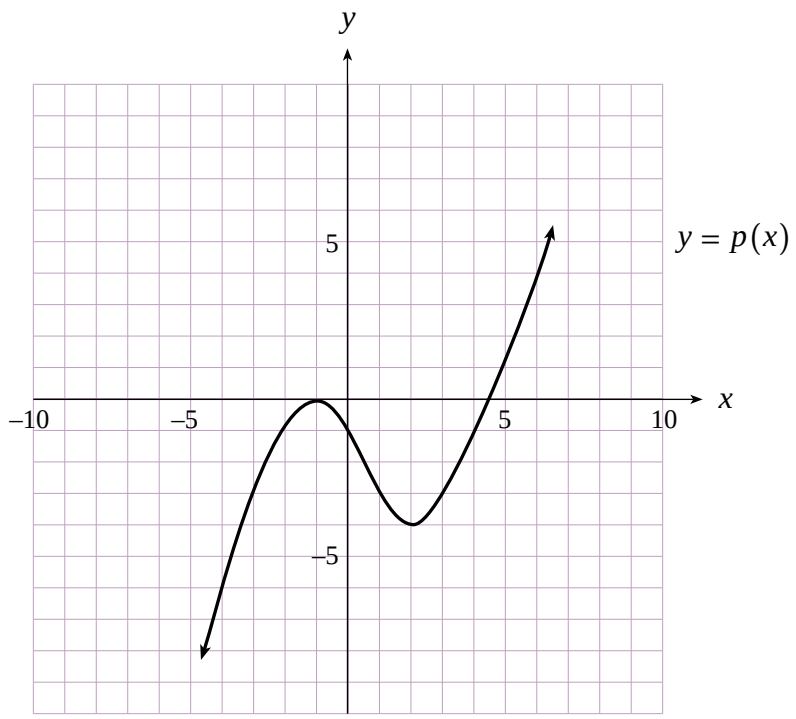
A. $-\frac{3}{2}$

B. $\frac{3}{2}$

C. $-\frac{3}{2}$, $\pm \sqrt{3}$

D. $\frac{3}{2}$, $\pm \sqrt{3}$

33. Use the graph of the function $y = p(x)$ shown below to solve the equation $p(x-5)+6=0$.



A. -4
B. -1
C. 1
D. 4

34. The general term of an arithmetic sequence is $t_n = 4n+3$. Determine the common difference.

A. -4
B. 3
C. 4
D. 7

35. Find the sum of the infinite geometric series: $-45+30-20+\ \ldots$

A. -135
B. -35
C. -27
D. there is no finite sum

OVER

36. Use sigma notation to represent the series: $20 + 30 + 40 + 50 + 60 + 70 + 80$

A. $\sum_{k=1}^{7}(10k+10)$

B. $\sum_{k=0}^{7}(10k+20)$

C. $\sum_{k=1}^{7}20(2)^{k-1}$

D. $\sum_{k=1}^{7}10(2)^{k}$

37. In a geometric sequence $t_3 = -36$ and $t_6 = 972$. Find the value of t_1.

A. -4
B. -3
C. 3
D. 4

38. Given that $2^x,\ 8^y,\ k$ is a geometric sequence, determine k.

A. 2^{6y-x}
B. 2^{3y-x}
C. 2^{10y-5x}
D. 2^{2y-2x}

39. Find $f'(x)$ if $f(x) = 3x^2 + 2$.

A. $6x$
B. $5x$
C. $6x + 2$
D. $5x + 2$

40. Evaluate: $\lim\limits_{x\to\infty} \dfrac{2x^4 - 5x^2}{3x^3 + 8x^2}$

A. $-\dfrac{3}{11}$

B. 0

C. $\dfrac{2}{3}$

D. limit does not exist (no finite limit)

41. At which of the following values of x is the function $g(x) = x^3 - 4x^2$ decreasing?

A. $x = -3$

B. $x = -1$

C. $x = 2$

D. $x = 4$

42. Evaluate: $\lim\limits_{x\to 1} \dfrac{x^2 + x - 2}{x^2 - 1}$

A. $-\dfrac{1}{2}$

B. 1

C. $\dfrac{3}{2}$

D. limit does not exist (no finite limit)

43. An object moves along the x-axis so that its position at time t is $x = t^2 - 3t + 5$, where x is in metres and t is in seconds. At what time(s) is its velocity 5 m/s ?

