JUNE 1996

PROVINCIAL EXAMINATION

MINISTRY OF EDUCATION, SKILLS AND TRAINING

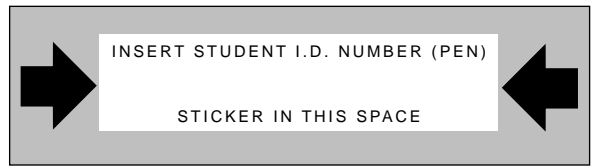# MATHEMATICS 12

GENERAL INSTRUCTIONS

1. Insert the stickers with your Student I.D. Number (PEN) in the allotted spaces above. **Under no circumstance is your name or identification, other than your Student I.D. Number, to appear on this paper.**

2. Take the separate Answer Sheet and follow the directions on its front page.

3. Be sure you have an **HB pencil** and an eraser for completing your Answer Sheet. Follow the directions on the Answer Sheet when answering multiple-choice questions.

4. For each of the written-response questions, write your answer in the space provided.

5. When instructed to open this booklet, **check the numbering of the pages** to ensure that they are numbered in sequence from page one to the last page, which is identified by

    $\boxed{\text{END OF EXAMINATION}}$.

6. At the end of the examination, place your Answer Sheet inside the front cover of this booklet and return the booklet and your Answer Sheet to the supervisor.

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MATHEMATICS 12 JUNE 1996 PROVINCIAL

Course Code = MA   Examination Type = P

1. _____
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MATHEMATICS 12 PROVINCIAL EXAMINATION

		Value	Suggested Time

1. This examination consists of **two** parts:

		Value	Suggested Time
PART A:	50 multiple-choice questions	50	75
PART B:	7 written-response questions	20	45

    2 questions worth **two** marks each,
    4 questions worth **three** marks each, and
    1 question worth **four** marks.

    Total: 70 marks   120 minutes

2. The last **three** pages inside the back cover contain **A Summary of Basic Identities and Formulae**, **Rough Work for Graphing**, and **Rough Work for Multiple Choice**. These pages may be detached for convenient reference prior to writing this examination.

3. You will not be provided with any additional paper since rough-work space for the written-response questions has been incorporated into the space allowed for answering each question. You may not need all of the space provided to answer each question.

4. An approved scientific calculator is essential for the examination. The calculator must be a hand-held device designed **only** for mathematical computations such as logarithmic and trigonometric functions. It **can be** programmable, but **must not** contain any graphing capabilities. You **must not** bring into the examination room any devices to support calculators such as manuals, printed or electronic cards, printers, memory expansion chips or cards, or keyboards.

5. You are permitted to use rulers, compasses, and protractors.

6. You have **two hours** to complete this examination.

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Value: 50 marks   **Suggested Time: 75 minutes**

INSTRUCTIONS: For each question, select the **best** answer and record your choice on the Answer Sheet provided. Using an HB pencil, completely fill in the circle that has the letter corresponding to your answer.
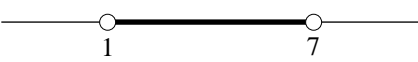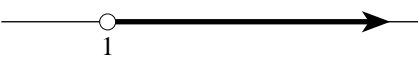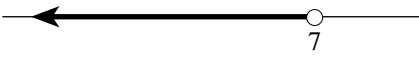
1. Determine the centre of the circle that has $(-8,\ 5)$ and $(6,\ -1)$ as endpoints of a diameter.

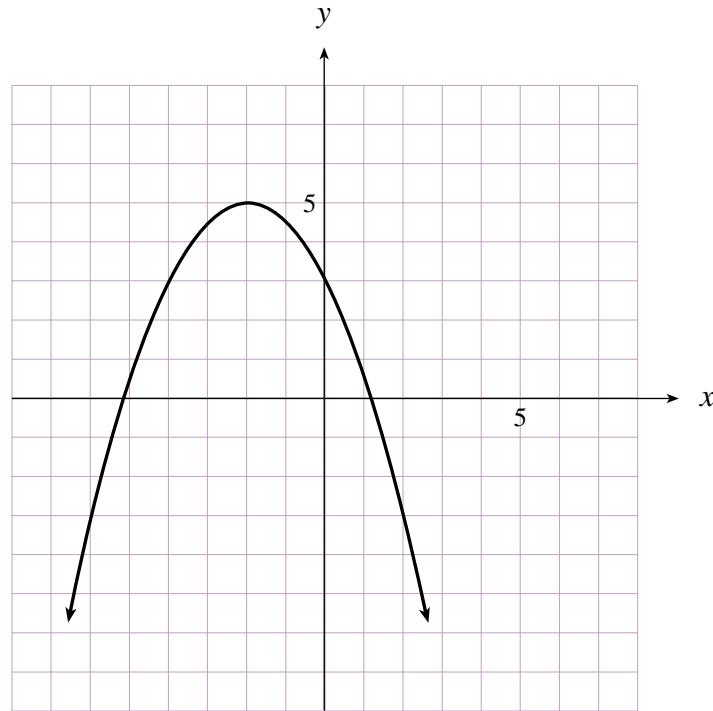   A. $(-7,\ 3)$
   B. $(-2,\ 4)$
   C. $(-1,\ 2)$
   D. $(7,\ -3)$

2. What is the vertex of the parabola $x = -2(y-8)^2 + 5$?

   A. $(-5,\ -8)$
   B. $(-5,\ 8)$
   C. $(5,\ -8)$
   D. $(5,\ 8)$

3. Solve: $|2x - 8| < 6$

   A.

   B.

   C.

   D.

