

**AUGUST 1995**

## **PROVINCIAL EXAMINATION**

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**MINISTRY OF EDUCATION**

# **MATHEMATICS 12**

### **GENERAL INSTRUCTIONS**

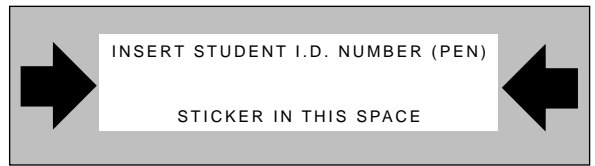
1. Insert the stickers with your Student I.D. Number (PEN) in the allotted spaces above. **Under no circumstance is your name or identification, other than your Student I.D. Number, to appear on this paper.**
2. Take the separate Answer Sheet and follow the directions on its front page.
3. Be sure you have an HB pencil and an eraser for completing your Answer Sheet. Follow the directions on the Answer Sheet when answering multiple-choice questions.
4. For each of the written-response questions, write your answer in the space provided.
5. When instructed to open this booklet, **check the numbering of the pages** to ensure that they are numbered in sequence from page one to the last page, which is identified by

**END OF EXAMINATION** .

6. At the end of the examination, place your Answer Sheet inside the front cover of this booklet and return the booklet and your Answer Sheet to the supervisor.

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**FOR OFFICE USE ONLY**



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**MATHEMATICS 12 AUGUST 1995 PROVINCIAL**

**Course Code = MA**

**Examination Type = P**

1.  $\frac{\quad}{(2)}$

5.  $\frac{\quad}{(3)}$

2.  $\frac{\quad}{(3)}$

6.  $\frac{\quad}{(3)}$

3.  $\frac{\quad}{(2)}$

7.  $\frac{\quad}{(2)}$

4.  $\frac{\quad}{(1)}$

8.  $\frac{\quad}{(4)}$

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## MATHEMATICS 12 PROVINCIAL EXAMINATION

		<b>Value</b>	<b>Suggested Time</b>
1.	This examination consists of <b>two</b> parts:		
	PART A 50 multiple-choice questions	50	75
	PART B 7 written-response questions	20	45
	2 questions worth <b>two</b> marks each, 4 questions worth <b>three</b> marks each, and 1 question worth <b>four</b> marks.		
	<b>Total:</b>	<b>70 marks</b>	<b>120 minutes</b>

- The last **three** pages inside the back cover contain **A Summary of Basic Identities and Formulae**, **Rough Work for Graphing**, and **Rough Work for Multiple-Choice**. These pages may be detached for convenient reference prior to writing this examination.
- You will not be provided with any additional paper since rough-work space for the written-response questions has been incorporated into the space allowed for answering each question. You may not need all of the space provided to answer each question.
- An approved scientific calculator is essential for the examination. The calculator must be a hand-held device designed **only** for mathematical computations such as logarithmic and trigonometric functions. It **can be** programmable, but **must not** contain any graphing capabilities. You **must not** bring into the examination room any devices to support calculators such as manuals, printed or electronic cards, printers, memory expansion chips or cards, or keyboards.
- You are permitted to use rulers, compasses, and protractors.
- You have **two hours** to complete this examination.

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## PART A: MULTIPLE-CHOICE

Value: 50 marks

Suggested Time: 75 minutes

**INSTRUCTIONS:** For each question, select the **best** answer and record your choice on the Answer Sheet provided. Using an HB pencil, completely fill in the circle that has the letter corresponding to your answer.

1. Find the distance between  $(7, -2)$  and  $(-3, -5)$ . (Accurate to 2 decimal places.)

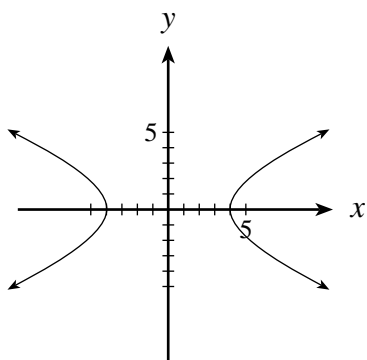
- A. 7.35
- B. 9.54
- C. 10.44
- D. 13.00

2. Which conic is described by the equation  $x - 4y^2 = 9$  ?

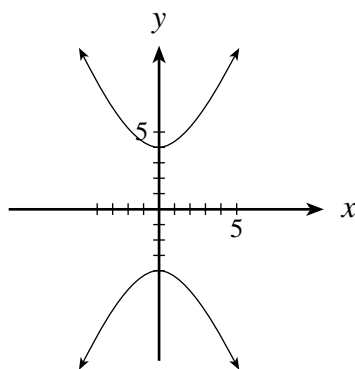
- A. circle
- B. ellipse
- C. parabola
- D. hyperbola

3. Which of the following graphs **best** represents  $\frac{x^2}{16} - \frac{y^2}{4} = -1$  ?

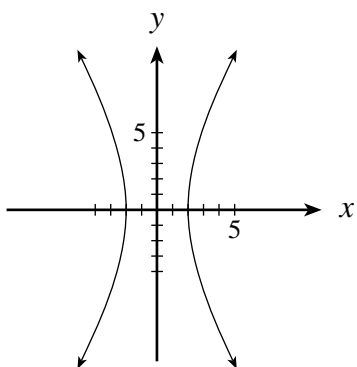
A.