A. $t = 1$

B. $t = 4$

C. $t = 7$

D. $t = 0$ or 3

OVER

44. Find $\frac{dy}{dx}$ if $y = 2\sqrt{x}$.

A. $\frac{1}{\sqrt{x}}$

B. $-\sqrt{x}$

C. $\frac{1}{2\sqrt{x}}$

D. $-\frac{1}{2}x$

45. The line $y = -4x + 18$ is tangent to the parabola $y = ax^2 + bx$ at the point where $x = 3$. If the parabola has a critical point at $x = 2$, determine the value of a.

A. -4

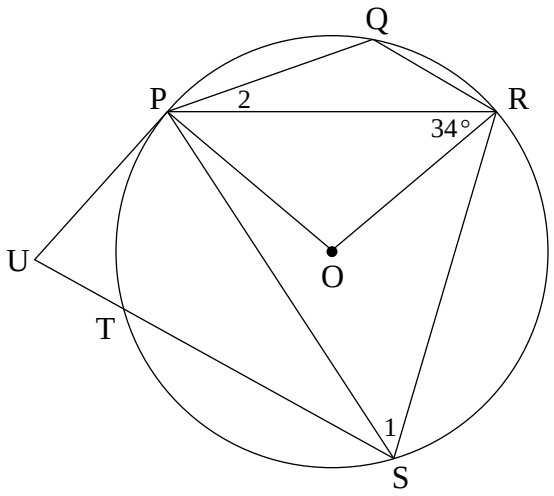
B. -2

C. -1

D. 2

Use the following diagram to answer questions 46 and 47.
Diagram is not drawn to scale.

Given: Circle with centre O
UP is a tangent



46. Determine the measure of $\angle 1$.

A. 54°
B. 56°
C. 58°
D. 68°

47. If $\angle UPQ = 150°$, determine the measure of $\angle 2$.

A. 20°
B. 26°
C. 30°
D. 34°

48. Solve over the real numbers: $\sin x = 0.5$

A. $0.52 + 2n$, $2.62 + 2n$ (n is an integer)
B. $0.52 + n\pi$, $2.62 + n\pi$ (n is an integer)
C. $0.52n + 2\pi$, $2.62n + 2\pi$ (n is an integer)
D. $0.52 + 2n\pi$, $2.62 + 2n\pi$ (n is an integer)

OVER

49. A point, P, divides the line segment AB in a ratio of $3:4$; that is, $\frac{AP}{PB} = \frac{3}{4}$.
If $A = (9, 12)$ and $B = (2, 3)$, find the *x*-coordinate of point P.

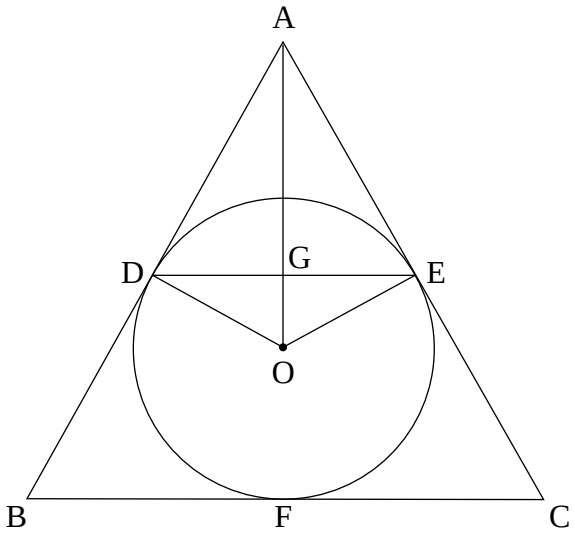
A. 5

B. 6

C. $\frac{29}{4}$

D. $\frac{44}{7}$

50. A circle with centre O has a radius of 4. AB, AC and BC are tangent to the circle at D, E and F respectively. If $AB = AC$, $AO \perp DE$ and $OG = 2$, determine the length of AE.