4. Determine an equation of the parabola in the diagram below.

A. $y = -2(x+2)^2 + 5$

B. $y = -\dfrac{1}{2}(x+2)^2 + 5$

C. $y = -2(x-2)^2 + 5$

D. $y = -\dfrac{1}{2}(x-2)^2 + 5$

5. Determine the slopes of the asymptotes of the hyperbola $\dfrac{x^2}{36} - \dfrac{y^2}{100} = 1$.

A. $\pm \dfrac{3}{5}$

B. $\pm \dfrac{9}{25}$

C. $\pm \dfrac{5}{3}$

D. $\pm \dfrac{25}{9}$

6. How many different real solutions does the following system have?

$$\frac{x^2}{25} + \frac{y^2}{4} = 1$$

$$(x - 2)^2 + y^2 = 9$$

A. 1
B. 2
C. 3
D. 4

7. Change to standard form: $2x^2 - 3y^2 - 12x - 6 = 0$

A. $\dfrac{(x-3)^2}{12} - \dfrac{y^2}{8} = 1$

B. $\dfrac{(x-3)^2}{\frac{15}{2}} - \dfrac{y^2}{5} = 1$

C. $\dfrac{(x-3)^2}{6} - \dfrac{y^2}{4} = -1$

D. $\dfrac{(x-6)^2}{21} - \dfrac{y^2}{14} = 1$

8. A point $P(x, y)$ moves in a path that is parallel to the graph of the relation $8x - 4y = 7$ and passes through $(-3, 5)$. Determine an equation of this locus.

A. $y = 2x + 11$
B. $y = 2x - 11$
C. $y = -2x + 11$
D. $y = -2x - 11$

9. A circle with centre $(2, k)$ is tangent to the lines $x = -4$ and $y = 2$. Determine all possible values of k.

A. $k = -4$ or 8
B. $k = -4$ or 6
C. $k = -6$ or 8
D. $k = 6$ or 8

OVER

- 3 -

10. Determine the shortest distance from the point $(8, 6)$ to the circle $x^2 + y^2 = 5$. (Accurate to 2 decimal places.)

    A. 5.00
    B. 7.76
    C. 8.32
    D. 8.66

11. Convert $310°$ to radians. (Accurate to 2 decimal places.)

    A. 1.72
    B. 2.71
    C. 4.15
    D. 5.41

12. Give the value of $\csc 2.24$. (Accurate to 2 decimal places.)

    A. −1.61
    B. 0.43
    C. 0.90
    D. 1.27

13. Simplify: $\cos(\pi + \theta) + \cos(\pi - \theta)$

    A. −2
    B. 1
    C. $2\cos\theta$
    D. $-2\cos\theta$

14. Give the range of the function $f(x) = 3\sin x + 7$.

    A. $-10 \le y \le -4$
    B. $-3 \le y \le 3$
    C. $0 \le y \le 6$
    D. $4 \le y \le 10$

15. Which expression is equivalent to $\dfrac{\cot\theta - 1}{1 - \tan\theta}$?

   A. $\cot\theta$
   B. $\tan\theta$
   C. $-\cot\theta$
   D. $-\tan\theta$

16. Solve: $\cos^2 x = \dfrac{3}{4}$,   where $0 \le x < 2\pi$.   (Accurate to 2 decimal places.)

   A.   0.52 , 5.76
   B.   1.05 , 5.24
   C.   0.52 , 2.62 , 3.67 , 5.76
   D.   1.05 , 2.09 , 4.19 , 5.24

17. Which expression is equivalent to $\dfrac{1 - \cos 2\theta}{2}$?

   A. $-\sin^2\theta$
   B. $-\cos^2\theta$
   C. $\sin^2\theta$
   D. $\cos^2\theta$

18. Solve: $2\sin x \cos x = 1$,   where $0 \le x < 2\pi$.   (Accurate to 2 decimal places.)

   A.   0.52
   B.   0.79
   C.   0.52 , 2.62
   D.   0.79 , 3.93

19. Change $p^r = q$ to logarithmic form.

    A. $\log_q p = r$

    B. $\log_p r = q$

    C. $\log_r q = p$

    D. $\log_p q = r$

20. Give the equation of the asymptote of the graph of $y = 3^{x-4} + 5$.

    A. $y = -5$

    B. $y = 0$

    C. $y = 4$

    D. $y = 5$

21. Express $2 + 3\log a - \log b$ as a single logarithm.

    A. $\log\left(\dfrac{100a^3}{b}\right)$

    B. $\log\left(100a^3 - b\right)$

    C. $\log\left(\dfrac{2a^3}{b}\right)$

    D. $\log\left(2a^3 - b\right)$

22. Given $f(x) = \dfrac{1}{3}x + 5$, determine $f^{-1}(x)$, the inverse of f.

    A. $f^{-1}(x) = 3x - 15$

    B. $f^{-1}(x) = 3x + \dfrac{1}{5}$

    C. $f^{-1}(x) = \dfrac{3}{x} + \dfrac{1}{5}$

    D. $f^{-1}(x) = -\dfrac{1}{3}x - 5$

23. Simplify: $\log_b\left(b\sqrt{b}\right)$

A. $\dfrac{1}{2}$

B. $\dfrac{3}{2}$

C. $b^{\frac{1}{2}}$

D. $b^{\frac{3}{2}}$

24. The mass of a radioactive substance is halved every 28 years. Determine an expression for the mass of the substance remaining from a 60 gram sample after t years.

A. $0.5(60)^{\frac{28}{t}}$

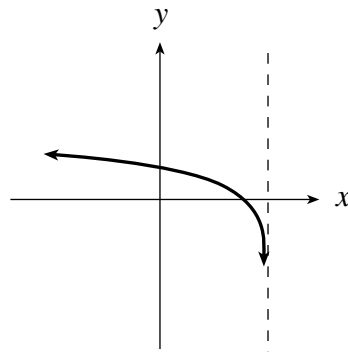B. $60(0.5)^{\frac{28}{t}}$

C. $0.5(60)^{\frac{t}{28}}$

D. $60(0.5)^{\frac{t}{28}}$

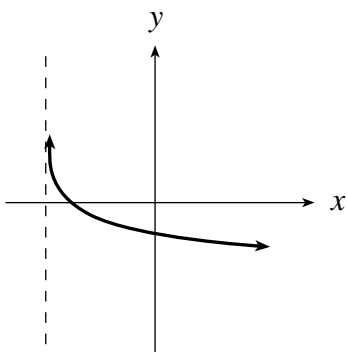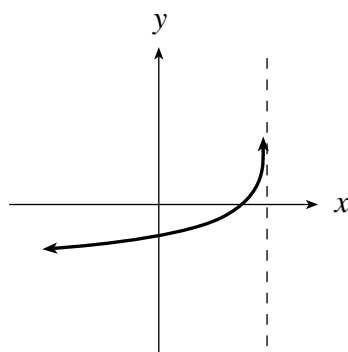25. Which graph **best** represents the function $y = -\log_3(x+5)$?