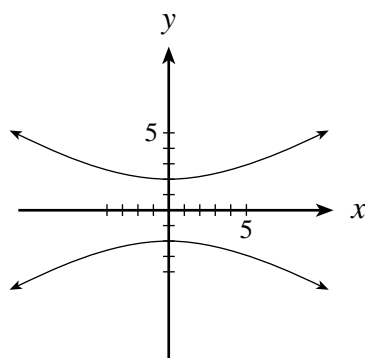
B.



C.



D.



4. What is the length of the major axis of the ellipse  $\frac{(x-3)^2}{9} + \frac{(y-4)^2}{16} = 1$  ?
- A. 3  
B. 4  
C. 6  
D. 8
5. Points ABCD are collinear with C as midpoint of AD, and B as midpoint of AC. Determine the coordinates of B if A has coordinates  $(-3, 7)$  and D has coordinates  $(3, -5)$ .
- A.  $(-1.5, 4)$   
B.  $(-1.5, 3)$   
C.  $(1.5, 1)$   
D.  $(1.5, 4)$
6. Write  $2x^2 + 2y^2 - 8x - 18 = 0$  in standard form.
- A.  $(x-2)^2 + y^2 = 9$   
B.  $(x-2)^2 + y^2 = 11$   
C.  $(x-2)^2 + y^2 = 13$   
D.  $(x-4)^2 + y^2 = 25$
7. Determine an equation for the set of all points which are 3 times as far from the point  $(0, 5)$  as they are from the point  $(-1, 2)$ .
- A.  $3\sqrt{x^2 + (y+5)^2} = \sqrt{(x-1)^2 + (y+2)^2}$   
B.  $3\sqrt{x^2 + (y-5)^2} = \sqrt{(x+1)^2 + (y-2)^2}$   
C.  $\sqrt{x^2 + (y+5)^2} = 3\sqrt{(x-1)^2 + (y+2)^2}$   
D.  $\sqrt{x^2 + (y-5)^2} = 3\sqrt{(x+1)^2 + (y-2)^2}$



8. A bridge over a river is supported by a parabolic arch which is 100 m wide at its base. If the maximum height of the arch is 10 m, determine which equation could represent the arch.

- A.  $y = -0.2x^2$
- B.  $y = -0.1x^2$
- C.  $y = -0.001x^2$
- D.  $y = -0.004x^2$

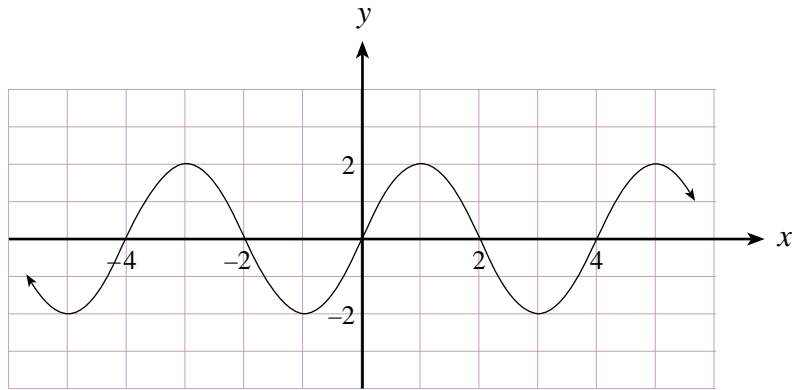
9. How many points of intersection are there for the following system?

$$x^2 + y^2 = 1$$

$$y = \sqrt{x}$$

- A. 1
  - B. 2
  - C. 3
  - D. 4
10. The equation  $Ax^2 + By^2 + Cy = 1$  represents an ellipse (not a circle). If  $A > 0$  and  $B > 0$ , what conditions **must** be satisfied if this ellipse has its minor axis on the  $x$ -axis?
- A.  $C \neq 0$  and  $A > B$
  - B.  $C \neq 0$  and  $A < B$
  - C.  $C = 0$  and  $A > B$
  - D.  $C = 0$  and  $A < B$

11. Determine the amplitude of the function graphed below.



- A. -4
- B. -2
- C. 2
- D. 4

12. Determine the period of the graph of  $y = 5 \cos 3\left(x - \frac{\pi}{2}\right)$ .

- A.  $\frac{2\pi}{5}$
- B.  $\frac{2\pi}{3}$
- C.  $2\pi$
- D.  $6\pi$

13. Evaluate:  $\cot 3.76$  (Accurate to 2 decimal places.)

- A. 0.71
- B. 0.76
- C. 1.31
- D. 1.41

14. Determine the reference angle for an angle of 4.45 radians.
- A. 0.26
  - B. 1.31
  - C. 1.83
  - D. 10.73
15. Determine all restrictions for  $\csc 2\theta$ .
- A.  $\sin \theta \neq 0$
  - B.  $\cos \theta \neq 0$
  - C.  $\sin \theta \neq 0, \cos \theta \neq 0$
  - D. no restrictions
16. Simplify:  $\frac{\csc^2 \theta - 2}{\csc^2 \theta}$
- A.  $-2 \sin^2 \theta$
  - B.  $2 \sin^2 \theta$
  - C.  $-\cos 2\theta$
  - D.  $\cos 2\theta$
17. How many solutions does  $\cos 4x = -1$  have over the interval  $0 \leq x < 2\pi$ ?
- A. 1
  - B. 2
  - C. 4
  - D. 8

18. Determine the value of  $\csc \theta$  if  $\tan \theta = a$ , where  $a > 0$ , and  $\theta$  terminates in quadrant 3.