A. 6

B. $4\sqrt{3}$

C. 8

D. $4\sqrt{5}$

This is the end of the multiple-choice section.
Answer the remaining questions directly in this examination booklet.

PART B: WRITTEN RESPONSE

Value: 20 marks **Suggested Time: 45 minutes**

INSTRUCTIONS:	Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided. **Full marks will NOT be given for the final answer only.**

OVER

1. Solve: $3\cos^2 x - 5\cos x - 2 = 0$, where $0 \le x < 2\pi$ (Accurate to at least 2 decimal places.)

(2 marks)

ANSWER:	Score for Question 1: 1. _____ (2)

OVER

2. The population of a city is increasing at a rate of 6.5% each year. If the present population is 12 000, how long will it take for the population to reach 32 000? (Accurate to at least 1 decimal place.) **(3 marks)**

ANSWER:	Score for Question 2: 2. _____ (3)

OVER

3. Determine the sum of the arithmetic series $3 + 17 + 31 + \ldots + 1151$. **(3 marks)**

ANSWER:	Score for Question 3: 3. _____ (3)

OVER

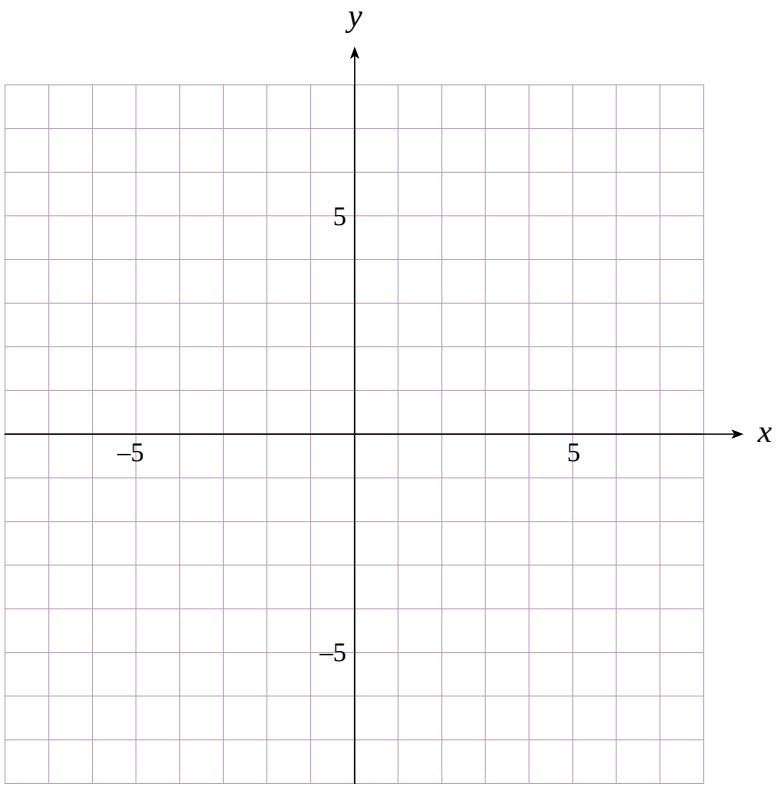
4. A point P moves such that it is always equidistant from the point $F(2, 5)$ and the line given by $y = 1$. Find an equation of this locus and write it in standard form. **(3 marks)**

ANSWER:	Score for Question 4: 4. _____ (3)

OVER

5. A function is defined by the equation $f(t) = t^2 + 6t + 7$. Sketch the graph of $f(x) + f(y) = 0$.

(2 marks)