A.

B.

C.

D.

OVER
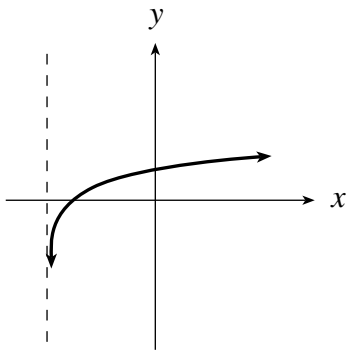
26. If the polynomial $p(x)$ is divided by $x-6$, which of the following represents the remainder?

   A. $p(6)$
   B. $p(-6)$
   C. $p(x)+6$
   D. $p(0)$

27. Determine the value of k if 2 is a zero of the function $p(x) = x^3 - 7x^2 + kx + 12$.

   A. $k = -16$
   B. $k = 4$
   C. $k = 5$
   D. $k = 16$

28. Determine the quotient when $x^3 - 12x^2 + 9x - 5$ is divided by $x - 3$.

   A. $x^2 - 9x - 16$
   B. $x^2 - 9x - 18$
   C. $x^2 - 15x + 54$
   D. $x^2 + 9x + 36$

29. If $x+4$ is a factor of the polynomial $mx^3 - 11x^2 - 10x + n$, where m and n are integers, according to the Rational Root Theorem, which of the following could be a value for n?

   A. 2
   B. 6
   C. 8
   D. 10

30. Solve: $x^3 - 4x^2 > 12x$

   A. $-2 < x < 6$
   B. $x < -2$ or $x > 6$
   C. $-2 < x < 0$ or $x > 6$
   D. $-6 < x < 0$ or $x > 2$

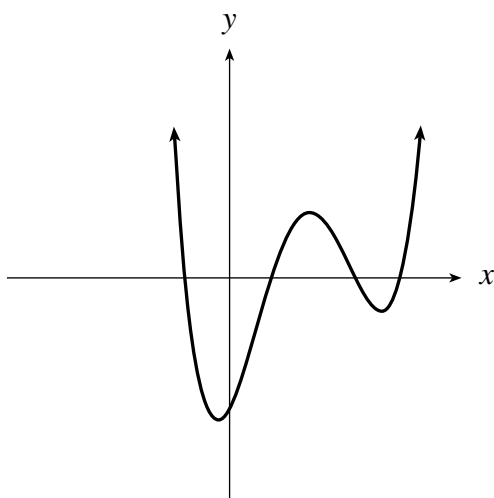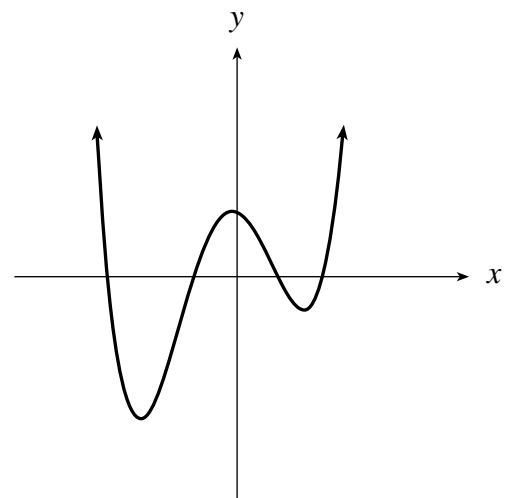31. Which graph is a possible representation of $y = ax^4 + bx^3 + cx - 6$, where a is a negative integer?