A.  $\frac{\sqrt{a^2 + 1}}{-a}$

B.  $\frac{\sqrt{a^2 + 1}}{a}$

C.  $\frac{a + 1}{-a}$

D.  $\frac{a + 1}{a}$

19. Write  $y = \log_3 x$  in exponential form.

A.  $y = 3^x$

B.  $y = x^3$

C.  $x = y^3$

D.  $x = 3^y$

20. Solve for  $x$ :  $3^x = \frac{1}{20}$  (Accurate to 2 decimal places.)

A.  $-0.44$

B.  $-2.73$

C.  $0.44$

D.  $2.73$

21. Determine the equation of the asymptote for the graph of  $y = 3^{x-1}$ .

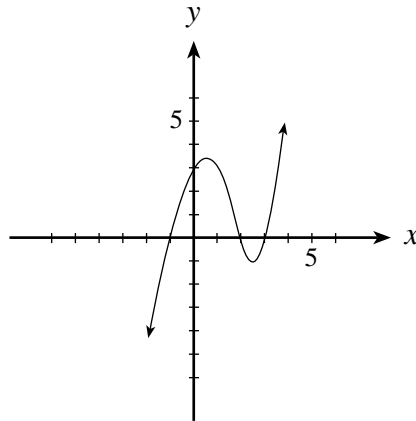
A.  $y = -1$

B.  $y = 0$

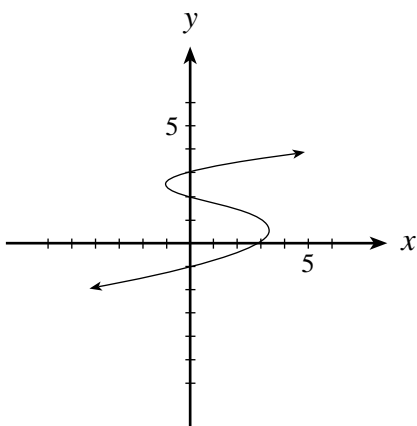
C.  $y = 1$

D.  $y = 3$

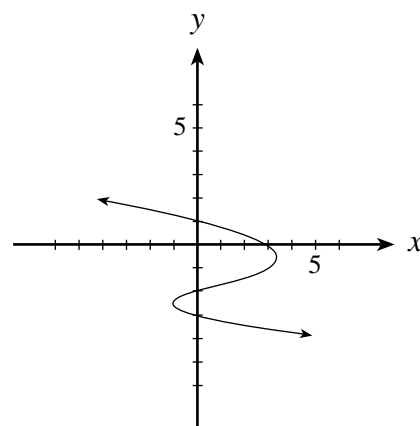
22. Which graph **best** represents the inverse relation of the graph shown.



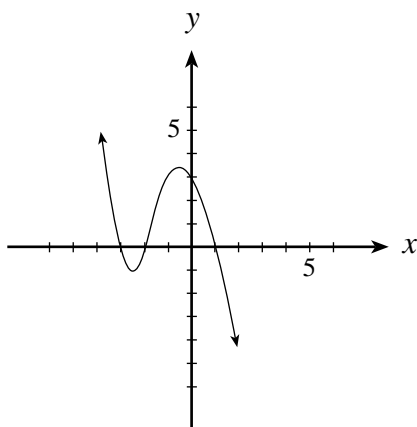
A.



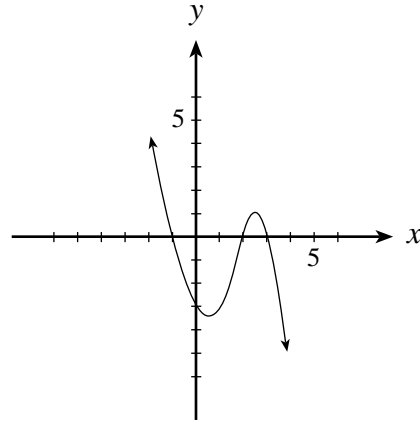
B.



C.



D.



23. Determine the  $x$ -intercept for the graph of  $y = \log_3(x+3)$ .

- A. -3
- B. -2
- C. 0
- D. 1

**OVER**

24. If  $\log x = a$  and  $\log y = b$ , express  $\log \left( \frac{10x^2}{y^2} \right)$  in terms of  $a$  and  $b$ .

- A.  $1 + a^2 - b^2$
- B.  $1 + 2a - 2b$
- C.  $10 + a^2 - b^2$
- D.  $10 + 2a - 2b$

25. Which expression is equivalent to  $\frac{\log_n p}{\log_n m}$  ?

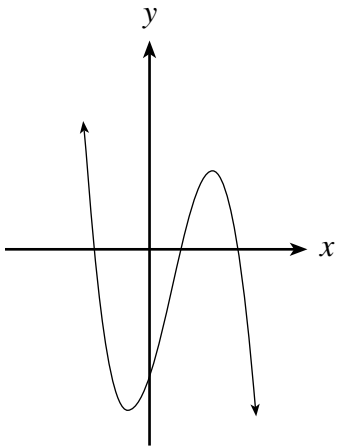
- A.  $\frac{p}{m}$
- B.  $\log_m p$
- C.  $\log_p m$
- D.  $\log p - \log m$