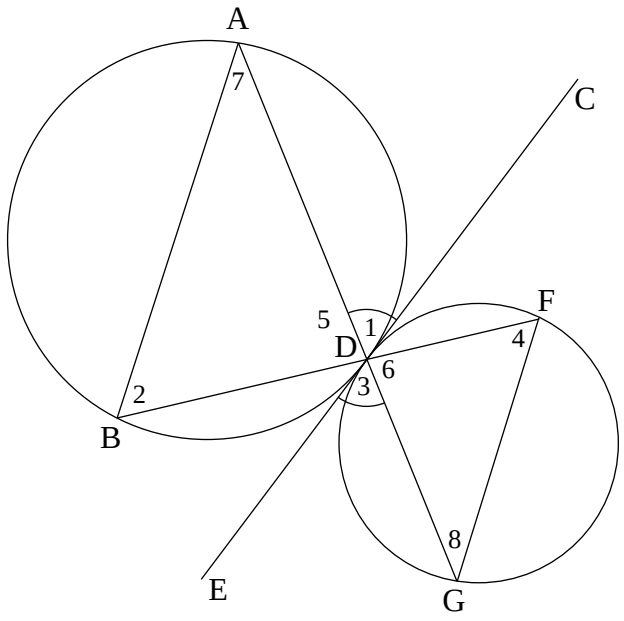
Score for Question 5:
5. _____ (2)

OVER

6. Complete the proof. **(4 marks)**

Given: AG and BF intersect at D
CE is tangent to both circles at D

Prove: AB || FG



Proof

Statement	Reason

Score for Question 6:
6. _____ (4)

OVER

7. Two numbers x and y differ by 50, where x is larger than y. If $R = x + y + xy$, determine the values of x and y such that R is a minimum. **(3 marks)**

ANSWER:	Score for Question 7: 7. _____ (3)

END OF EXAMINATION

THIS PAGE INTENTIONALLY BLANK

A SUMMARY OF BASIC IDENTITIES AND FORMULAE

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$ $\qquad 1 + \tan^2\theta = \sec^2\theta$ $\qquad 1 + \cot^2\theta = \csc^2\theta$

Reciprocal and Quotient Identities

$\sec\theta = \dfrac{1}{\cos\theta}$ $\qquad \csc\theta = \dfrac{1}{\sin\theta}$ $\qquad \cot\theta = \dfrac{1}{\tan\theta}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\qquad \cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Addition Identities

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\qquad \tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\qquad \tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

Double-Angle Identities

$\cos 2\theta = \cos^2\theta - \sin^2\theta$ $\qquad \sin 2\theta = 2\sin\theta\cos\theta$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$ $\qquad \tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

Formulae

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\qquad t_n = a + (n-1)d$ $\qquad t_n = ar^{n-1}$

$\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$ $\qquad S_n = \dfrac{n}{2}\left[2a + (n-1)d\right]$ $\qquad S_n = \dfrac{a\left(1 - r^n\right)}{1 - r}$

$c^2 = a^2 + b^2 - 2ab\cos C$ $\qquad S_n = \dfrac{n}{2}(a + \ell)$ $\qquad S_n = \dfrac{a - \ell r}{1 - r}$