A. y

B. y

C. y

D. y
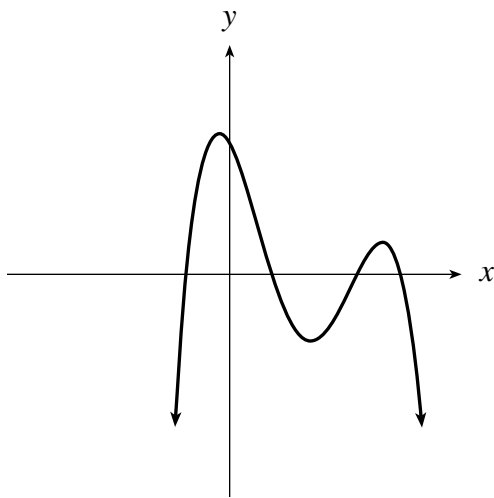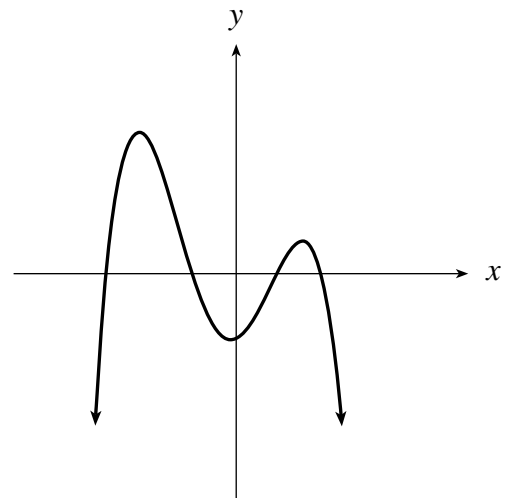
32. Determine a polynomial equation that has the following roots: $2, \pm\sqrt{5}$
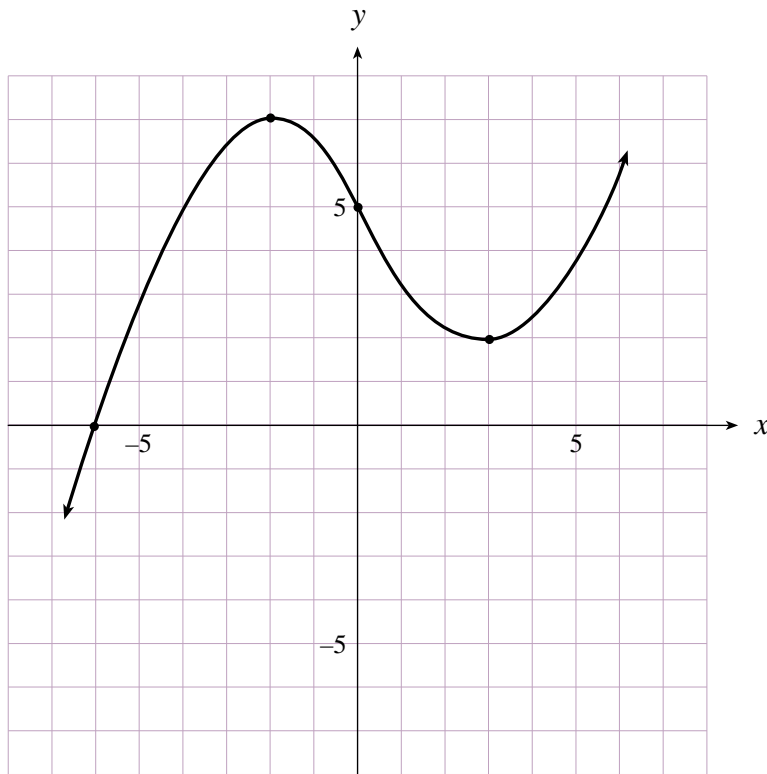
A. $x^3 + 2x^2 - 5x - 10 = 0$

B. $x^3 - 2x^2 + 5x - 10 = 0$

C. $x^3 - 2x^2 - 5x + 10 = 0$

D. $x^3 + 2x^2 + 5x + 10 = 0$

OVER

33. The graph of the cubic polynomial function $p(x)$ is given below. Which of the following functions must have 3 unequal real zeros?

A. $p(x) - 7$

B. $p(x) - 3$

C. $p(x - 3)$

D. $p(x - 7)$

34. Determine the 200th term of the arithmetic sequence -12, 10, 32, ...

A. $-4\,390$

B. $4\,366$

C. $4\,388$

D. $4\,410$

35. If an infinite geometric series has a finite sum, which of the following could be the common ratio r?

    A.   −1.2
    B.   0.6
    C.   1.0
    D.   1.5

36. What is the 3rd term of the sequence given by the following recursive definition?

$$t_1 = 3$$

$$t_n = (n+1)t_{n-1} + 12, \quad n > 1$$

    A.   75
    B.   80
    C.   96
    D. 132

37. A geometric sequence of positive terms has $t_1 = 320$ and $t_7 = 78\,125$. Find t_4.

    A.   2 000
    B.   5 000
    C. 12 500
    D. 39 222.5

38. Which expression represents the sum of the series given by $\displaystyle\sum_{k=0}^{15} 16(2)^{k+1}$?

    A. $16\left(2^{15} - 1\right)$
    B. $16\left(2^{16} - 1\right)$
    C. $32\left(2^{15} - 1\right)$
    D. $32\left(2^{16} - 1\right)$

39. Give the derivative of $f(x) = -3x$.

    A. -3

    B. 0

    C. $-3x^{-1}$

    D. $-3x^2$

40. Evaluate: $\displaystyle \lim_{x \to \infty} \frac{5x^2 + 3x - 2}{3x^2 - 4x + 7}$

    A. $-\dfrac{2}{7}$

    B. 1

    C. $\dfrac{5}{3}$

    D.   does not exist (no finite limit)

41. Find the slope of the line tangent to the graph of $y = x^3 - 4x^2 + 2$ at the point where $x = 2$.

    A. -10

    B. -6

    C. -4

    D. -2

42. Evaluate: $\displaystyle \lim_{x \to 2} \frac{x^2 - x - 2}{x^2 + x - 6}$

    A. $\dfrac{1}{3}$

    B. $\dfrac{3}{5}$

    C. 1

    D.   does not exist (no finite limit)

43. Determine all values of x such that the function $f(x) = x^3 - 3x^2 + 5$ is decreasing.

    A. $x < 2$

    B. $x > 2$

    C. $0 < x < 2$

    D. $x < 0$ or $x > 2$

44. Find the minimum value of the function $f(x) = 2x^2 - 12x + 6$.

    A. -24

    B. -12

    C.   3

    D.   6

45. If $f(x) = k\sqrt{x}$, determine the value of the constant k so that $f'(4) = 6$.

    A. $k = 3$

    B. $k = 6$

    C. $k = 12$

    D. $k = 24$

Use the following diagram and proof to answer questions 46 and 47.

