INSERT STUDENT I.D. NUMBER (PEN)

STICKER IN THIS SPACE

APRIL 1996

PROVINCIAL EXAMINATION

MINISTRY OF EDUCATION

MATHEMATICS 12

GENERAL INSTRUCTIONS

1. Insert the stickers with your Student I.D. Number (PEN) in the allotted spaces above. **Under no circumstance is your name or identification, other than your Student I.D. Number, to appear on this paper.**

2. Take the separate Answer Sheet and follow the directions on its front page.

3. Be sure you have an **HB pencil** and an eraser for completing your Answer Sheet. Follow the directions on the Answer Sheet when answering multiple-choice questions.

4. For each of the written-response questions, write your answer in the space provided.

5. When instructed to open this booklet, **check the numbering of the pages** to ensure that they are numbered in sequence from page one to the last page, which is identified by

   END OF EXAMINATION.

6. At the end of the examination, place your Answer Sheet inside the front cover of this booklet and return the booklet and your Answer Sheet to the supervisor.

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FOR OFFICE USE ONLY

INSERT STUDENT I.D. NUMBER (PEN)

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MATHEMATICS 12 APRIL 1996 PROVINCIAL

Course Code = MA Examination Type = P

1. _____ (3)

2. _____ (2)

3. _____ (3)

4. _____ (3)

5. _____ (2)

6. _____ (1)

7. _____ (4)

8. _____ (2)

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MATHEMATICS 12 PROVINCIAL EXAMINATION

		Value	Suggested Time
1.	This examination consists of **two** parts:		
	PART A: 50 multiple-choice questions	50	75
	PART B: 7 written-response questions 2 questions worth **two** marks each, 4 questions worth **three** marks each, and 1 question worth **four** marks.	20	45
	Total:	**70 marks**	**120 minutes**

2. The last **three** pages inside the back cover contain **A Summary of Basic Identities and Formulae**, **Rough Work for Graphing**, and **Rough Work for Multiple-Choice**. These pages may be detached for convenient reference prior to writing this examination.

3. You will not be provided with any additional paper since rough-work space for the written-response questions has been incorporated into the space allowed for answering each question. You may not need all of the space provided to answer each question.

4. An approved scientific calculator is essential for the examination. The calculator must be a hand-held device designed **only** for mathematical computations such as logarithmic and trigonometric functions. It **can be** programmable, but **must not** contain any graphing capabilities. You **must not** bring into the examination room any devices to support calculators such as manuals, printed or electronic cards, printers, memory expansion chips or cards, or keyboards.

5. You are permitted to use rulers, compasses, and protractors.

6. You have **two hours** to complete this examination.

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PART A: MULTIPLE CHOICE

Value: 50 marks **Suggested Time: 75 minutes**

INSTRUCTIONS: For each question, select the **best** answer and record your choice on the Answer Sheet provided. Using an HB pencil, completely fill in the circle that has the letter corresponding to your answer.

1. Which conic is described by $x^2 + 3y^2 = 6$?

   A. circle
   B. ellipse
   C. parabola
   D. hyperbola

2. Determine the distance between the points $\mathrm{A}(5, -2)$ and $\mathrm{B}(-3, 6)$.
   (Accurate to 2 decimal places.)

   A. 4.47
   B. 8.25
   C. 8.94
   D. 11.31

3. Determine the midpoint of the line segment joining points $\mathrm{A}(-9, -3)$ and $\mathrm{B}(5, 9)$.

   A. $(-2, 3)$
   B. $(-3, 1)$
   C. $(-4, 6)$
   D. $(-7, -6)$

OVER

4. What is the range of the quadratic relation $(x+3)^2+(y-1)^2=4$?

A. $-5 \le y \le 3$
B. $-3 \le y \le 5$
C. $-3 \le y \le 1$
D. $-1 \le y \le 3$

5. Determine the solution of $\left|\frac{x}{3}-1\right|>2$.

A. $x<-6$ or $x>7$
B. $x<-3$ or $x>9$
C. $x<-1$ or $x>3$
D. $x<1$ or $x>5$

6. Which of the following is the standard form of the equation $x^2-2x-2y^2-3=0$?

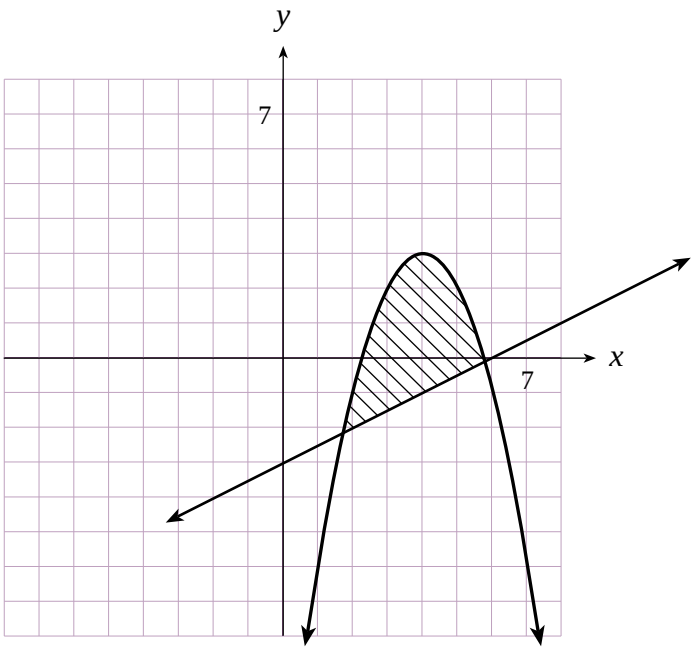
A. $\frac{(x-1)^2}{4}-\frac{y^2}{2}=1$

B. $\frac{(x-1)^2}{2}-y^2=1$

C. $y^2-\frac{(x-1)^2}{2}=1$

D. $\frac{y^2}{2}-\frac{(x-1)^2}{4}=1$

7. Which system describes the shaded region in the diagram below if the equation of the parabola is $y = -(x-4)^2 + 3$ and the equation of the line is $y = \frac{1}{2}x - 3$?



