

JUNE 1995

PROVINCIAL EXAMINATION

MINISTRY OF EDUCATION

MATHEMATICS 12

GENERAL INSTRUCTIONS

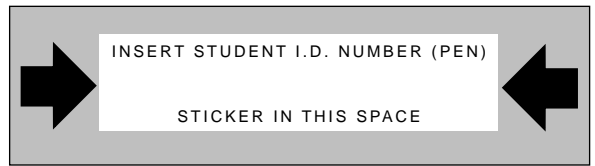
1. Insert the stickers with your Student I.D. Number (PEN) in the allotted spaces above. **Under no circumstance is your name or identification, other than your Student I.D. Number, to appear on this paper.**
2. Take the separate Answer Sheet and follow the directions on its front page.
3. Be sure you have an HB pencil and an eraser for completing your Answer Sheet. Follow the directions on the Answer Sheet when answering multiple-choice questions.
4. For each of the written-response questions, write your answer in the space provided.
5. When instructed to open this booklet, **check the numbering of the pages** to ensure that they are numbered in sequence from page one to the last page, which is identified by

END OF EXAMINATION .

6. At the end of the examination, place your Answer Sheet inside the front cover of this booklet and return the booklet and your Answer Sheet to the supervisor.

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FOR OFFICE USE ONLY



MATHEMATICS 12 JUNE 1995 PROVINCIAL

Course Code = MA Examination Type = P

1. $\frac{\quad}{(3)}$

5. $\frac{\quad}{(2)}$

2. $\frac{\quad}{(3)}$

6. $\frac{\quad}{(3)}$

3. $\frac{\quad}{(4)}$

7. $\frac{\quad}{(2)}$

4. $\frac{\quad}{(3)}$

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MATHEMATICS 12 PROVINCIAL EXAMINATION

		Value	Suggested Time
1.	This examination consists of two parts:		
	PART A 50 multiple-choice questions	50	75
	PART B 7 written-response questions	20	45
	2 questions worth two marks each, 4 questions worth three marks each, and 1 question worth four marks.		
	Total:	70 marks	120 minutes

- The last **three** pages inside the back cover contain **A Summary of Basic Identities and Formulae**, **Rough Work for Graphing**, and **Rough Work for Multiple-Choice**. These pages may be detached for convenient reference prior to writing this examination.
- You will not be provided with any additional paper since rough-work space for the written-response questions has been incorporated into the space allowed for answering each question. You may not need all of the space provided to answer each question.
- An approved scientific calculator is essential for the examination. The calculator must be a hand-held device designed **only** for mathematical computations such as logarithmic and trigonometric functions. It **can be** programmable, but **must not** contain any graphing capabilities. You **must not** bring into the examination room any devices to support calculators such as manuals, printed or electronic cards, printers, memory expansion chips or cards, or keyboards.
- You are permitted to use rulers, compasses, and protractors.
- You have **two hours** to complete this examination.

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PART A: MULTIPLE-CHOICE

Value: 50 marks

Suggested Time: 75 minutes

INSTRUCTIONS: For each question, select the **best** answer and record your choice on the Answer Sheet provided. Using an HB pencil, completely fill in the circle that has the letter corresponding to your answer.

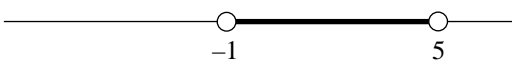
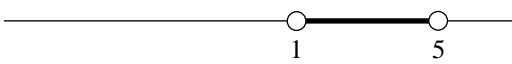
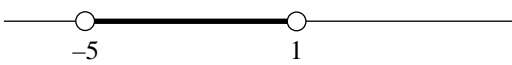
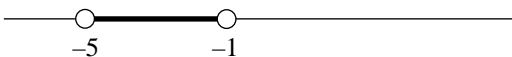
1. Determine the midpoint of the line segment that has endpoints $(-12, 7)$ and $(8, -5)$.

- A. $(-10, 6)$
- B. $(-5, 3)$
- C. $(-4, 2)$
- D. $(-2, 1)$

2. Give the vertex of the parabola $x = (y + 3)^2 - 4$.

- A. $(-4, -3)$
- B. $(-4, 3)$
- C. $(4, -3)$
- D. $(4, 3)$

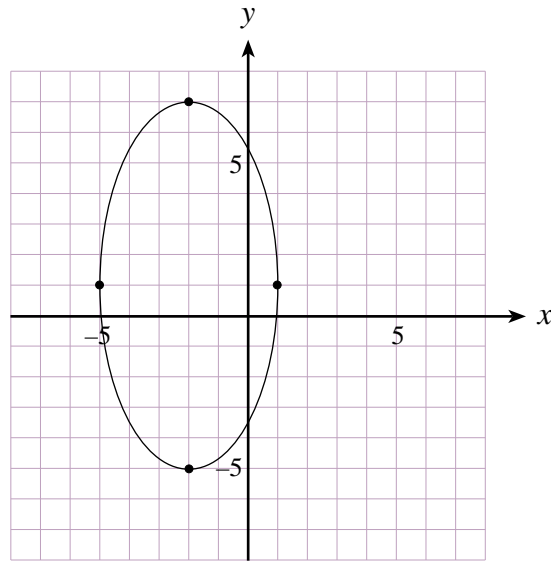
3. Which graph shows the correct solution to $|x - 3| < 2$?