Given: E is the centre
         A, E, C are collinear
         D, E, B are collinear
         $\angle 4 = \angle 6$

Prove: $\angle 1 = \angle 9$
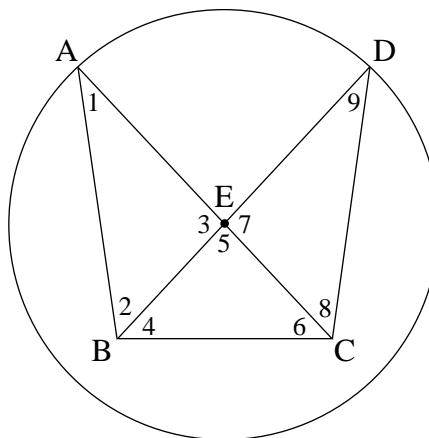
Proof

Statement		Reason
	$\angle 4 = \angle 6$	given
(a)	BE = CE	sides opposite = $\angle$s are =
(b)	BC = BC	same side
(c)	$\angle 3 = \angle 7$	vertically opposite $\angle$s are =
(d)	EA = ED	radii
(e)	$\triangle ABE \cong \triangle DCE$	_____
	$\angle 1 = \angle 9$	CPCTC

46. Which line is **not** necessary in the given proof?

    A. a
    B. b
    C. c
    D. d

47. Give the reason for line (e).

    A. SAS
    B. AAS
    C. SSS
    D. ASA

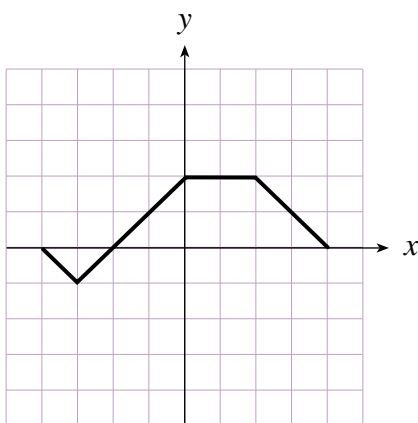48. Given the graph of the function $f(x)$ below, which of the following is the graph of $g(x) = f(|x|)$?
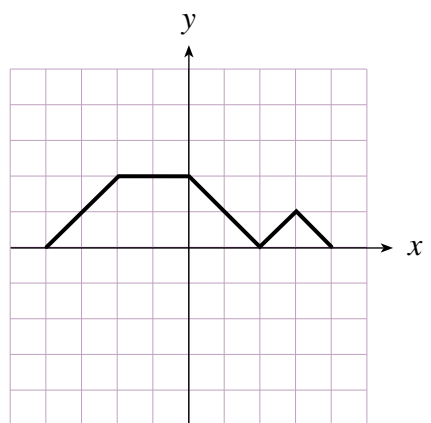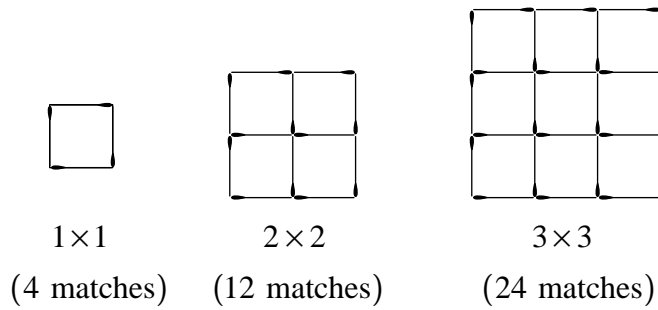

A.

B.

C.

D.

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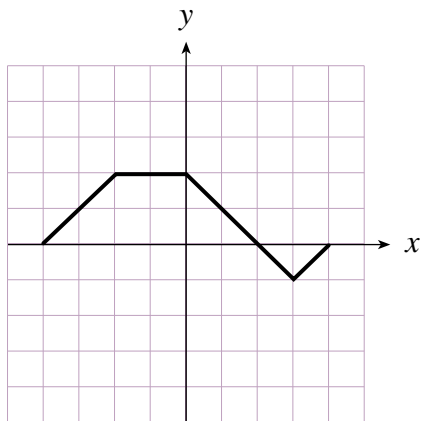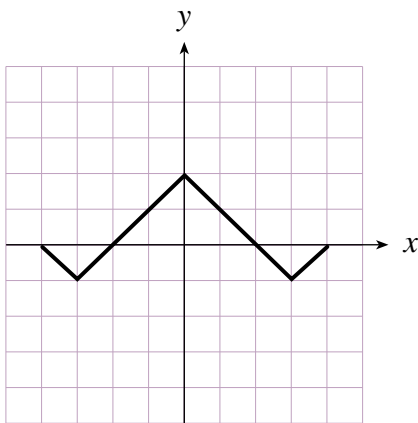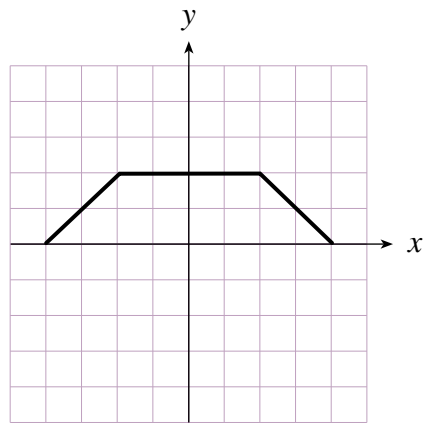
49. A fixed point of a function is a real number, x, for which $f(x) = x$. Which function has **no** fixed points?

   A. $f(x) = \cos x$
   B. $f(x) = x^2$
   C. $f(x) = x^3$
   D. $f(x) = \log_2 x$

50. How many matches are required to make a grid which is 30×30 square?

1×1	2×2	3×3
(4 matches)	(12 matches)	(24 matches)

   A. 1 740
   B. 1 860
   C. 2 400
   D. 2 700

This is the end of the multiple-choice section.
Answer the remaining questions directly in this examination booklet.

**Value: 20 marks**   **Suggested Time: 45 minutes**

INSTRUCTIONS:	Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.
	Full marks will NOT be given for the final answer only.