A. $y \le -(x-4)^2 + 3$
$y \le \frac{1}{2}x - 3$

B. $y \le -(x-4)^2 + 3$
$y \ge \frac{1}{2}x - 3$

C. $y \ge -(x-4)^2 + 3$
$y \le \frac{1}{2}x - 3$

D. $y \ge -(x-4)^2 + 3$
$y \ge \frac{1}{2}x - 3$

8. Solve the system:

$$x^2 + y^2 = 1$$
$$x - 1 = y^2$$

A. $(1,\ 0)$

B. $(2,\ 1)\ ,\ (2,\ -1)$

C. $(1,\ 0)\ ,\ \left(-2,\ \sqrt{3}\right)\ ,\ \left(-2,\ -\sqrt{3}\right)$

D. $(-1,\ 0)\ ,\ (0,\ 1),\ (0,\ -1)$

OVER

9. Determine **all** values of k such that the following system has exactly four different real solutions.

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

$$\frac{y^2}{k} - \frac{x^2}{16} = 1$$

A. $k > 9$
B. $k > 25$
C. $0 < k < 9$
D. $0 < k < 25$

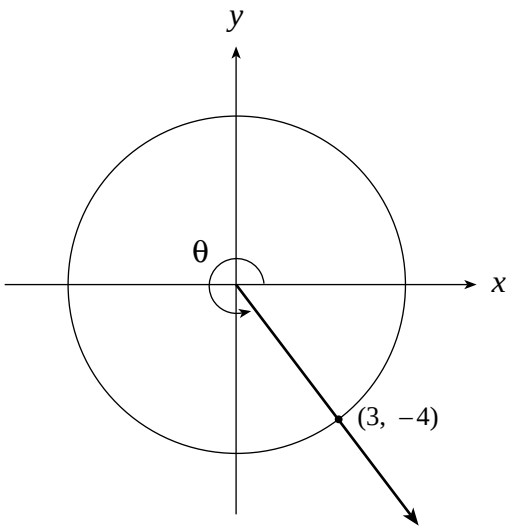
10. An ellipse is defined by $8x^2 + 4y^2 = p$, and its **major** axis is 6 units long. Find the value of p.

A. 12
B. 24
C. 36
D. 72

11. Convert $124°$ to radians. (Accurate to 2 decimal places.)

A. 2.07
B. 2.16
C. 2.18
D. 2.48

12. Given standard position angle θ as shown in the diagram, find $\cot\theta$.



A. $-\frac{4}{3}$

B. $-\frac{3}{4}$

C. $\frac{3}{4}$

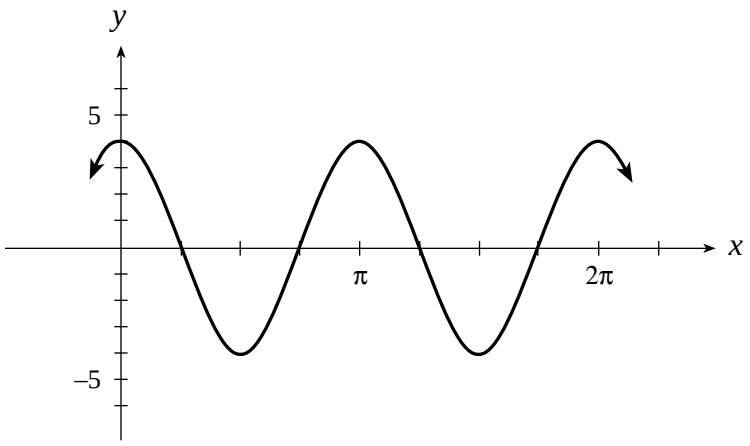
D. $\frac{4}{3}$

13. Determine the minimum value of the function $y = -3\sin x + 2$.

A. –5
B. –3
C. –1
D. 5

OVER

14. Determine an equation for the following graph.



A. $y = 4\cos 2x$

B. $y = 8\cos 2x$

C. $y = 4\cos \frac{1}{2}x$

D. $y = 8\cos \frac{1}{2}x$

15. Given that $\sin\theta = \frac{4}{5}$, where $90° < \theta < 180°$, evaluate $\sin(\theta - 270°)$.

A. $-\frac{4}{5}$

B. $-\frac{3}{5}$

C. $\frac{3}{5}$

D. $\frac{4}{5}$

16. Solve for θ: $\csc\theta - 4 = 0$, $0° \le \theta < 360°$. (Accurate to the nearest degree.)

A. 14° , 104°
B. 14° , 166°
C. 76° , 104°
D. 76° , 284°

17. Solve for θ: $\sin\theta = \cos\theta$, $0 \le \theta < 2\pi$. (Accurate to 2 decimal places.)

A. 0.00 , 3.14
B. 0.79 , 2.36
C. 0.79 , 3.93
D. 0.79 , 2.36 , 3.93 , 5.50

18. Which of the given values of b will make the graph of $y = 3\sin\frac{\pi}{4}(x-b)$ identical to the graph of $y = -3\cos\frac{\pi}{4}x$?

A. 2
B. 4
C. 6
D. 8

19. Evaluate: $\log_2 16$

A. $\frac{1}{4}$
B. 4
C. 8
D. 256

20. Determine an equation of the asymptote of the graph of $y = 2^x - 1$.

A. $y = -1$
B. $y = 0$
C. $y = 1$
D. $y = 2$

21. Give the equation of the inverse function of $y = 4^x$.

A. $x = \log_4 y$
B. $x = \log_y 4$
C. $y = \log_x 4$
D. $y = \log_4 x$

OVER

22. Determine an expression for $\log 45$ if $\log 5 = a$ and $\log 3 = b$.

A. $a + b^2$
B. $a + 2b$
C. $2a + 2b$
D. $3a + 3b$

23. Solve for x: $\log_2(\log_{16} x) = -1$

A. 2
B. 4
C. 8
D. 16

24. If $(p,\ q)$ is a point on the graph of $y = \log_b x$, then which point must be on the graph of $y = -\log_b x$?

A. $(p,\ -q)$
B. $(p,\ q)$
C. $(-p,\ q)$
D. $(-p,\ -q)$

25. Solve for x: $a^{3x} = b^{2x+1}$

A. 1

B. $\log\left(\frac{1}{a^3 b}\right)$

C. $\frac{\log b}{3\log a - 2}$

D. $\frac{\log b}{3\log a - 2\log b}$

26. According to the Rational Root Theorem, determine all possible rational roots of $8x^4 - 3x^2 + 4x - 1 = 0$.

A. $1\ ,\ 2\ ,\ 4\ ,\ 8$

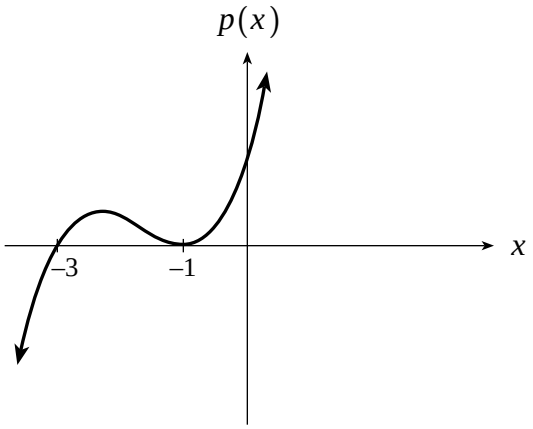
B. $1\ ,\ \frac{1}{2}\ ,\ \frac{1}{4}\ ,\ \frac{1}{8}$