26. According to the Rational Root Theorem, list all possible rational roots of  $3x^4 - 5x^3 + 7x^2 - 4 = 0$ .

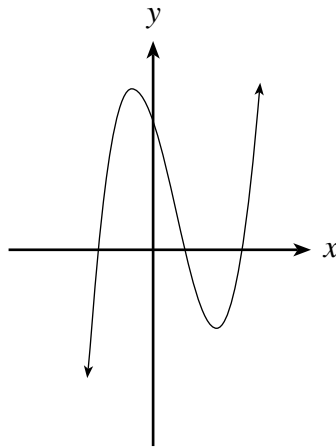
- A.  $\pm 1, \pm 3$
- B.  $\pm 1, \pm 2, \pm 4$
- C.  $\pm 1, \pm 3, \pm \frac{1}{4}, \pm \frac{1}{2}, \pm \frac{3}{4}, \pm \frac{3}{2}$
- D.  $\pm 1, \pm 2, \pm 4, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}$

27. Which graph **best** represents the function  $y = -(x - 3)(x + 2)(x - 1)$ .

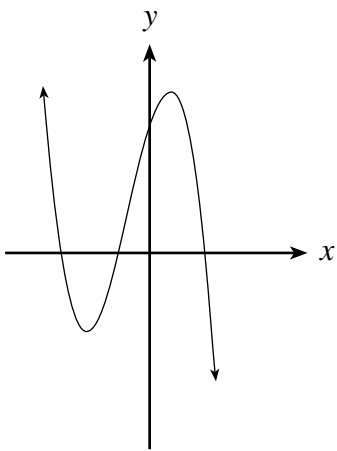
A.



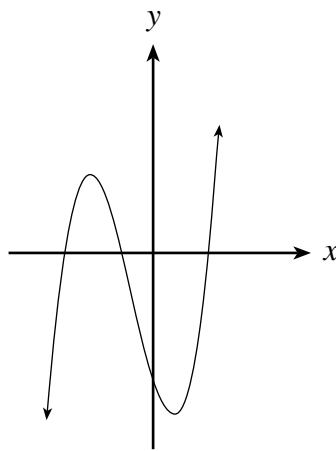
B.



C.



D.



28. Given the following table of values for the polynomial function  $y = f(x)$ , determine the minimum number of zeros for  $f(x)$ .

$x$	$y$
-3	-15
-2	-12
-1	2
0	3
1	5
2	-7
3	-19

- A. 1
- B. 2
- C. 3
- D. 4

29. Write a polynomial equation which has the following roots:  $0, \pm\sqrt{2}, \pm 3$
- A.  $x^5 - 11x^3 + 18x = 0$
  - B.  $x^5 + 11x^3 + 18x = 0$
  - C.  $x^5 - 7x^3 - 18x = 0$
  - D.  $x^5 + 7x^3 - 18x = 0$
30. Solve for  $x$ :  $x^3 - 4x^2 - 7x + 10 = 0$
- A.  $-5, -1, 2$
  - B.  $-5, 1, 2$
  - C.  $-2, 1, 5$
  - D.  $-2, -1, 5$
31. Determine  $k$  such that  $2$  is a root of the polynomial equation  $x^3 + 3x^2 + (5k)x + 2 = 0$ .
- A.  $-11$
  - B.  $-\frac{11}{5}$
  - C.  $\frac{11}{5}$
  - D.  $11$
32. Determine the remainder when  $x^3 + 2x^2 - 3x + 5$  is divided by  $x^2 - x - 3$ .
- A.  $14$
  - B.  $-7x + 2$
  - C.  $-3x - 4$
  - D.  $3x + 14$



33. Which absolute value inequality represents the solution of  $x^2 + 2x - 8 \leq 0$ ?
- A.  $|x+1| \leq 3$
  - B.  $|x+1| \geq 3$
  - C.  $|x-1| \leq 3$
  - D.  $|x-1| \geq 3$
34. Determine a single geometric mean between 4 and 16.
- A. 6
  - B. 8
  - C. 10
  - D. 12
35. Three consecutive terms of an arithmetic sequence are  $a$ ,  $b$ , and  $c$ . Which of the following statements will **always** be true?
- A.  $\frac{b}{a} = \frac{b}{c}$
  - B.  $\frac{b}{a} = \frac{c}{b}$
  - C.  $b - a = b - c$
  - D.  $b - a = c - b$
36. Find the sum of the infinite geometric series  $1 - 2 + 4 - 8 + \dots$
- A.  $-5$
  - B.  $-1$
  - C.  $\frac{1}{3}$
  - D. no finite sum