OVER

1. Graph at least one period of $y = 4\sin 2\left(x - \dfrac{\pi}{4}\right).$ **(2 marks)**
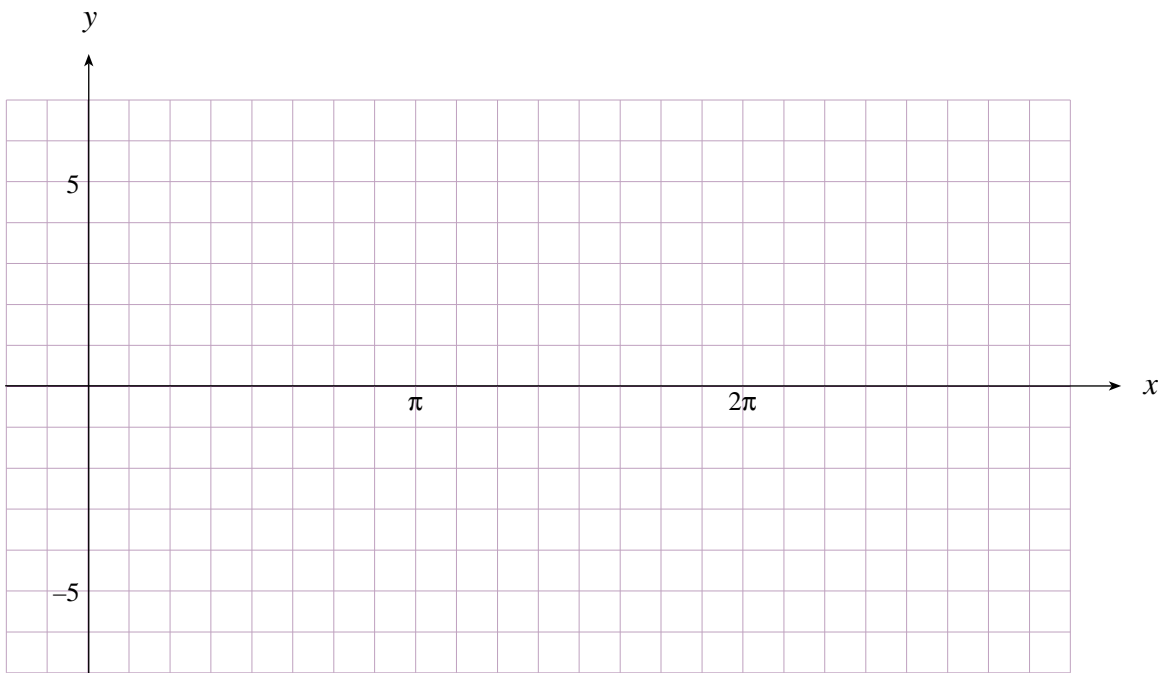
OVER

2. Three consecutive terms in an arithmetic sequence are $8x + 7$, $2x + 5$ and $2x^2 + x$.
   Determine the values of the three terms for all such sequences. **(3 marks)**

ANSWER:	Score for Question 2:
	2. _____ (3)

OVER

3. Solve: $\log_2(x+7) + \log_2(x+5) = 3$ **(3 marks)**

ANSWER:	Score for Question 3:
	3. _____ (3)

4. Solve the following system **for x only**. (Accurate to at least 2 decimal places.) **(3 marks)**

$$xy = 6$$

$$x^2 + y^2 = 15$$

ANSWER:	Score for Question 4:
	4. _____ (3)

OVER

5. Complete the proof. **(4 marks)**

Given: AC is a diameter of the circle
with centre O
EF is a tangent at B
CD ⊥ EF

Prove: BC bisects ∠DCA
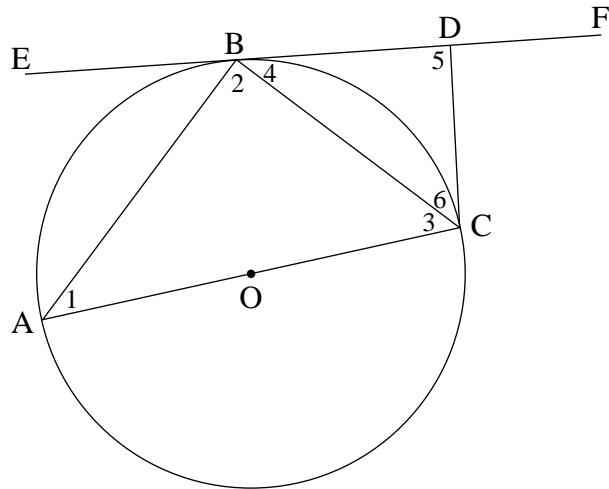
Proof

Statement	Reason

OVER

6. Given $f(x) = 3x^2$, use the definition of derivative to show that $f'(x) = 6x$. **(3 marks)**

ANSWER:	Score for Question 6:
	6. _____ (3)

OVER

7. Determine the domain of the relation $\log_{x+4} y = \log_{x+4} x^2$. **(2 marks)**

ANSWER:	Score for Question 7:
	7. _____ (2)

END OF EXAMINATION

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A SUMMARY OF BASIC IDENTITIES AND FORMULAE

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

Reciprocal and Quotient Identities

$$\sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Addition Identities

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \qquad\qquad \tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \qquad\qquad \tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

Double-Angle Identities

$$\cos 2\theta = \cos^2\theta - \sin^2\theta \qquad\qquad \sin 2\theta = 2\sin\theta\cos\theta$$

$$= 2\cos^2\theta - 1$$

$$= 1 - 2\sin^2\theta \qquad\qquad \tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Formulae

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad t_n = a + (n-1)d \qquad t_n = ar^{n-1}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \qquad S_n = \frac{n}{2}\left[2a + (n-1)d\right] \qquad S_n = \frac{a(1 - r^n)}{1 - r}$$