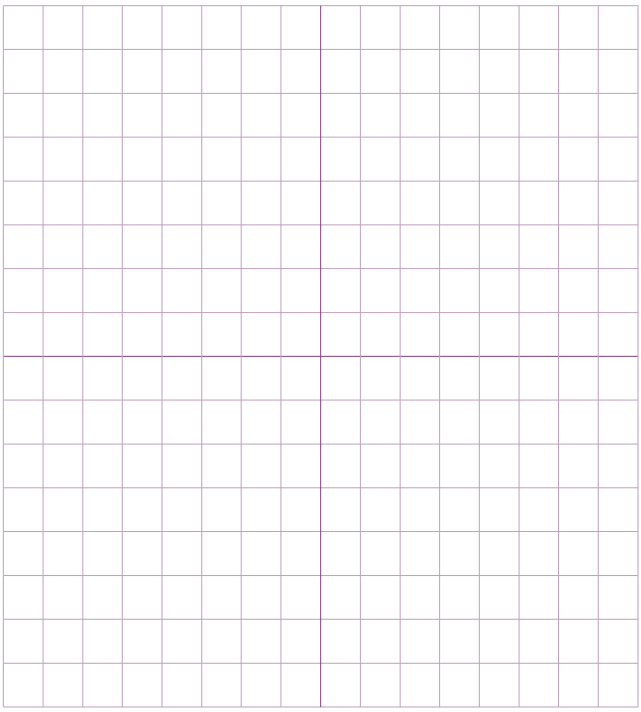
$S = \dfrac{a}{1 - r}$

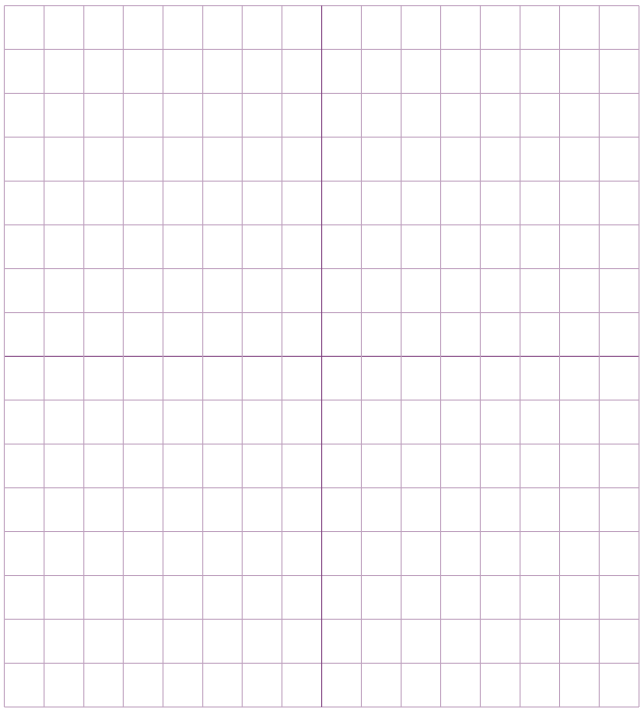
You may detach this page for convenient reference.
Exercise care when tearing along perforations.