C. $\pm 1\ ,\ \pm 2\ ,\ \pm 4\ ,\ \pm 8$

D. $\pm 1\ ,\ \pm\frac{1}{2}\ ,\ \pm\frac{1}{4}\ ,\ \pm\frac{1}{8}$

27. Select the graph of the polynomial function $p(x) = -(x-3)(x-1)^2$.

A.



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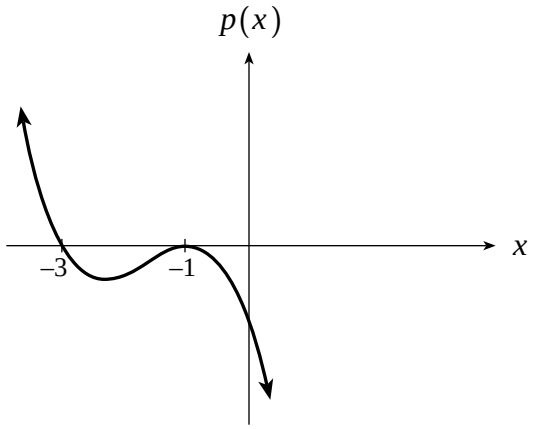

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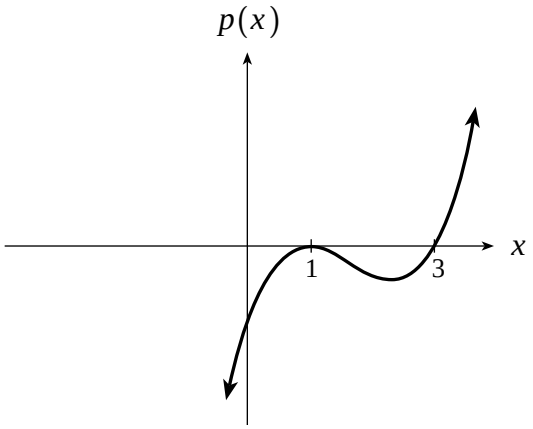

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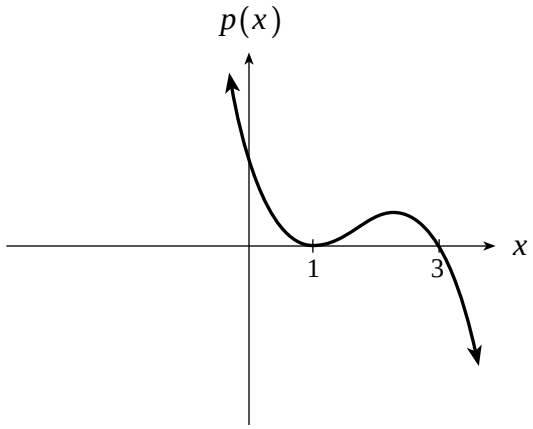


OVER

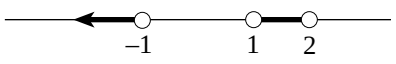
28. Determine the number of real roots of the equation $6x(x+7)\left(x^2-9\right)\left(x^2+4\right)=0$.

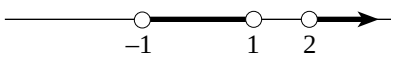
A. 3
B. 4
C. 5
D. 6

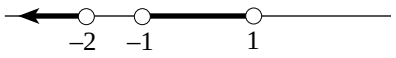
29. One root of $2x^3-7x^2+2x+3=0$ is $x=-\frac{1}{2}$. Determine the other roots.

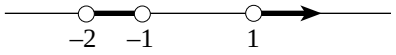
A. -1 , 1
B. -1 , -3
C. -3 , 3
D. 1 , 3

30. Determine the solution of $x^3-2x^2-x+2<0$.

A. 

B. 

C. 

D. 

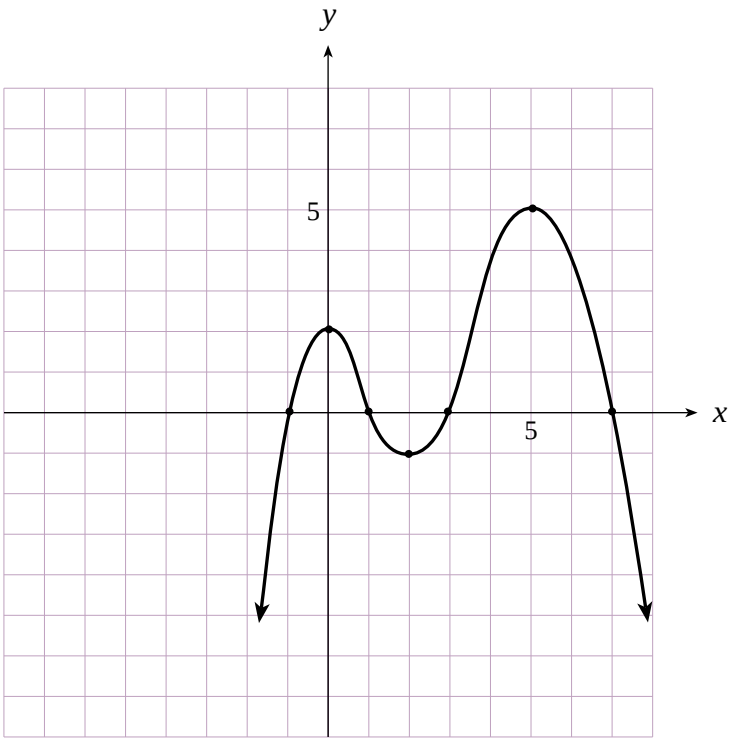
31. If $x-3$ is a factor of $x^3-13x+k$, then the value of k is:

A. -48
B. -12
C. 12
D. 30

32. Between which **two** consecutive x-values must a real zero be located for $p(x) = 2x^3 - x^2 + 6$?

A. -2 and -1
B. -1 and 0
C. 0 and 1
D. 1 and 2

33. The graph of the function $y = f(x)$ is given below. How many **positive** zeros has the function $y = f(x+5) - 4$?



A. 1
B. 2
C. 3
D. 4

34. Which of the following is an arithmetic sequence?

A. 1 , 2 , 3 , 5 , 7
B. 1 , 2 , 4 , 8 , 16
C. 1 , 3 , 4 , 7 , 11
D. 1 , 3 , 5 , 7 , 9

OVER

35. If two arithmetic means are located between 12 and 324, which of the following would be one of them?

A. 36
B. 104
C. 108
D. 116

36. Evaluate: $\sum_{k=1}^{200}(4k-3)$

A. 79 600
B. 79 700
C. 79 800
D. 80 200

37. Determine the 18th term of the geometric sequence: $\frac{1}{x}$, x, …

A. x^{16}
B. x^{17}
C. x^{33}
D. x^{35}

38. Determine the sum of the following infinite geometric series:

$$\sin\frac{\pi}{2} + \frac{1}{4}\sin\frac{\pi}{4} + \frac{1}{16}\sin\frac{\pi}{6} + \cdots$$

A. 1.14
B. 1.21
C. 1.28
D. 1.33

39. Find $f'(x)$, if $f(x) = 3x^4$.

A. $3x$

B. $12x$

C. $12x^3$

D. $12x^5$

40. Find: $\lim_{n\to\infty}\left(\frac{2-n^2}{3n^3+6}\right)$

A. $-\frac{1}{3}$

B. 0

C. $\frac{1}{9}$

D. no limit exists

41. Determine the slope of the line tangent to the graph of $y = \frac{1}{x}$ at $x = 4$.

A. -16

B. $-\frac{1}{16}$

C. $\frac{1}{16}$

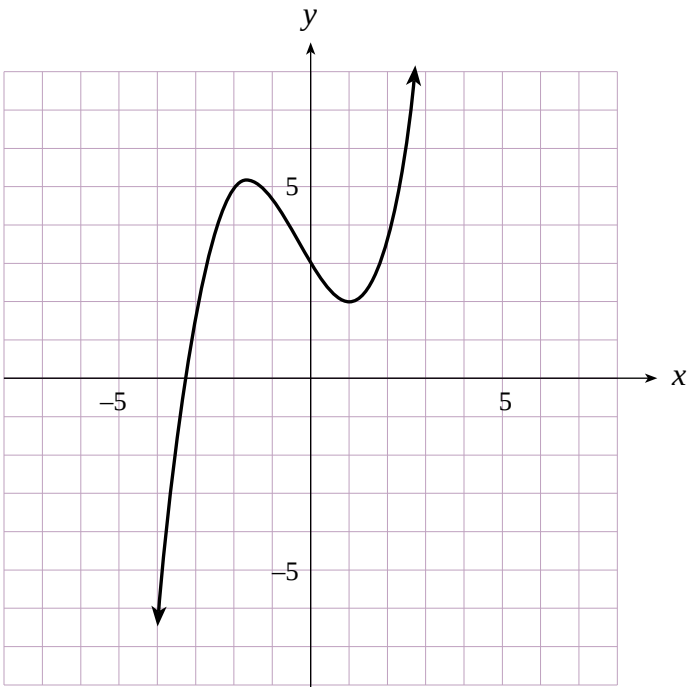
D. 16

42. Determine the maximum value of the function $f(x) = -2x^2 - x + 6$.

A. -0.25
B. 0
C. 5.625
D. 6.125

OVER

43. The graph of the function $y = f(x)$ is given below. Determine $\lim_{x \to 0} f(x)$.

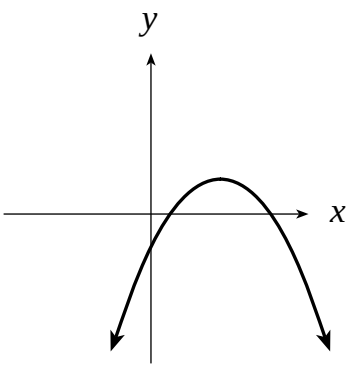


A. 0
B. 1
C. 2
D. 3

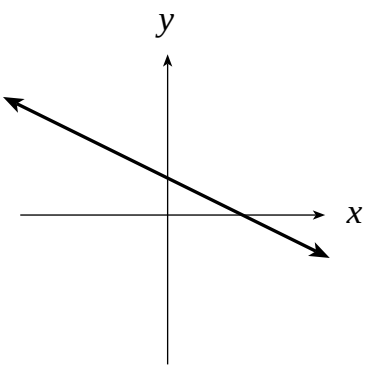
44. A particle moves along the *x*-axis so that its position at time t is $x(t) = 2t^2 - 7t + 3$ (x in cm and t in seconds). What is the velocity (in cm/sec) at time $t = 2$ seconds?

A. -6
B. -3
C. 1
D. 4

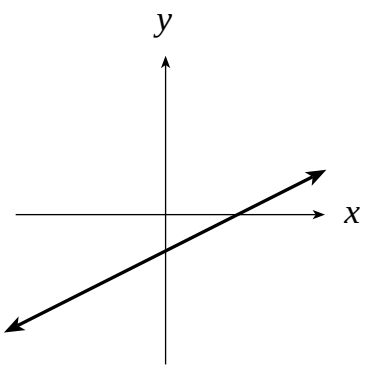
45. The graph of $y = f(x)$ is shown below. Select the graph which **best** represents $y = f'(x)$.



A.



B.