$$c^2 = a^2 + b^2 - 2ab\cos C \qquad S_n = \frac{n}{2}(a + \ell) \qquad S_n = \frac{a - \ell r}{1 - r}$$
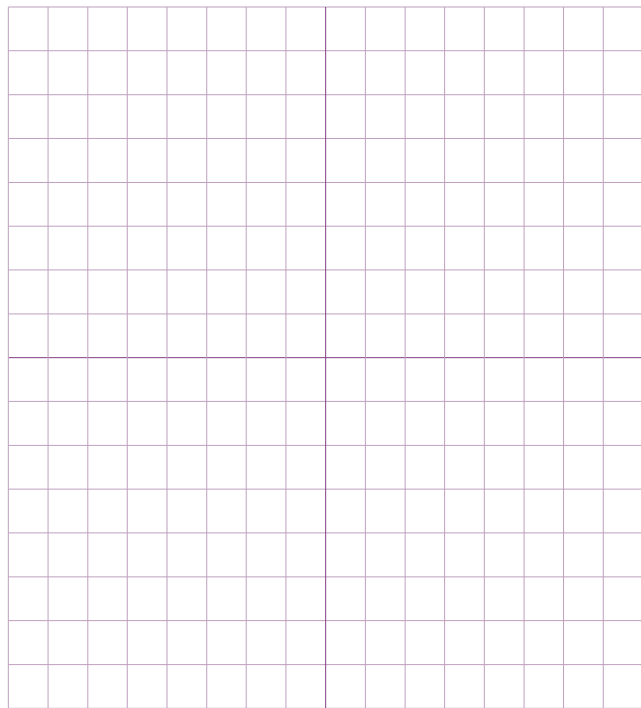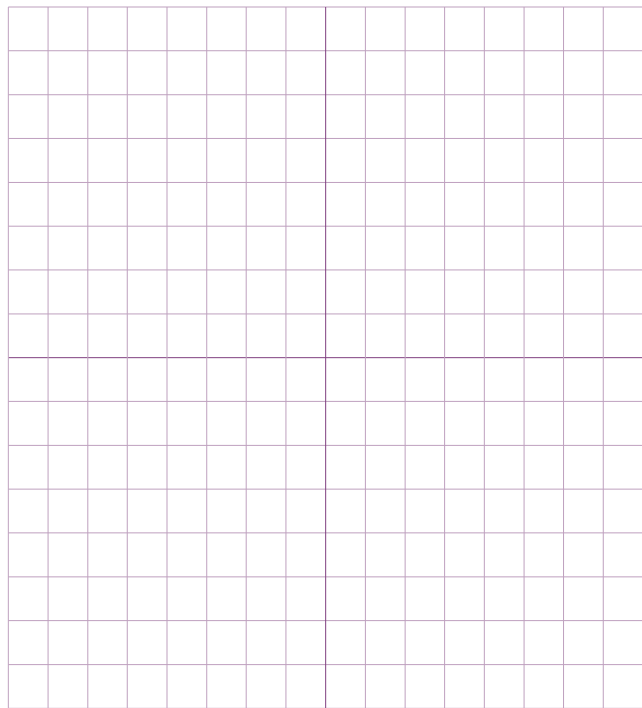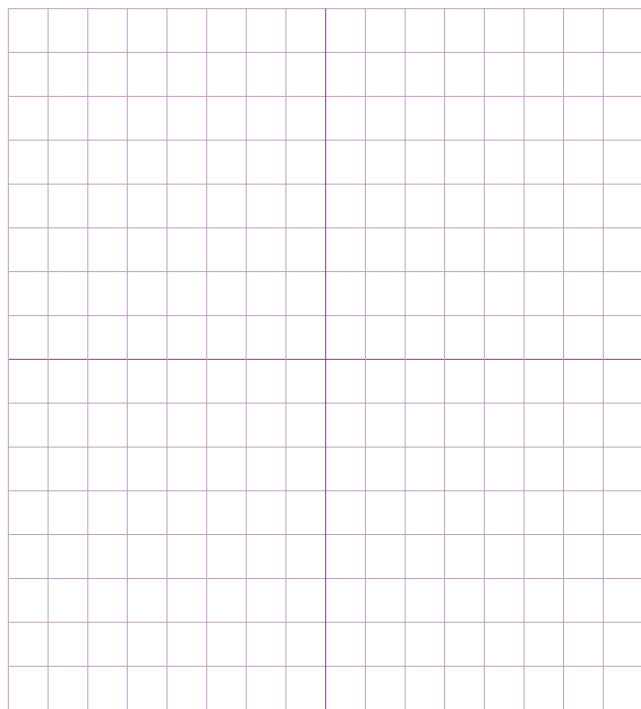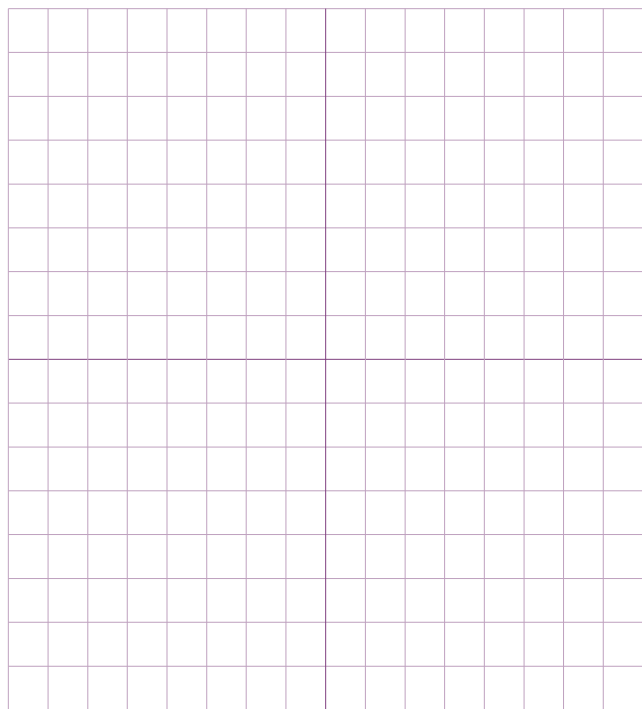
$$S = \frac{a}{1 - r}$$

You may detach this page for convenient reference.
Exercise care when tearing along perforations.