- A. 
- B. 
- C. 
- D. 

4. Determine the radius of the circle $x^2 + y^2 + 10y + 9 = 0$.

- A. 3
- B. $\sqrt{14}$
- C. 4
- D. $\sqrt{34}$

5. Determine the equation of the ellipse graphed below.



A. $\frac{(x-2)^2}{36} + \frac{(y+1)^2}{9} = 1$

B. $\frac{(x-2)^2}{9} + \frac{(y+1)^2}{36} = 1$

C. $\frac{(x+2)^2}{36} + \frac{(y-1)^2}{9} = 1$

D. $\frac{(x+2)^2}{9} + \frac{(y-1)^2}{36} = 1$

6. Determine an equation of the parabola with vertex $(-2, -6)$ that opens up and contains the point $(0, -3)$.

A. $y = -\frac{9}{4}(x+2)^2 + 6$

B. $y = (x+2)^2 - 3$

C. $y = (x+2)^2 - 6$

D. $y = \frac{3}{4}(x+2)^2 - 6$

7. A point P (x, y) moves such that it is always equidistant from the point (2, 3) and the line $x = -4$. Which equation represents this locus?

A. $(x-2)^2 + (y-3)^2 = (x+4)^2$

B. $(x-2)^2 + (y-3)^2 = (x-4)^2$

C. $(x+2)^2 + (y+3)^2 = (x+4)^2$

D. $(x+2)^2 + (y+3)^2 = (x-4)^2$

8. A **rectangular** hyperbola has centre (0, 0) and vertices on the y-axis. If (5, 7) and (-10, k) are points on the graph of the hyperbola, determine a value of k . (Accurate to 2 decimal places.)

A. 5.83

B. 8.72

C. 11.14

D. 26.00

9. The transverse axis of a hyperbola has endpoints (-2, 2) and (10, 2). If one of the asymptotes has a slope of $\frac{2}{3}$, determine an equation of this hyperbola.

A. $\frac{(x-4)^2}{36} - \frac{(y-2)^2}{16} = 1$

B. $\frac{(x-4)^2}{16} - \frac{(y-2)^2}{36} = 1$

C. $\frac{(x+4)^2}{36} - \frac{(y+2)^2}{16} = 1$

D. $\frac{(x+4)^2}{16} - \frac{(y+2)^2}{36} = 1$

10. A hyperbola and a parabola both have $x = 0$ as an axis of symmetry. If (m, n) is one intersection point of these two curves, then which other point **must** also be an intersection point?

A. (n, m)

B. ($m, -n$)

C. ($-m, n$)

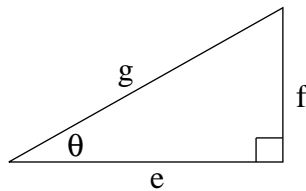
D. ($-m, -n$)

OVER

11. Convert 250° to radians. (Accurate to 2 decimal places.)

- A. 0.44
- B. 1.39
- C. 4.36
- D. 4.48

12. Using the diagram below, determine an expression for $\sec \theta$.



- A. $\frac{g}{f}$
- B. $\frac{g}{e}$
- C. $\frac{f}{e}$
- D. $\frac{e}{g}$

13. Determine the period of the function $y = 5 \sin(3x - \pi)$.

- A. $\frac{\pi}{3}$
- B. $\frac{2\pi}{5}$
- C. $\frac{2\pi}{3}$
- D. 2π

14. Which expression is equivalent to $\csc^2 \theta - 1$?

- A. $\tan^2 \theta$
- B. $\csc^2 \theta \sin^2 \theta$
- C. $\cot^2 \theta - 1$
- D. $\csc^2 \theta \cos^2 \theta$

15. Determine the smallest positive value for θ such that $\csc \theta = -3$. (Accurate to nearest degree.)
- 19°
 - 161°
 - 199°
 - 341°
16. Solve: $5 \sin^2 \theta - 2 \sin \theta = 0$ where $0 \leq \theta < 2\pi$. (Accurate to 2 decimal places.)
- 0.00, 0.41
 - 0.41, 2.73
 - 0.00, 0.41, 2.73, 3.14
 - 0.00, 3.14, 3.55, 5.87
17. A cosine curve has a maximum point at $(3, 20)$ and the nearest minimum point to the right of this point is $(8, 4)$. Which of the following is an equation for this curve?
- $y = 8 \cos \frac{2\pi}{5}(x + 3) + 12$
 - $y = 8 \cos \frac{2\pi}{5}(x - 3) + 12$
 - $y = 8 \cos \frac{\pi}{5}(x + 3) + 12$
 - $y = 8 \cos \frac{\pi}{5}(x - 3) + 12$
18. If the point $(1, 2)$ lies on the terminal arm of angle θ in standard position, determine the value of $\cos(\pi + \theta)$.
- $\frac{-2}{\sqrt{5}}$
 - $\frac{-1}{\sqrt{5}}$
 - $\frac{1}{\sqrt{5}}$
 - $\frac{2}{\sqrt{5}}$