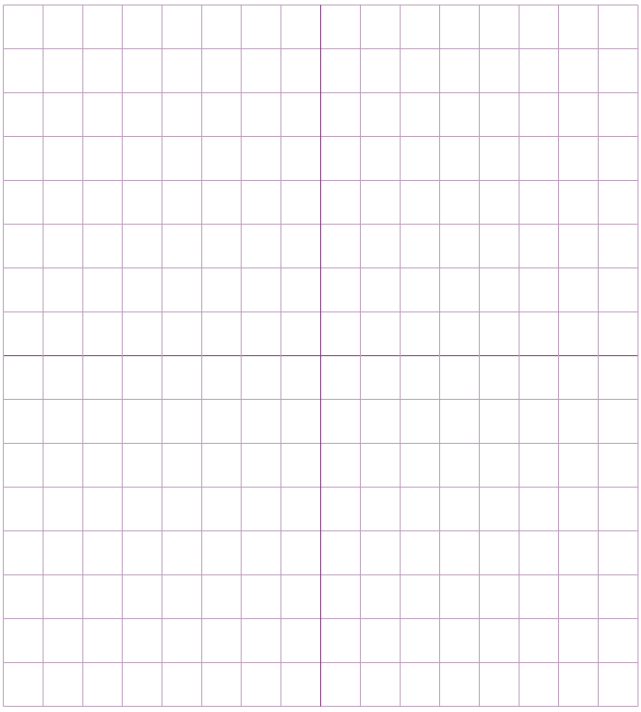
THIS PAGE INTENTIONALLY BLANK

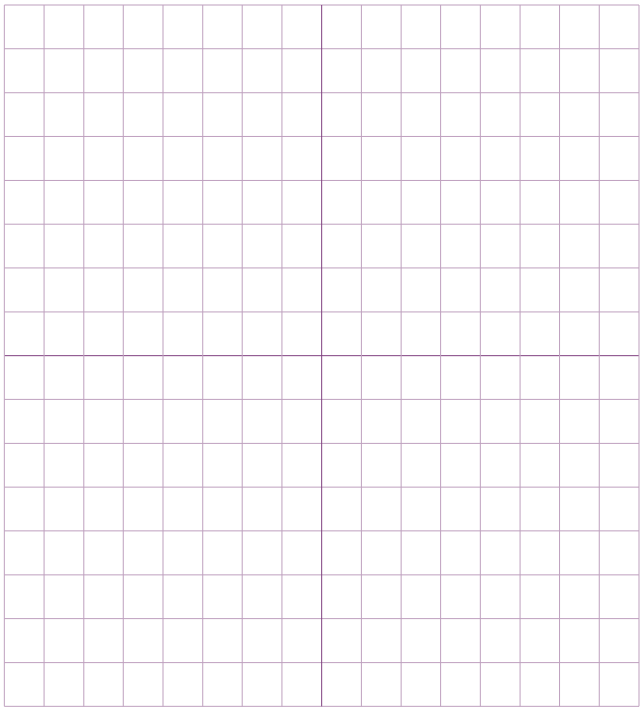
ROUGH WORK FOR GRAPHING

(No marks will be given for work done on this page.)