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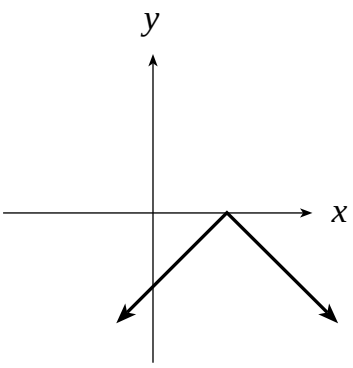
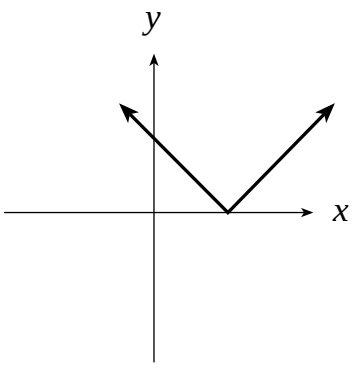
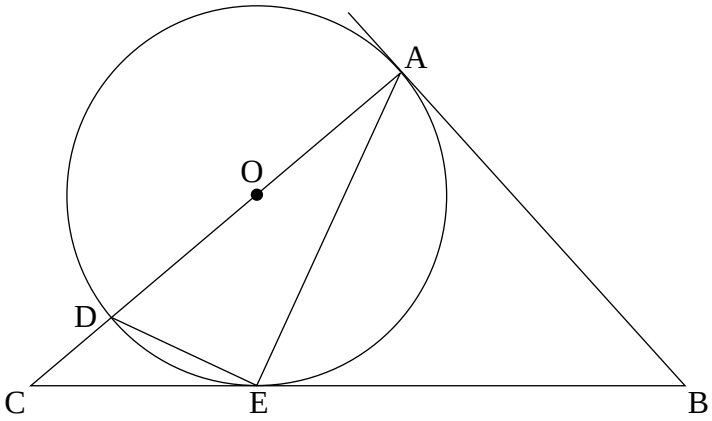


D.



OVER

46. AB and BC are tangents to the circle with centre O. $\angle CED = 20°$. Determine the measure of $\angle DCE$. (Diagram is not drawn to scale.)

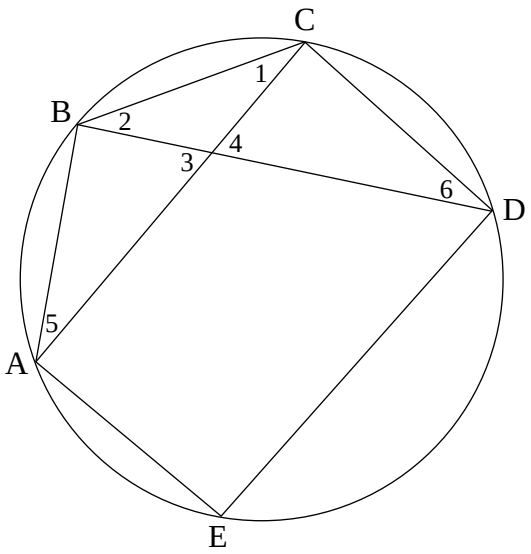


A. 40°
B. 50°
C. 60°
D. 70°

Use the following diagram and proof to answer question 47.

Given: AB = DC

Prove: AC = DB



Proof

	Statement	Reason
	AB = DC	given
(a)	$\angle 1 = \angle 2$	inscribed ∠s on = chords are =
(b)	$\angle 3 = \angle 4$	vertically opposite ∠s are =
(c)	$\angle 5 = \angle 6$	inscribed ∠s of same chord are =
(d)	$\angle ABC = \angle DCB$	3rd ∠s of Δs are =
	$\Delta ABC \cong \Delta DCB$	ASA
	AC = DB	CPCTC

47. Which line is **not** necessary in the given proof?

A. a
B. b
C. c
D. d

OVER

48. Consider a probability experiment conducted with three fair regular dice. What is the probability of rolling three dice with a sum of 4?

A. $\frac{1}{9}$

B. $\frac{1}{54}$

C. $\frac{1}{72}$

D. $\frac{1}{216}$

49. A fixed point of a function is a real number, x, for which $f(x) = x$. Find the fixed points for $f(x) = -x^2 + 12$.

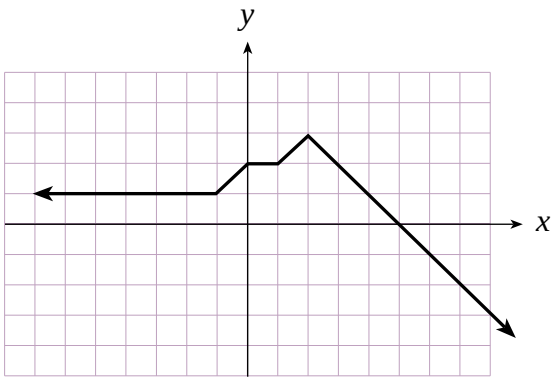
A. $x = -4$, $x = 3$

B. $x = -4$, $x = -3$

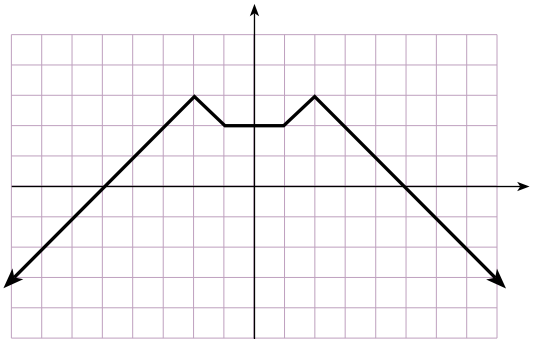
C. $x = 4$, $x = 3$

D. $x = 4$, $x = -3$

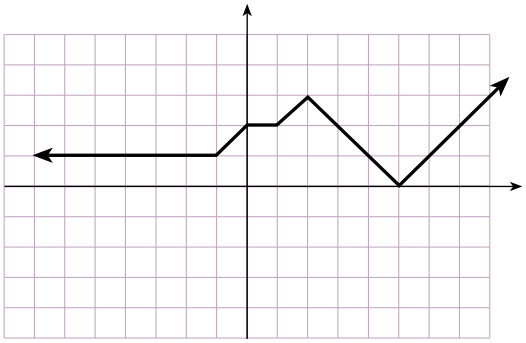
50. Given the graph of the function $f(x)$ below, select the graph of the function $g(x) = f(|x|)$.



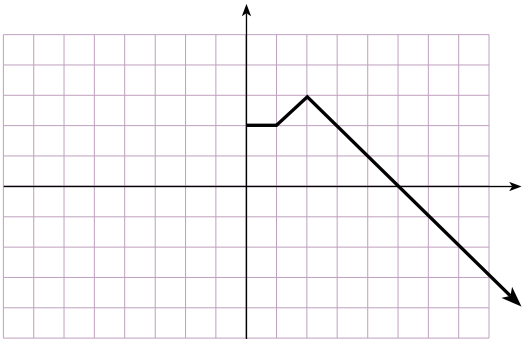
A.