19. Change to exponential form: $\log_a y = x$

- A. $x = a^y$
- B. $x = y^a$
- C. $y = x^a$
- D. $y = a^x$

20. Write as a single logarithm: $3 \log b - \log 2 - \log c$

- A. $\log\left(\frac{b^3}{2c}\right)$
- B. $\log\left(\frac{3b}{2c}\right)$
- C. $\log\left(\frac{b^3c}{2}\right)$
- D. $\log(3b - 2 - c)$

21. Give an equation of the asymptote of the graph of $y = 2 \log(x + 5) - 7$.

- A. $x = -7$
- B. $x = -5$
- C. $x = 5$
- D. $x = 7$

22. Given $f(x) = 5^x$, determine $f^{-1}(x)$, the inverse of $f(x)$.

- A. $f^{-1}(x) = \left(\frac{1}{5}\right)^x$
- B. $f^{-1}(x) = x^5$
- C. $f^{-1}(x) = \log_x 5$
- D. $f^{-1}(x) = \log_5 x$

23. One centillion is defined as the 100th power of one million. What is the common logarithm of one centillion?
- A. 12
 - B. 100
 - C. 106
 - D. 600
24. Simplify: $\log_a(\sqrt{a})^x$
- A. $\frac{x}{2}$
 - B. $\left(\frac{1}{2}\right)^x$
 - C. x^2
 - D. \sqrt{x}
25. Given that $y_1 = \log_a 5$ and $y_2 = \log_a 3$ where $0 < a < 1$, which of the following **must** be true?
- A. $y_1 > 5$
 - B. $y_1 < y_2$
 - C. $y_1 > y_2$
 - D. $3 < y_2 < 5$
26. If $x + 8$ is a factor of the polynomial $P(x)$, which of the following **must** be true?
- A. $P(-8) = 0$
 - B. $P(8) = 0$
 - C. $P(x) = 8$
 - D. $P(x) = -8$

27. What is the maximum number of real roots that a polynomial equation can have if its degree is 6?

- A. 3
- B. 5
- C. 6
- D. 7

28. According to the Rational Zero Theorem, which number is a **possible** zero of the function $f(x) = 6x^3 + 7x^2 - 3x + 4$?

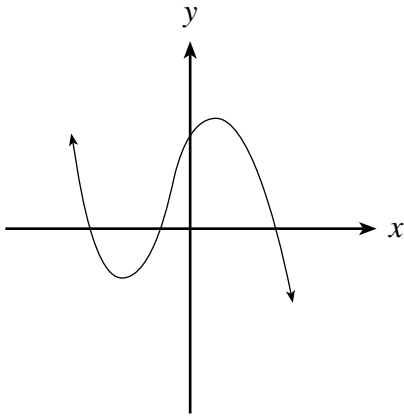
- A. $-\frac{3}{2}$
- B. $\frac{1}{4}$
- C. $\frac{1}{3}$
- D. 3

29. Determine the remainder when $2x^4 + 4x^3 - 5x^2 + 8$ is divided by $x - 2$.

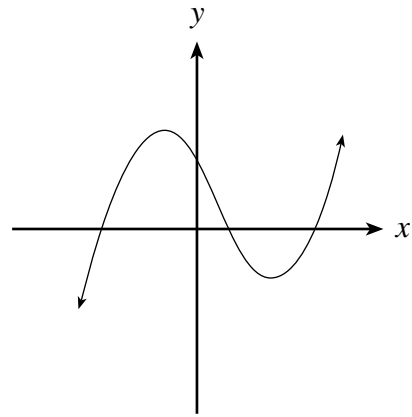
- A. -12
- B. 18
- C. 30
- D. 52

30. Which graph is the **best** representation of $y = ax^3 + bx^2 + cx - 24$ where $a > 0$?

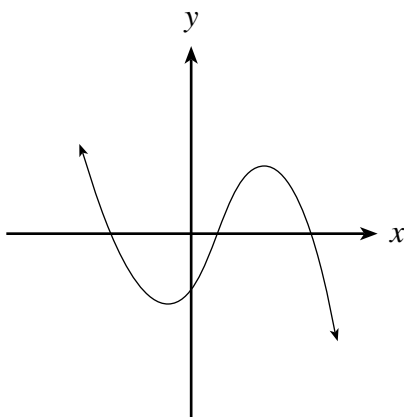
A.



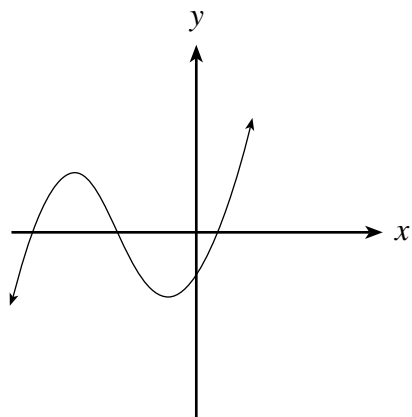
B.



C.



D.