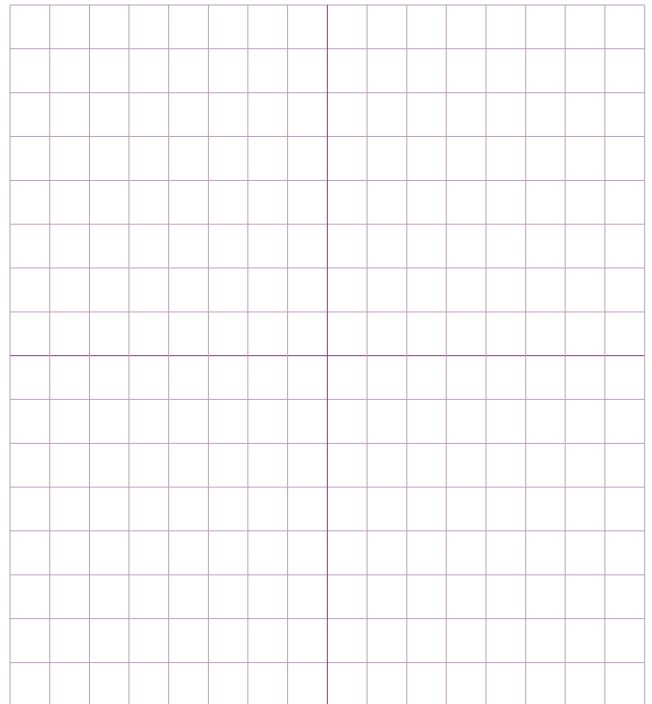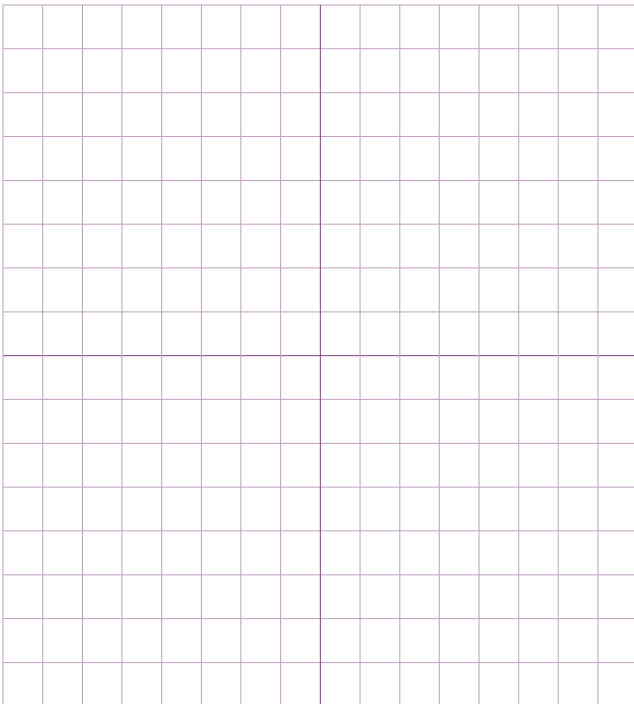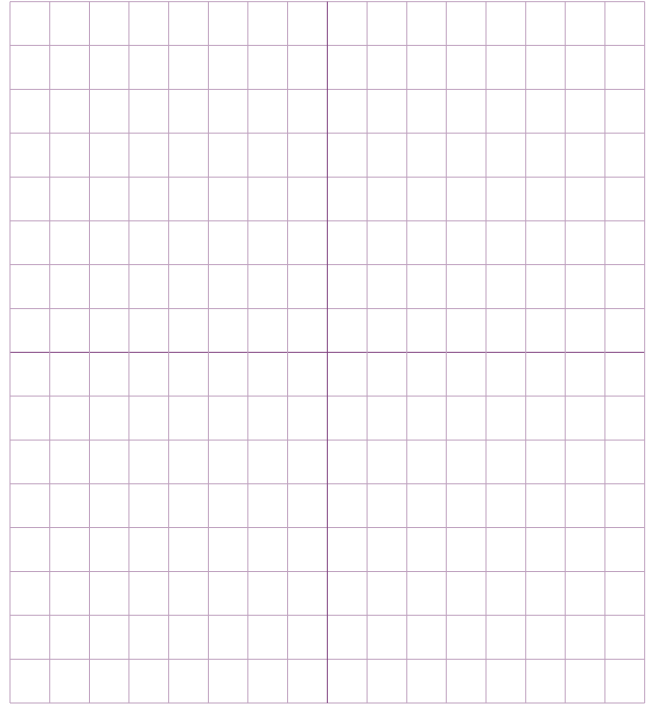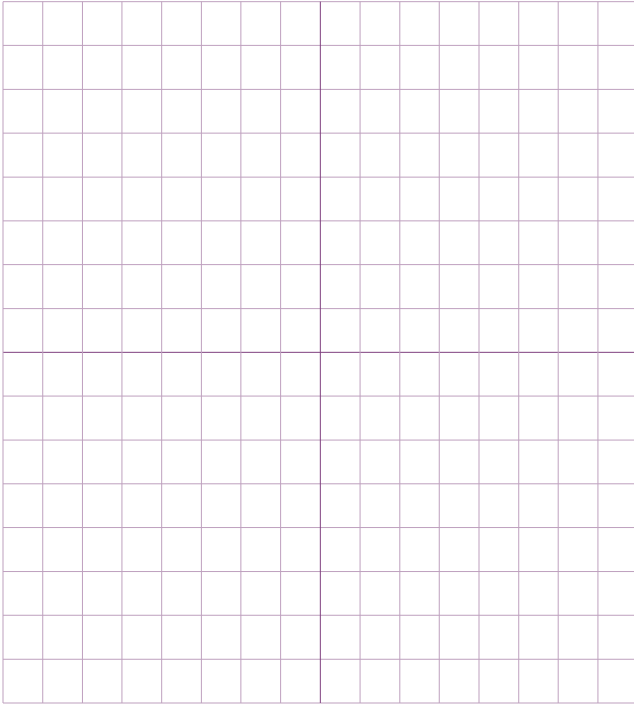THIS PAGE INTENTIONALLY BLANK

Rough Work for Graphing

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Rough Work for Multiple Choice