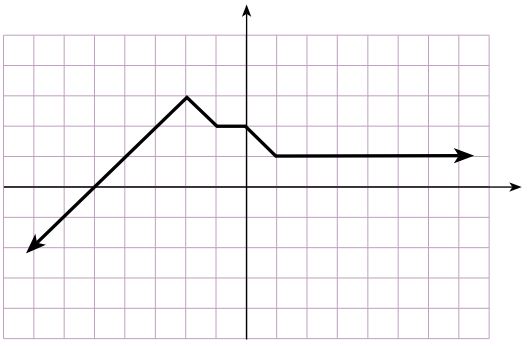
B.



C.



D.



This is the end of the multiple-choice section.
Answer the remaining questions directly in this examination booklet.

OVER

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PART B: WRITTEN RESPONSE

Value: 20 marks **Suggested Time: 45 minutes**

INSTRUCTIONS:	Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided. **Full marks will NOT be given for the final answer only.**

OVER

1. Solve: $\log_4(x-3)+\log_4(2x-4)=1$ **(3 marks)**

ANSWER:	Score for Question 1: 1. _____ (3)

OVER

2. Prove the following identity: **(2 marks)**

$$\sin\theta + \cos\theta \cot\theta = \csc\theta$$

Left side	Right side

Score for Question 2:
2. _____ (2)

OVER

3. A point $P(x, y)$ moves in such a way that it is always the same distance from $A(-5, 2)$ as it is from $B(2, -4)$. Determine the equation of this locus. Write the answer in a simplified form.

(3 marks)

ANSWER:	Score for Question 3: 3. _____ (3)

OVER

4. In a geometric series, the first term is 640 and the fourth term is 1080. Find the sum of the first 20 terms of this series. (Accurate to the nearest whole number.) **(3 marks)**

ANSWER:	Score for Question 4: 4. _____ (3)

OVER

5. a) Determine the x values of the critical points of $f(x) = x^4 - 8x^2$. **(2 marks)**

ANSWER:	Score for Question 5a: 5. _____ (2)

b) For what values of x is $f(x) = x^4 - 8x^2$ **decreasing**? **(1 mark)**

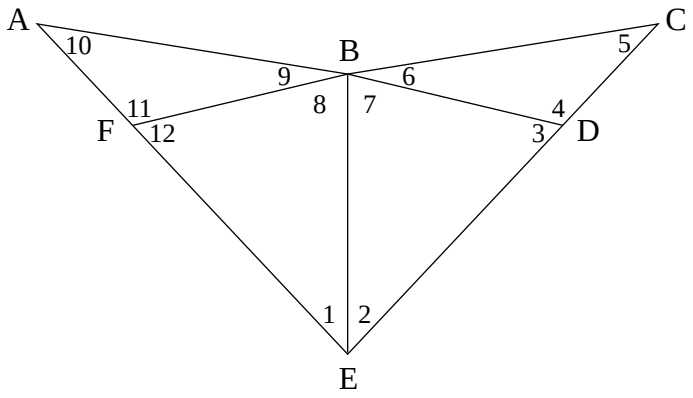
ANSWER:	Score for Question 5b: 6. _____ (1)

OVER

6. Complete the proof. **(4 marks)**

Given: $\angle 1 = \angle 2$
$EF = ED$

Prove: $\angle 11 = \angle 4$



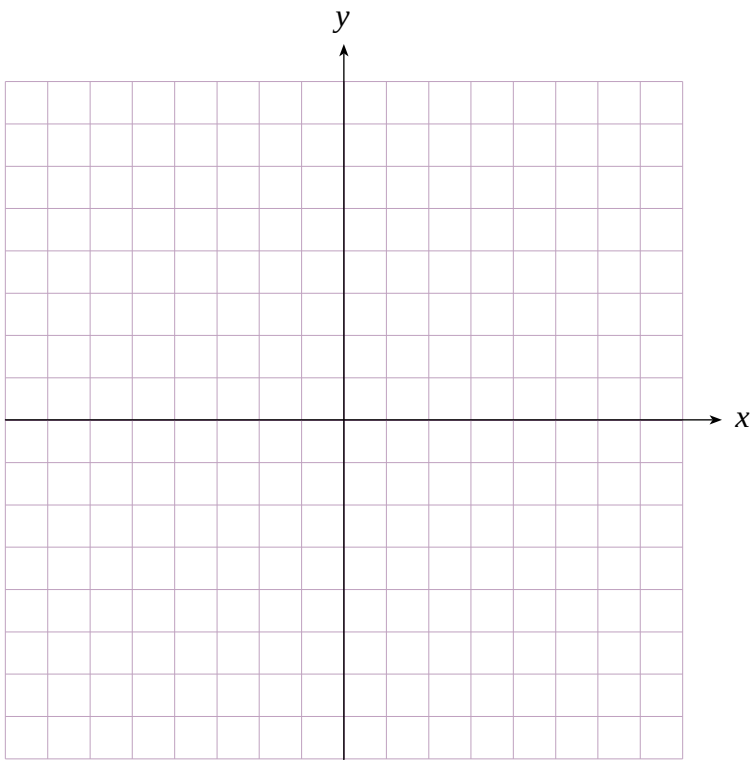
Proof

Statement	Reason

Score for Question 6:
7. _____ (4)

OVER

7. Graph: $\log_{x+1} y = \log_{x+1} x^2$ **(2 marks)**



Score for Question 7:

8. _____ (2)

END OF EXAMINATION

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A SUMMARY OF BASIC IDENTITIES AND FORMULAE

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$ $\qquad 1 + \tan^2\theta = \sec^2\theta$ $\qquad 1 + \cot^2\theta = \csc^2\theta$

Reciprocal and Quotient Identities

$\sec\theta = \dfrac{1}{\cos\theta}$ $\qquad \csc\theta = \dfrac{1}{\sin\theta}$ $\qquad \cot\theta = \dfrac{1}{\tan\theta}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\qquad \cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Addition Identities

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\qquad \tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\qquad \tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

Double-Angle Identities

$\cos 2\theta = \cos^2\theta - \sin^2\theta$ $\qquad \sin 2\theta = 2\sin\theta\cos\theta$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$ $\qquad \tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