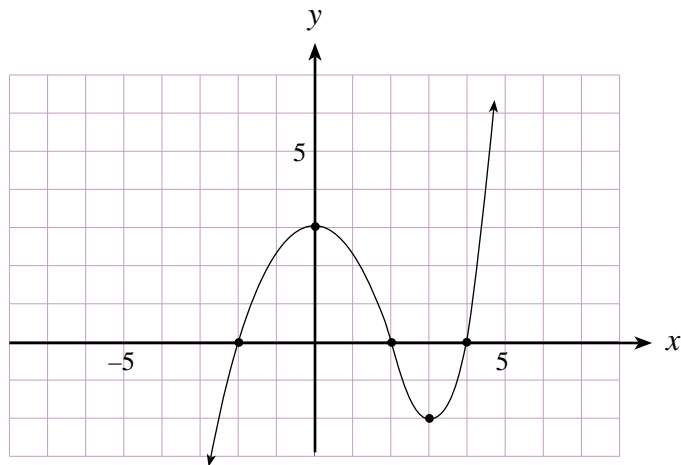
31. Determine all the real zeros of the function $P(x) = 2x(x^2 + 9)(x^2 - 2)$.

- A. $0, \pm\sqrt{2}$
- B. $0, \pm 3$
- C. $0, \sqrt{2}, 3$
- D. $0, \pm\sqrt{2}, \pm 3$

32. Solve the inequality: $(x + 2)^2(x - 2)(x - 4) < 0$

- A. $x < -2$
- B. $-2 < x < 4$
- C. $2 < x < 4$
- D. $x < 2$ or $x > 4$

33. The function $f(x)$ is graphed below. If $g(x) = 3f(x)$, determine the zeros of $g(x)$.



- A. -2, 2, 4
- B. -6, 6, 12
- C. -6, 9
- D. -2, 3

34. Determine the next term in the geometric sequence 2, 4, ...

- A. 6
- B. 7
- C. 8
- D. 16

35. Determine the first term of the infinite geometric series that has a sum of 24 and a common ratio of $\frac{1}{8}$.

- A. 3
- B. 18
- C. 21
- D. 27

36. Write the series given by $\sum_{j=2}^4 (5j + 3)$.

- A. $8 + 13 + 18$
- B. $13 + 18 + 23$
- C. $8 + 13 + 18 + 23$
- D. $13 + 18 + 23 + 28$

37. If 10 is a single geometric mean of x and y , express y in terms of x .

- A. $y = \frac{100}{x}$
- B. $y = \frac{x}{100}$
- C. $y = 20 - x$
- D. $y = 100 - x$

38. For a certain series $S_n = 3^{n+1}$, determine t_3 .

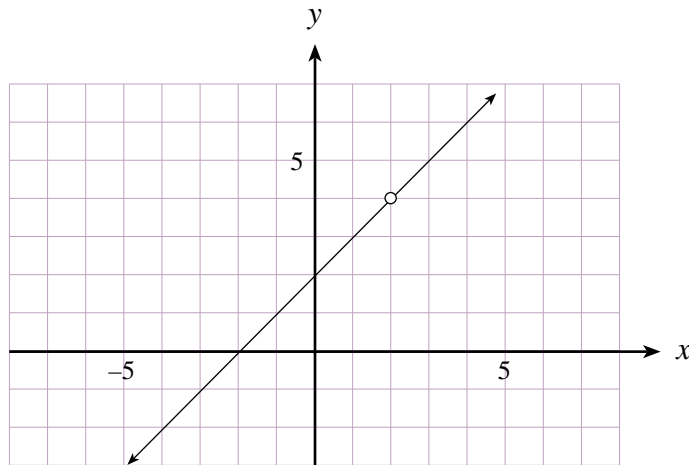
- A. 18
- B. 27
- C. 54
- D. 81

39. Give the derivative of $y = 6x^2$.

- A. 12
- B. $6x$
- C. $12x$
- D. $12x^2$

40. Evaluate: $\lim_{x \rightarrow \infty} \frac{x+5}{x+2}$
- A. 0
B. 1
C. $\frac{5}{2}$
D. limit does not exist (no finite limit)
41. Find k such that the function $f(x) = kx^2 + 12x - 4$ has a critical point at $x = 4$.
- A. $k = -6$
B. $k = -\frac{3}{2}$
C. $k = \frac{3}{2}$
D. $k = 6$
42. Find all values of x such that the function $f(x) = 2x^3 - 3x^2$ is increasing.
- A. $x < 1$
B. $x > 0$
C. $0 < x < 1$
D. $x < 0$ or $x > 1$
43. Determine the slope of the line tangent to the graph of $f(x) = \sqrt{x}$ at $x = 9$.
- A. $\frac{1}{6}$
B. $\frac{1}{3}$
C. $\frac{3}{2}$
D. 3