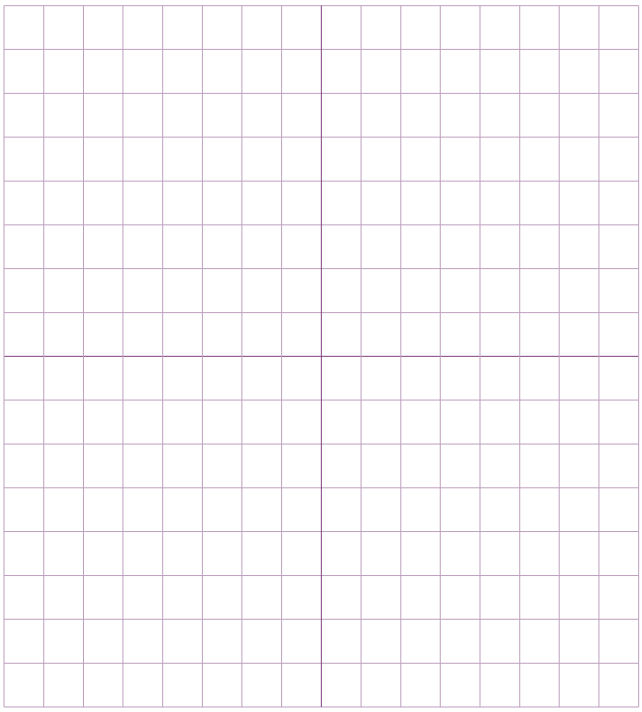


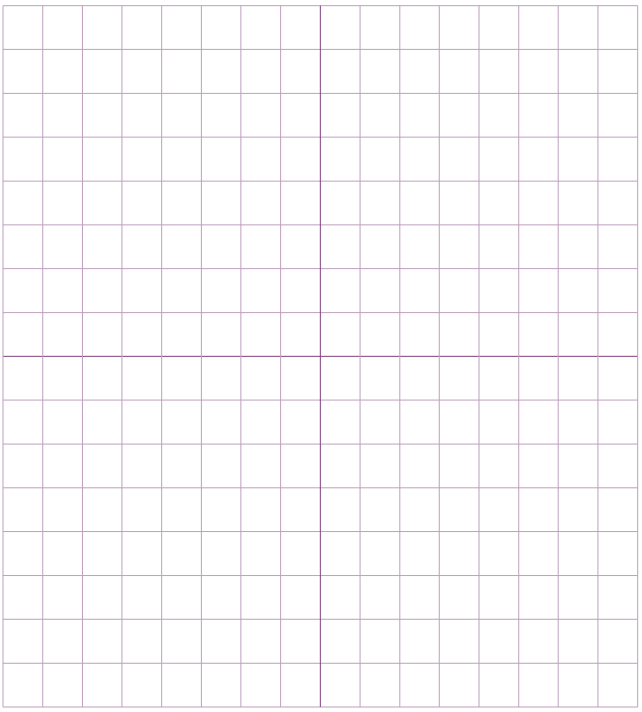


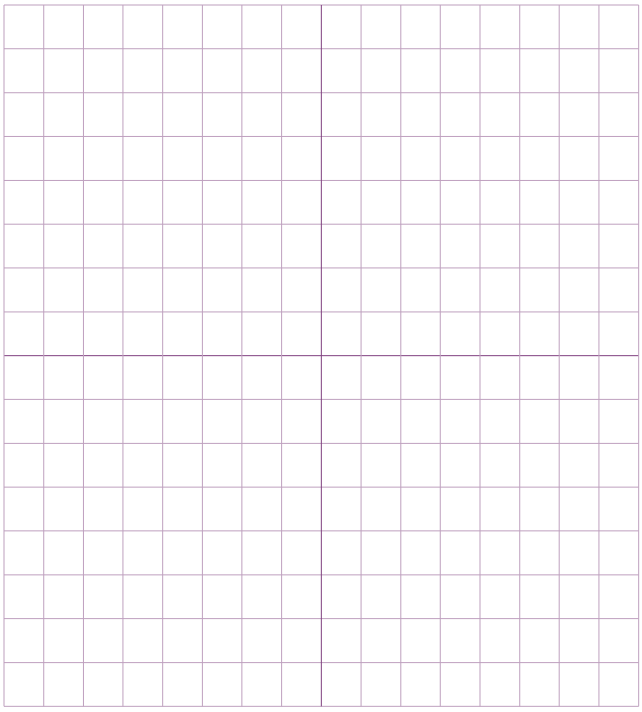
You may detach this page for convenient reference.
Exercise care when tearing along perforations.

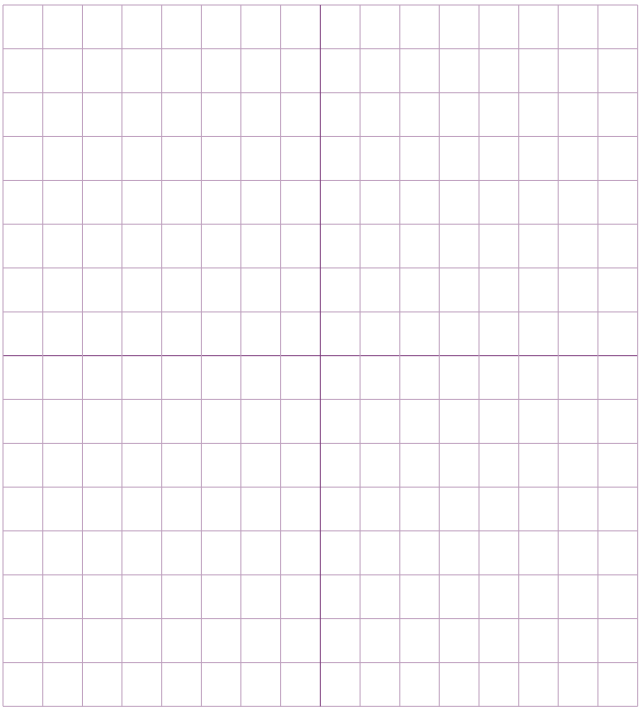
ROUGH WORK FOR GRAPHING

(No marks will be given for work done on this page.)









ROUGH WORK FOR MULTIPLE-CHOICE

You may detach this page for convenient reference.
Exercise care when tearing along perforations.

ROUGH WORK FOR MULTIPLE-CHOICE