37. Use sigma notation to write the geometric series  $2 - 6 + 18 - \dots + 1458$ .

A.  $\sum_{k=1}^7 2(-3)^{k-1}$

B.  $\sum_{k=1}^7 2(3)^{k-1}$

C.  $\sum_{k=1}^7 2(-3)^k$

D.  $\sum_{k=1}^7 2(3)^k$

38. The recursive formula for a sequence is:  $t_1 = 5$

$$t_n = t_{n-1} - 3, \quad n > 1$$

Determine:  $t_{n+1} - t_{n-1}$

A.  $-6$

B.  $-3$

C.  $2$

D.  $6$

39. Evaluate:  $\lim_{n \rightarrow \infty} \frac{2n^3 - 1}{3n^3}$

A.  $0$

B.  $\frac{1}{3}$

C.  $\frac{2}{3}$

D. limit does not exist

40. Find the slope of the line tangent to the graph of  $f(x) = x^2 + 3$  at the point where  $x = -1$ .

- A. -2
- B. 1
- C. 2
- D. 4

41. Evaluate:  $\lim_{x \rightarrow 7} \frac{x^2 - 6x - 7}{x^2 - 5x - 14}$

- A. -1
- B.  $\frac{1}{2}$
- C.  $\frac{8}{9}$
- D. 1

42. Which expression represents the derivative of  $f(x) = \frac{1}{x^3}$  ?

A.  $\lim_{h \rightarrow 0} \frac{\frac{1}{(x+h)^3} + \frac{1}{x^3}}{h}$

B.  $\lim_{h \rightarrow 0} \frac{\frac{1}{(x+h)^3} - \frac{1}{x^3}}{h}$

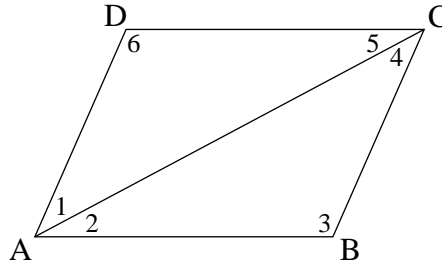
C.  $\lim_{x \rightarrow 0} \frac{\frac{1}{(x+h)^3} + \frac{1}{x^3}}{h}$

D.  $\lim_{x \rightarrow 0} \frac{\frac{1}{(x+h)^3} - \frac{1}{x^3}}{h}$

43. A regular polygon with  $n$  sides is inscribed in a circle of radius 5. As  $n \rightarrow \infty$ , what is the limit of the perimeter of the polygon?
- A.  $5\pi$
  - B.  $10\pi$
  - C.  $25\pi$
  - D.  $50\pi$
44. If  $f'(x) = -6x$ , determine all values of  $x$  such that  $f(x)$  is decreasing.
- A.  $x > 0$
  - B.  $x < 0$
  - C.  $-6 < x < 0$
  - D. all real numbers
45. If  $y = 2ax + bx^2$ , and  $a$  and  $b$  are positive constants, determine the minimum value of  $y$ .
- A.  $-\frac{a}{b}$
  - B.  $\frac{a}{b}$
  - C.  $-\frac{a^2}{b}$
  - D.  $-\frac{3a^2}{b}$

Use the following diagram and proof to answer questions 46 and 47.

Given:  $DC \parallel AB$   
 $DC = AB$   
 Prove:  $AD = BC$



Proof	
Statement	Reason
$DC \parallel AB$	given
(a) $\angle 2 = \angle 5$	alternate interior $\angle$ s are =
(b) $\angle 1 = \angle 4$	alternate interior $\angle$ s are =
(c) $DC = AB$	given
(d) $AC = AC$	same side
(e) $\triangle ADC \cong \triangle CBA$	_____

46. Which line is **incorrect** in the given proof?

- A. a
- B. b
- C. c
- D. d

47. Determine the correct reason for line (e).

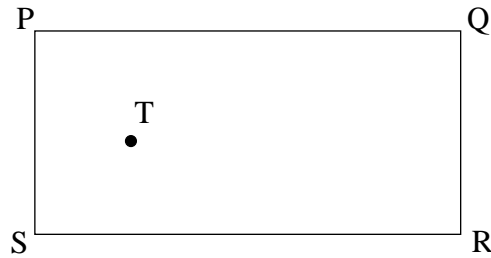
- A. SSS
- B. ASA
- C. AAS
- D. SAS

48. Determine the range of  $y = \sin x \cos x$ .

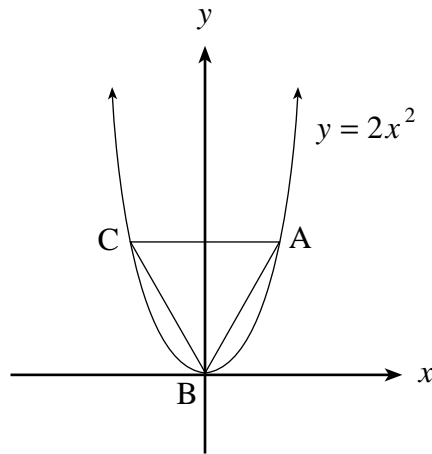
- A.  $-2 \leq y \leq 2$
- B.  $-1 \leq y \leq 1$
- C.  $-\frac{1}{2} \leq y \leq \frac{1}{2}$
- D.  $-\pi \leq y \leq \pi$

**OVER**

49. Point  $T$  is located inside rectangle  $PQRS$  such that  $TP = TS = 5$  and  $TQ = TR = 12$ . If  $PS = 6$ , determine  $PQ$ . (Accurate to 2 decimal places.)



- A. 14.91  
 B. 15.23  
 C. 15.62  
 D. 16.37
50. Points  $A$ ,  $B$ , and  $C$  are on the parabola  $y = 2x^2$  and  $\triangle ABC$  is equilateral. Determine the  $x$ -coordinate of point  $A$ .



- A.  $\frac{\sqrt{3}}{2}$   
 B.  $\sqrt{3}$   
 C. 2  
 D.  $2\sqrt{3}$

**This is the end of the multiple-choice section.  
 Answer the remaining questions directly in this examination booklet.**

**PART B: WRITTEN-RESPONSE**

**Value: 20 marks**

**Suggested Time: 45 minutes**

**INSTRUCTIONS:** Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

**Full marks will NOT be given for the final answer only.**

1. Determine the sum of the arithmetic series  $-6 - 3 + 0 + \dots + 621$ .

**(2 marks)**



ANSWER:

Score for  
Question 1:

1.  $\frac{\quad}{(2)}$

**OVER**

2. Solve for  $x$ :  $\log_2(3x+2) + \log_2(x-1) = 1$

**(3 marks)**

ANSWER:

Score for  
Question 2:

2.  $\frac{\quad}{(3)}$

**OVER**

3. Given the function  $f(x) = 2x^3 - 3x^2 - 12x + 4$ ,

a) determine the coordinates of the critical points of  $f(x)$ .