44. The graph of the function $f(x) = \frac{x^2 - 4}{x - 2}$ is given below. Determine $\lim_{x \rightarrow 2} f(x)$.



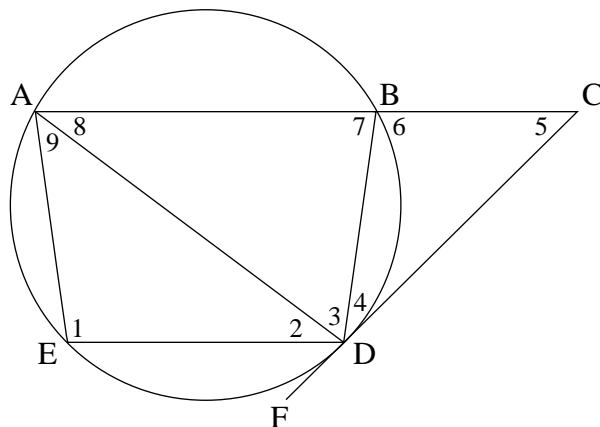
- A. 0
 B. 2
 C. 4
 D. limit does not exist
45. Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two points on the graph of a polynomial function. Which expression represents the derivative at point P?
- A. $\lim_{x_2 \rightarrow 0} \frac{y_2 - x_2}{y_1 - x_1}$
 B. $\lim_{x_2 \rightarrow x_1} \frac{y_2 - x_2}{y_1 - x_1}$
 C. $\lim_{x_2 \rightarrow 0} \frac{y_2 - y_1}{x_2 - x_1}$
 D. $\lim_{x_2 \rightarrow x_1} \frac{y_2 - y_1}{x_2 - x_1}$

Use the following diagram to answer questions 46 and 47.

Given: $\angle 9 = 65^\circ$

$AC \parallel DE$

CF is tangent to the circle at D



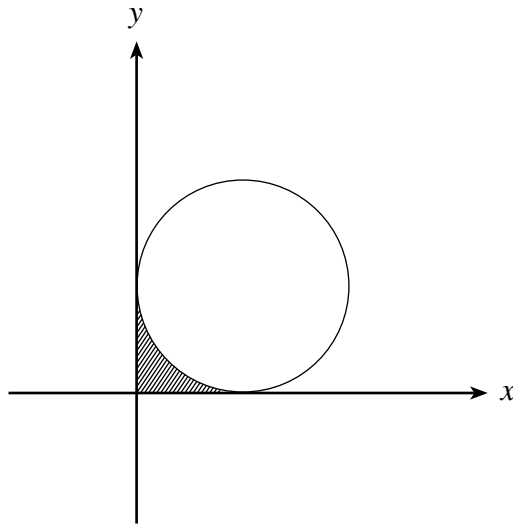
46. If the measure of $\angle 1 = x$, determine an expression for the measure of $\angle 8$ in terms of x .

- A. $\frac{1}{3}x$
- B. $115^\circ - x$
- C. $x - 65^\circ$
- D. $x + 65^\circ$

47. If $\angle 1 = 100^\circ$, determine the measure of $\angle 5$. (Diagram is not to scale.)

- A. 55°
- B. 65°
- C. 75°
- D. 85°

48. Determine the area of the shaded region below if the equation of the circle is $(x - 7)^2 + (y - 7)^2 = 49$. (Accurate to 2 decimal places.)



- A. 10.52
B. 11.46
C. 27.97
D. 38.00
49. Find a polynomial equation of lowest degree with integral coefficients such that one root is $1 + \sqrt[3]{2}$.
- A. $x^3 - 3 = 0$
B. $x^3 - 3x^2 + 3x - 3 = 0$
C. $x^3 + 3x^2 + 3x - 1 = 0$
D. $x^3 - x^2 + x - 3 = 0$
50. Give the amplitude of the function $y = 6 \sin x \cos x$.
- A. 2
B. 3
C. 6
D. 12

**This is the end of the multiple-choice section.
Answer the remaining questions directly in this examination booklet.**

OVER

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PART B: WRITTEN-RESPONSE

Value: 20 marks

Suggested Time: 45 minutes

INSTRUCTIONS: Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

Full marks will NOT be given for the final answer only.

1. The sum of the squares of two positive numbers is 914. Find the two numbers if the difference of their squares is 336. **(3 marks)**

ANSWER:

Score for
Question 1:

1.
(3)

OVER

2. Solve: $\log_4(7 - 3x) + \log_4(x + 4) = 2$

(3 marks)

ANSWER:

Score for
Question 2:

2. $\frac{\quad}{(3)}$

OVER

3. Complete the proof.

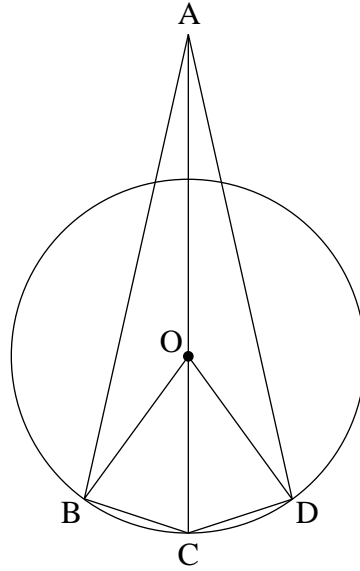
(4 marks)

Given: Circle with centre at O

$BC = CD$

Prove: $AB = AD$

Note: Students are encouraged to number angles.