Formulae

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\qquad t_n = a + (n-1)d$ $\qquad t_n = ar^{n-1}$

$\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$ $\qquad S_n = \dfrac{n}{2}\left[2a + (n-1)d\right]$ $\qquad S_n = \dfrac{a\left(1 - r^n\right)}{1 - r}$

$c^2 = a^2 + b^2 - 2ab\cos C$ $\qquad S_n = \dfrac{n}{2}(a + \ell)$ $\qquad S_n = \dfrac{a - \ell r}{1 - r}$

$S = \dfrac{a}{1 - r}$

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ROUGH WORK FOR GRAPHING

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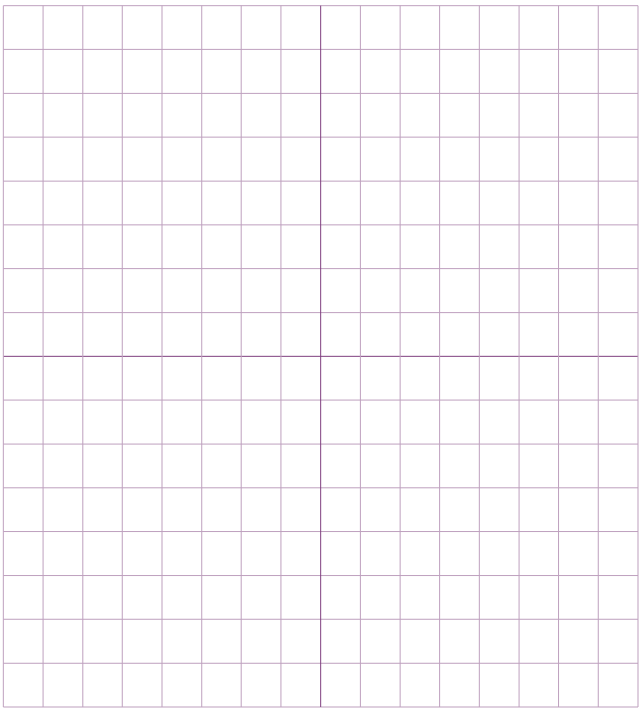