**(2 marks)**

ANSWER:	Score for Question 3a:  3. <u>        </u> (2)
---------	--

b) determine where  $f(x)$  is increasing.

**(1 mark)**

ANSWER:

Score for  
Question 3b:

4.           
(1)

**OVER**

4. Determine all real ordered pairs that satisfy the following system:

**(3 marks)**

$$y^2 - x^2 = 16$$

$$y = \frac{6}{x}$$

(Give answers that are exact **or** accurate to 2 decimal places.)

ANSWER:

Score for  
Question 4:

5.  $\frac{\quad}{(3)}$

**OVER**

5. Prove the identity.

**(3 marks)**

$$\tan^2 \theta - \sin^2 \theta = \tan^2 \theta \sin^2 \theta$$

Left side	Right side

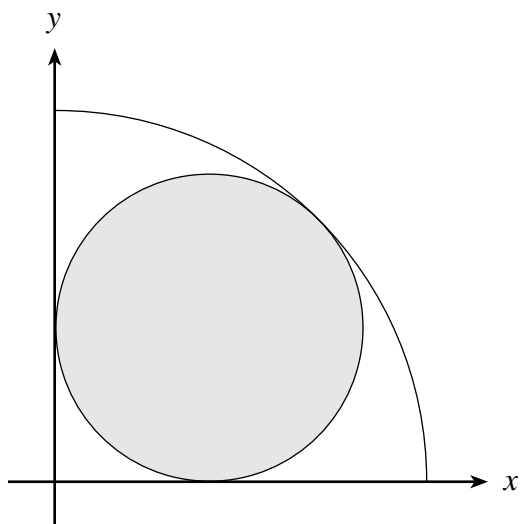


Score for  
Question 5:

6.  $\frac{\quad}{(3)}$

**OVER**

6. A circle is inscribed in the quadrant I sector of  $x^2 + y^2 = 121$ . Determine the area of the inscribed circle that is shaded. (Accurate to 1 decimal place.) **(2 marks)**



ANSWER:

Score for  
Question 6:

7.  $\frac{\quad}{(2)}$

**OVER**

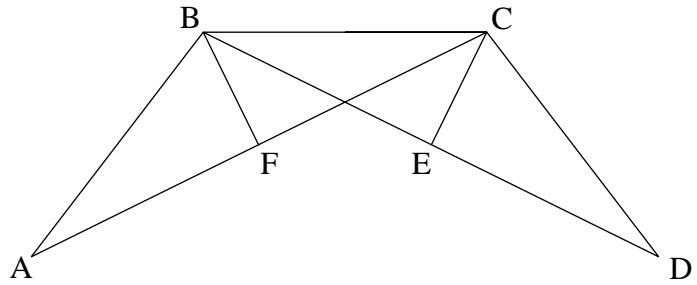
7. Complete the proof.

(4 marks)

Given: BF is the perpendicular bisector  
of AC  
CE is the perpendicular bisector  
of BD

Prove:  $AB = CD$

Note: Students are encouraged  
to number angles.



Statement	Proof	Reason

Score for  
Question 7:

8. \_\_\_\_\_  
(4)

**END OF EXAMINATION**

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## A SUMMARY OF BASIC IDENTITIES AND FORMULAE

### Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

### Reciprocal and Quotient Identities

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

### Addition Identities

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

### Double-Angle Identities

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - 2 \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

### Formulae

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t_n = a + (n-1)d$$

$$t_n = ar^{n-1}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$S_n = \frac{n}{2}(a + \ell)$$