Statement	Proof	Reason

Score for
Question 3:

3. $\frac{\quad}{(4)}$

OVER

4. Find the sum of all multiples of 6 between 100 and 1 000.

(3 marks)

ANSWER:

Score for
Question 4:

4. $\frac{\quad}{(3)}$

OVER

5. Prove the identity.

(2 marks)

$$\frac{1}{1 + \sin \theta} = \sec^2 \theta - \frac{\tan \theta}{\cos \theta}$$

Left Side	Right Side

Score for
Question 5:

5. $\frac{\quad}{(2)}$

OVER

6. The distance D travelled by an aircraft along a runway before takeoff is given by $D = \frac{3}{5}t^2 + 8t$, where D is measured in metres from the start of the takeoff run, and t is measured in seconds. If the aircraft becomes airborne when its velocity reaches 44 m/s, what distance (in metres) will it travel along the runway before becoming airborne? **(3 marks)**

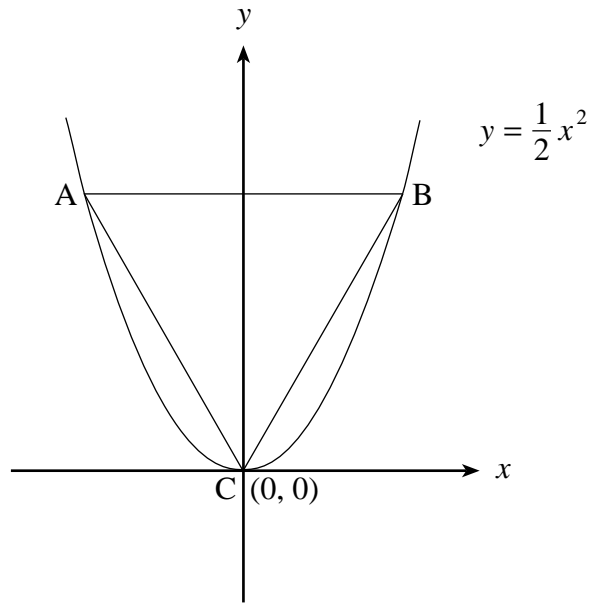
ANSWER:

Score for
Question 6:

6. $\frac{\quad}{(3)}$

OVER

7. Points A, B, and C are on the parabola $y = \frac{1}{2}x^2$ as shown in the diagram. If $\triangle ABC$ is equilateral, determine the x -coordinate of point B. (Give answers that are exact **or** accurate to 2 decimal places.) **(2 marks)**



ANSWER:

Score for
Question 7:

7.
(2)

END OF EXAMINATION

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A SUMMARY OF BASIC IDENTITIES AND FORMULAE

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Reciprocal and Quotient Identities

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Addition Identities

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

Double-Angle Identities

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - 2 \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Formulae

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t_n = a + (n-1)d$$

$$t_n = ar^{n-1}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$S_n = \frac{n}{2}(a + \ell)$$