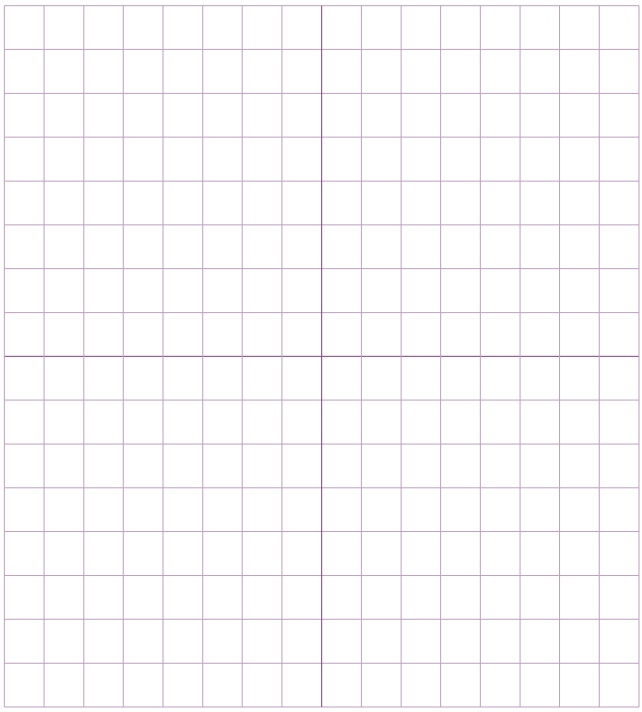

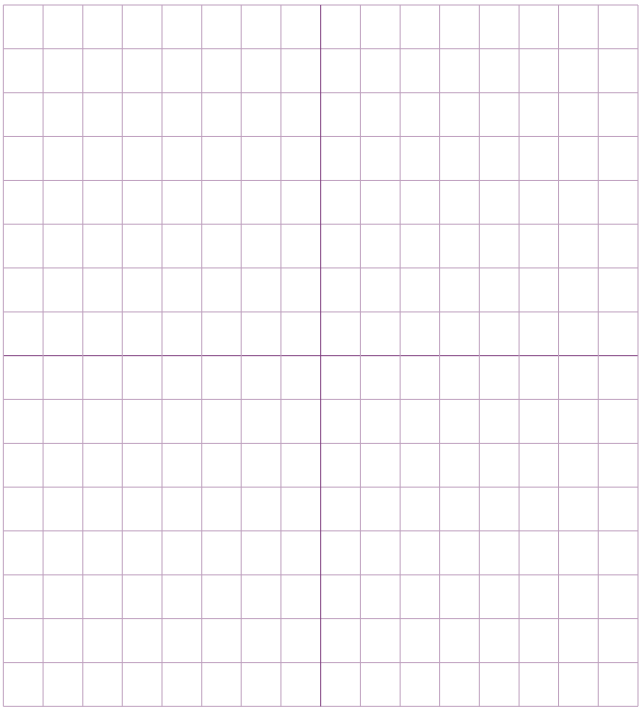

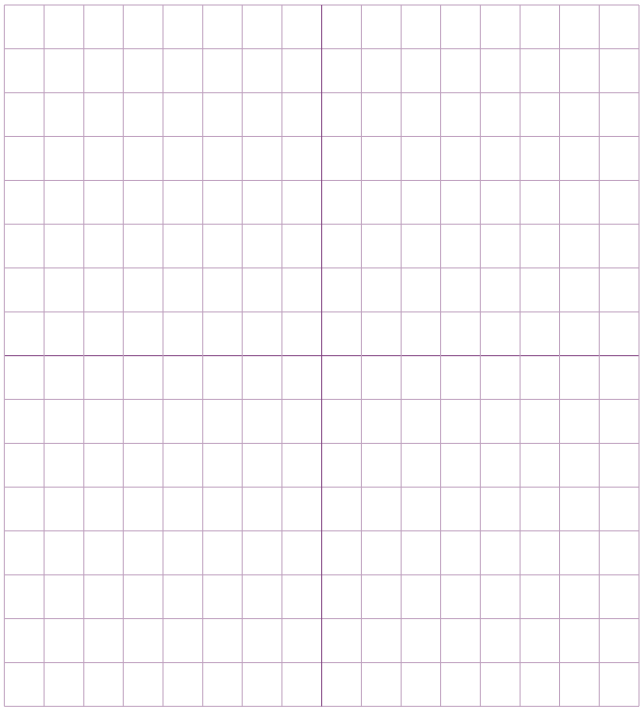

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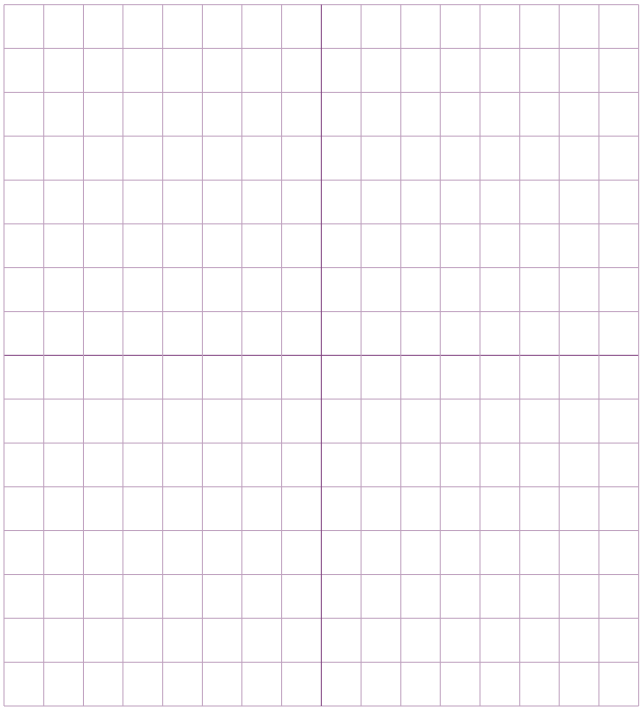

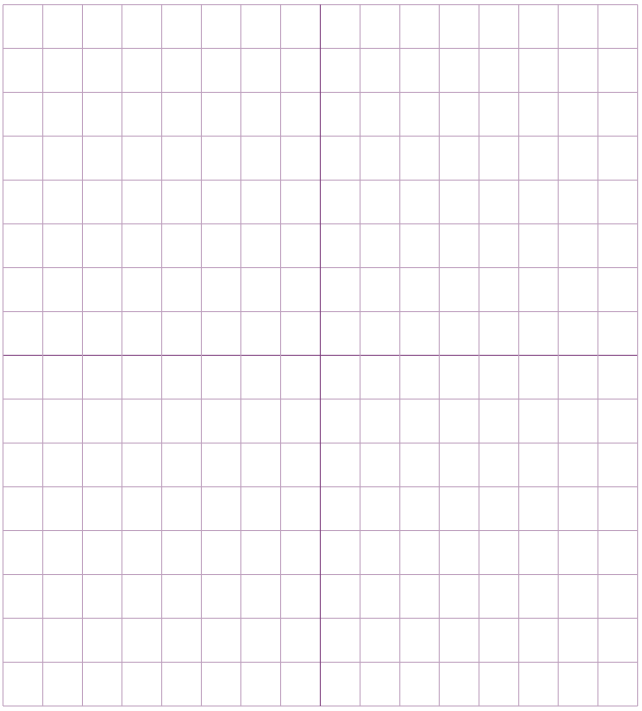

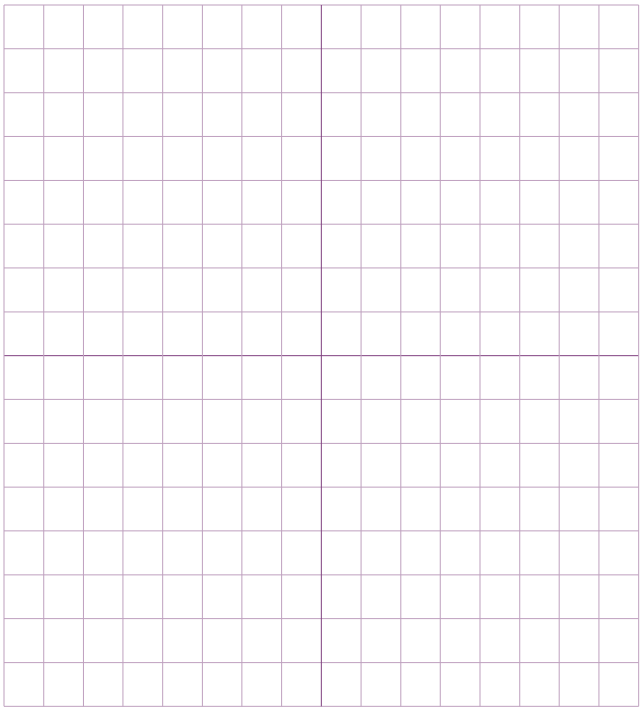

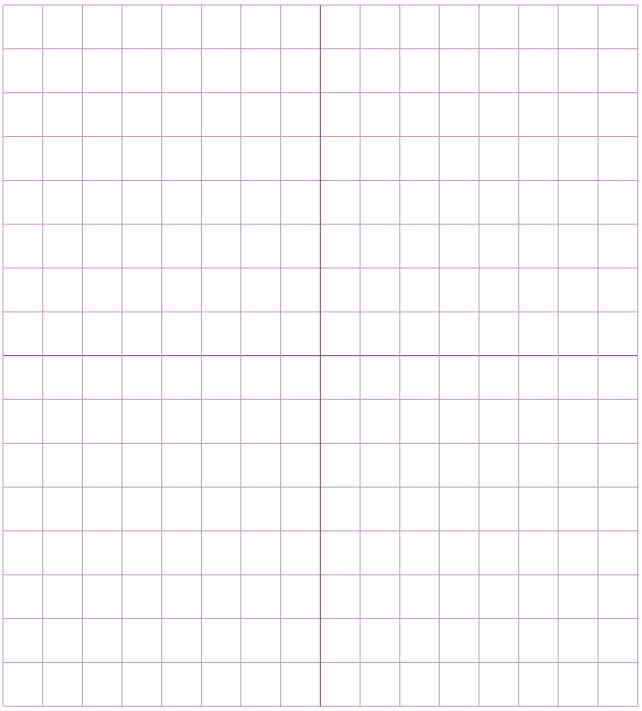

ROUGH WORK FOR MULTIPLE-CHOICE

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ROUGH WORK FOR MULTIPLE-CHOICE