$$S_n = \frac{a - \ell r}{1-r}$$

$$S = \frac{a}{1-r}$$

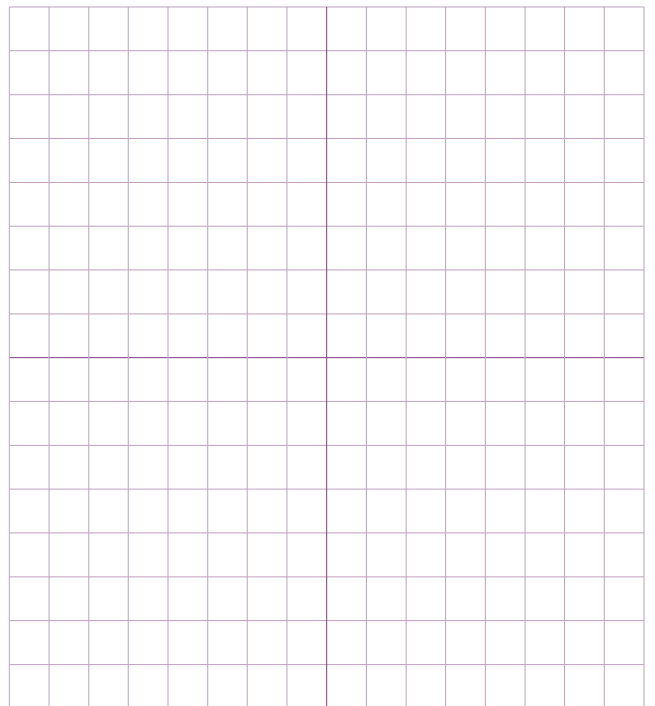
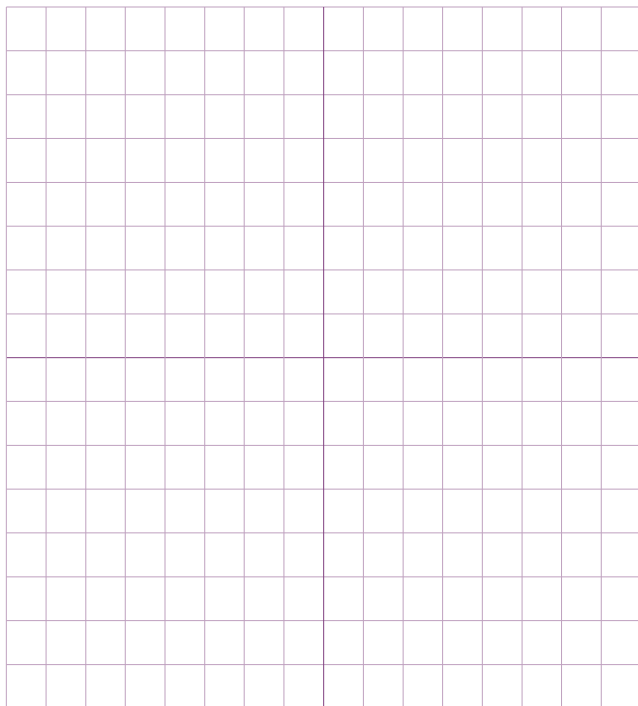
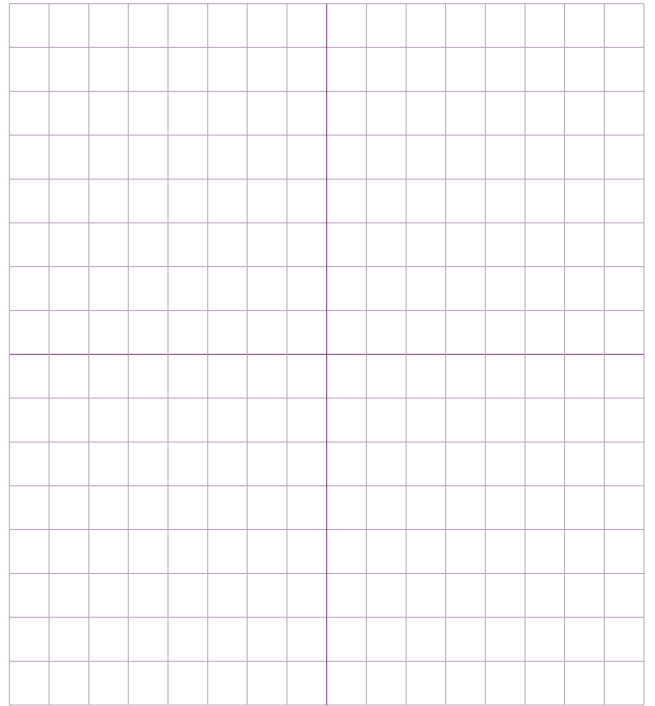
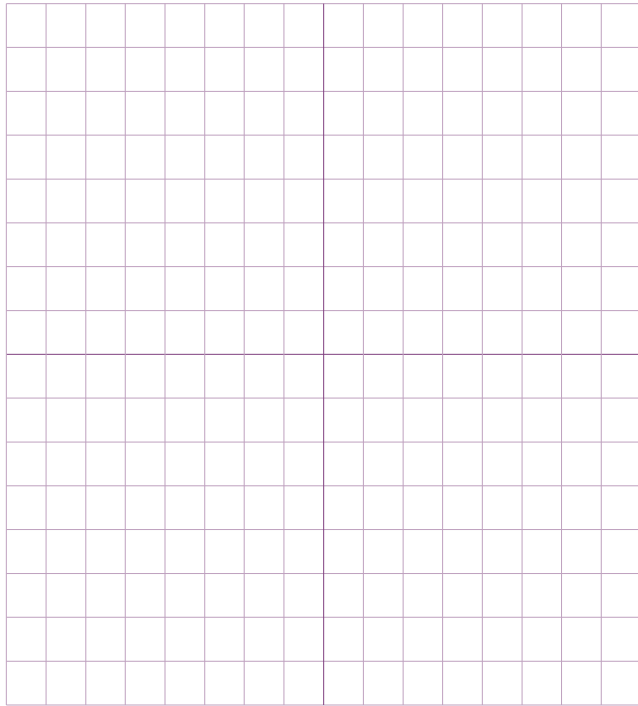
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