$$S_n = \frac{a - \ell r}{1-r}$$

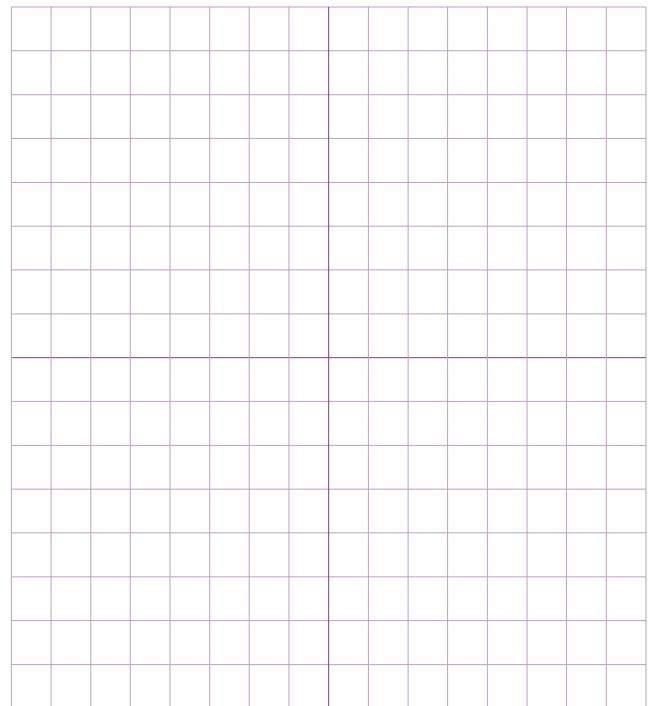
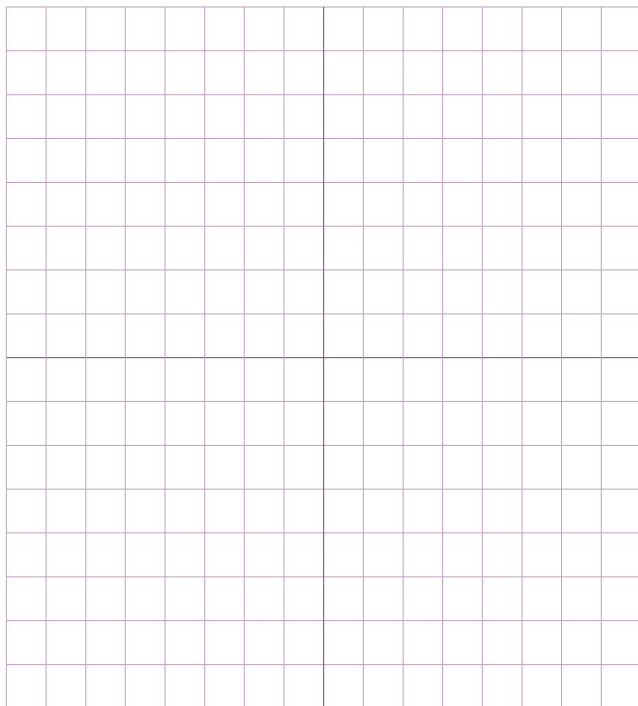
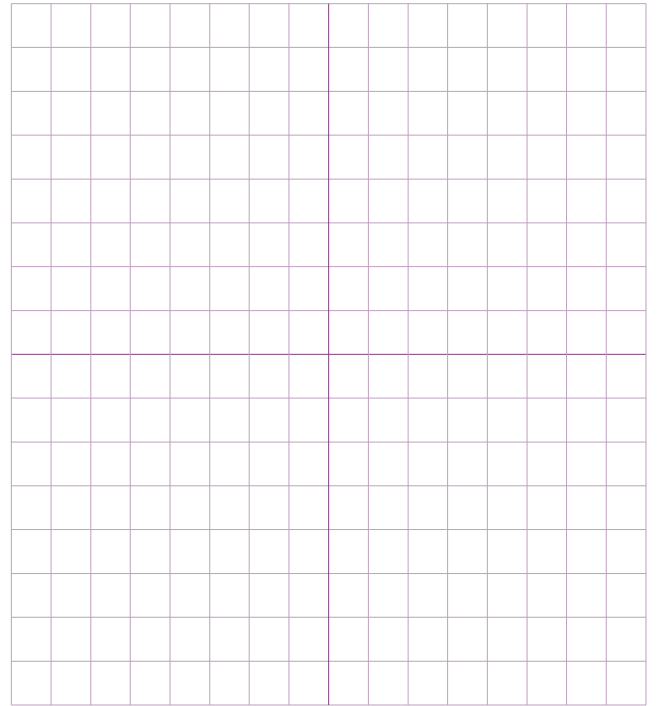
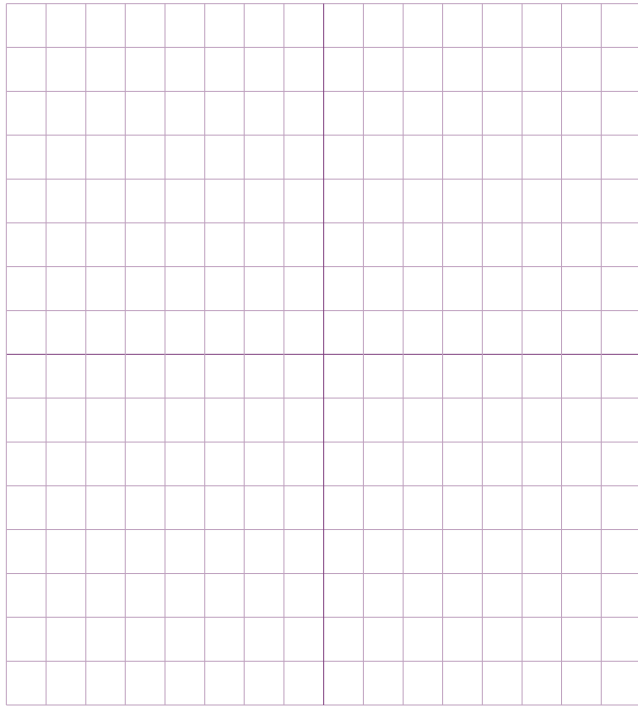
$$S = \frac{a}{1-r}$$

**You may detach this page for convenient reference.
Exercise care when tearing along perforations.**

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ROUGH WORK FOR GRAPHING

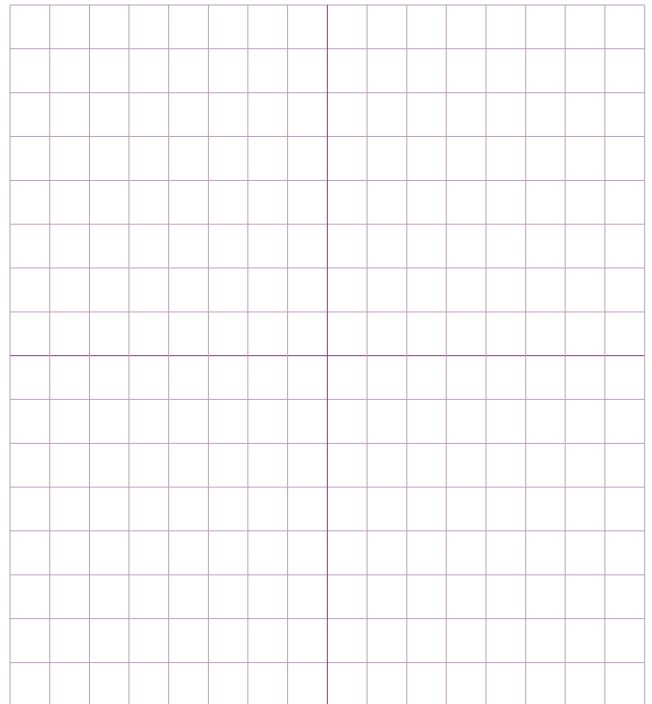
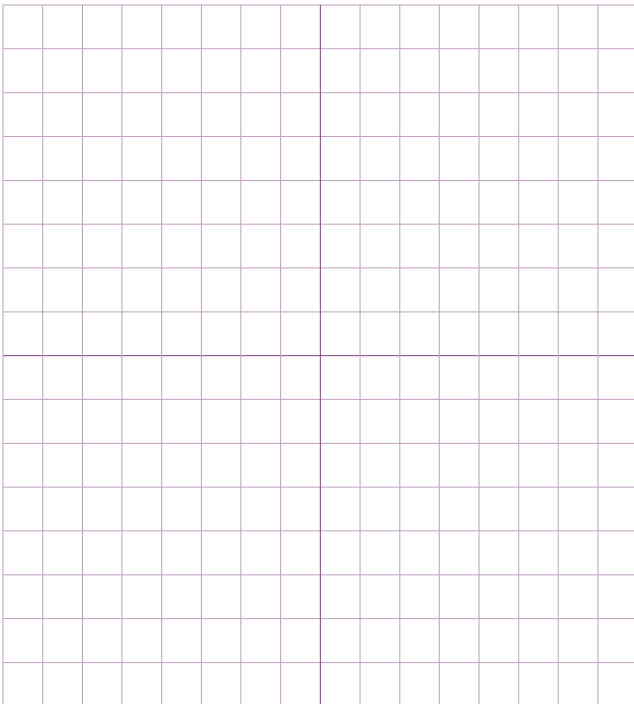
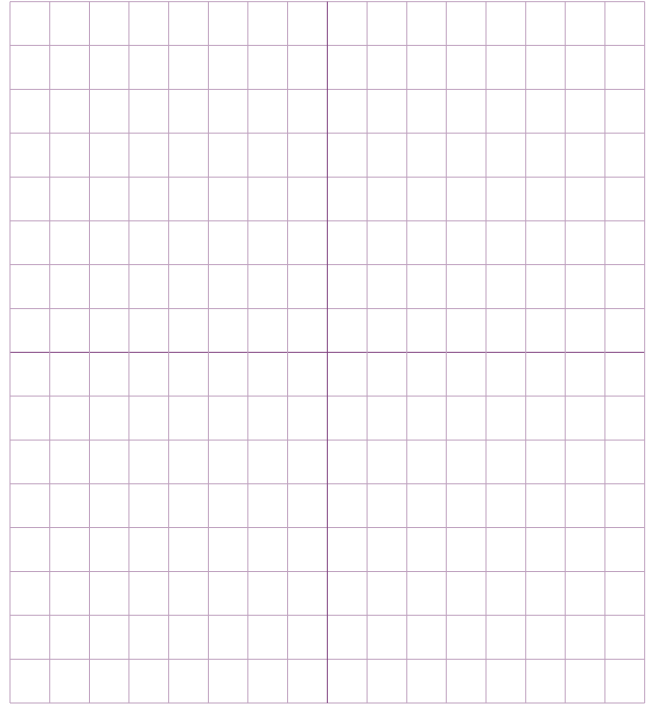
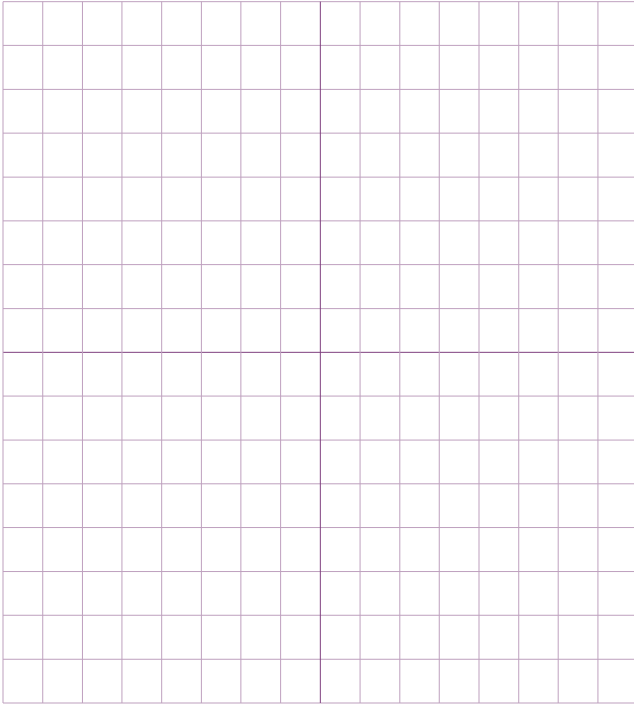
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ROUGH WORK FOR GRAPHING

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ROUGH WORK FOR MULTIPLE-CHOICE

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ROUGH WORK FOR MULTIPLE-CHOICE