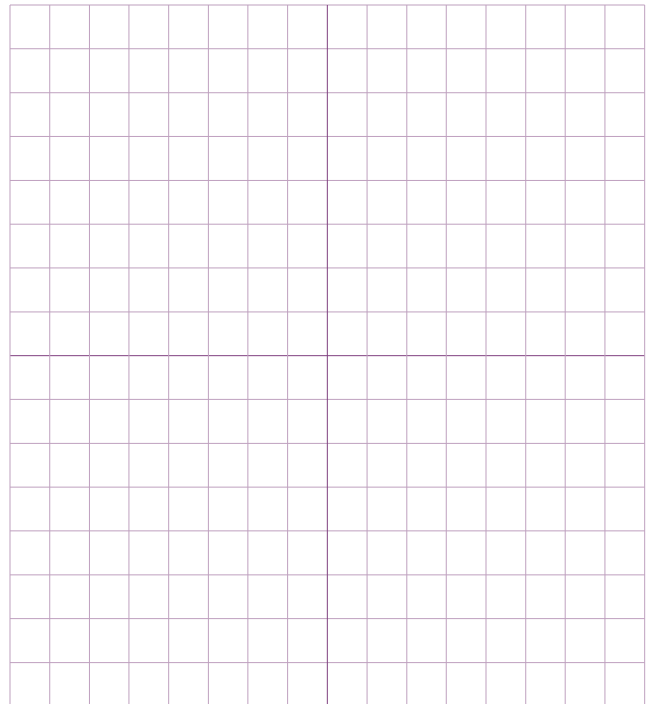
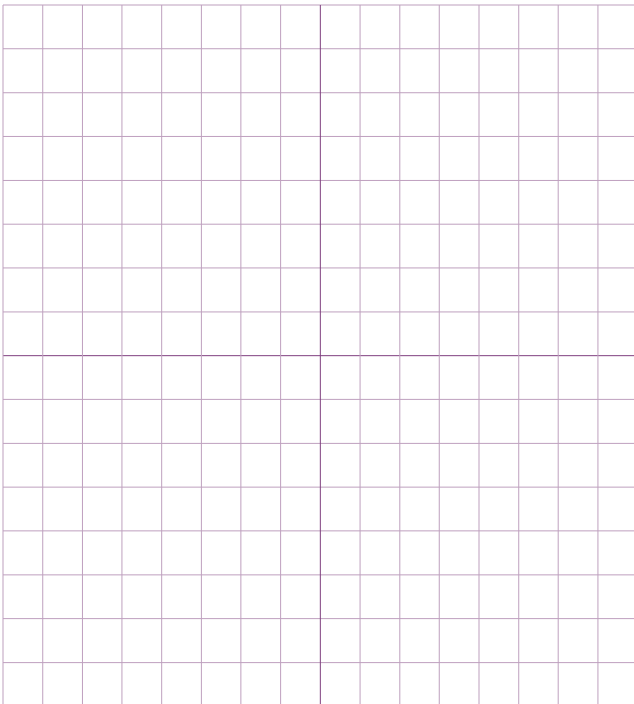
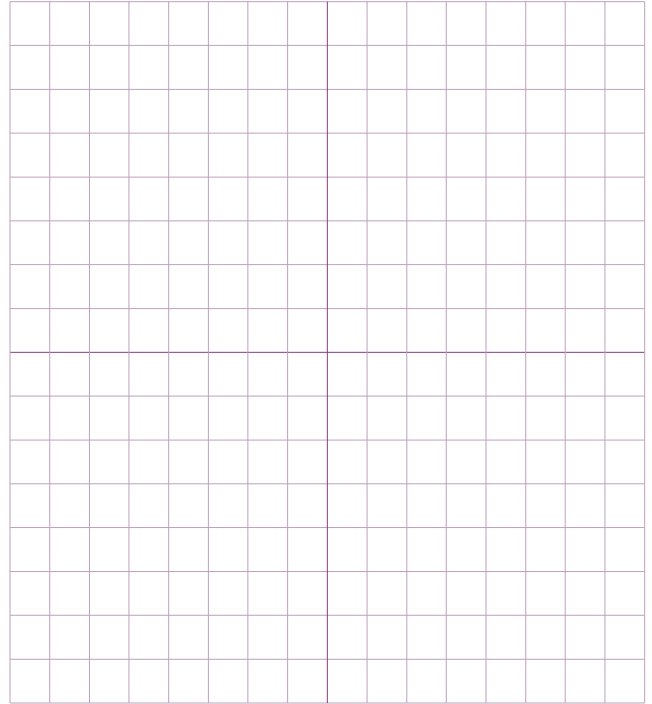
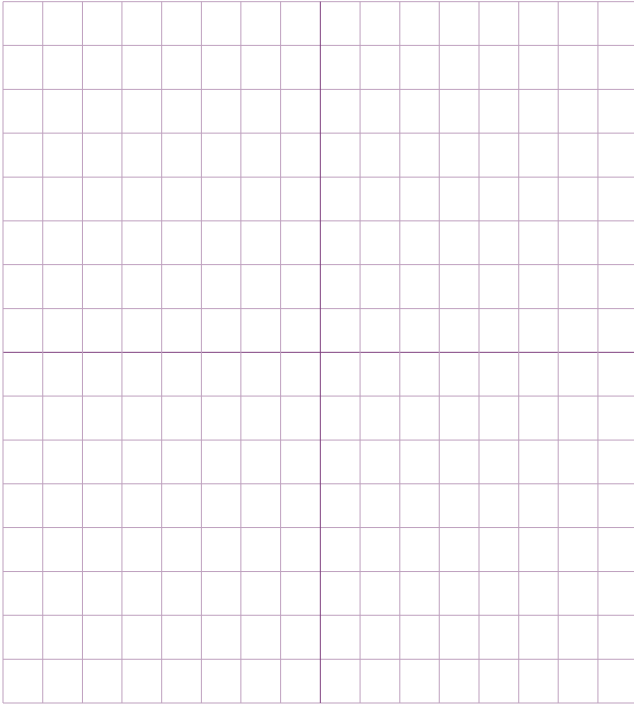
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## ROUGH WORK FOR MULTIPLE-CHOICE