INSERT STUDENT I.D. NUMBER (PEN)

STICKER IN THIS SPACE

APRIL 1995

PROVINCIAL EXAMINATION

MINISTRY OF EDUCATION

MATHEMATICS 12

GENERAL INSTRUCTIONS

1. Insert the stickers with your Student I.D. Number (PEN) in the allotted spaces above. **Under no circumstance is your name or identification, other than your Student I.D. Number, to appear on this paper.**

2. Take the separate Answer Sheet and follow the directions on its front page.

3. Be sure you have an HB pencil and an eraser for completing your Answer Sheet. Follow the directions on the Answer Sheet when answering multiple-choice questions.

4. For each of the written-response questions, write your answer in the space provided.

5. When instructed to open this booklet, **check the numbering of the pages** to ensure that they are numbered in sequence from page one to the last page, which is identified by

   END OF EXAMINATION.

6. At the end of the examination, place your Answer Sheet inside the front cover of this booklet and return the booklet and your Answer Sheet to the supervisor.

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FOR OFFICE USE ONLY

INSERT STUDENT I.D. NUMBER (PEN)

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MATHEMATICS 12 APRIL 1995 PROVINCIAL
(MAP)

1. _____ (3)

2. _____ (2)

3. _____ (1)

4. _____ (3)

5. _____ (2)

6. _____ (3)

7. _____ (4)

8. _____ (2)

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MATHEMATICS 12 PROVINCIAL EXAMINATION

		Value	Suggested Time
1. This examination consists of **two** parts:			
PART A	50 multiple-choice questions	50	75
PART B	7 written-response questions 2 questions worth **two** marks each, 4 questions worth **three** marks each, and 1 question worth **four** marks.	20	45
	Total:	**70 marks**	**120 minutes**

2. The last **three** pages inside the back cover contain **A Summary of Basic Identities and Formulae**, **Rough Work for Graphing**, and **Rough Work for Multiple-Choice**. These pages may be detached for convenient reference prior to writing this examination.

3. You will not be provided with any additional paper since rough-work space for the written-response questions has been incorporated into the space allowed for answering each question. You may not need all of the space provided to answer each question.

4. An approved scientific calculator is essential for the examination. The calculator must be a hand-held device designed **only** for mathematical computations such as logarithmic and trigonometric functions. It **can be** programmable, but **must not** contain any graphing capabilities. You **must not** bring into the examination room any devices to support calculators such as manuals, printed or electronic cards, printers, memory expansion chips or cards, or keyboards.

5. You are permitted to use rulers, compasses, and protractors.

6. You have **two hours** to complete this examination.

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PART A: MULTIPLE-CHOICE

Value: 50 marks **Suggested Time: 75 minutes**

INSTRUCTIONS: For each question, select the **best** answer and record your choice on the Answer Sheet provided. Using an HB pencil, completely fill in the circle that has the letter corresponding to your answer.

1. Which conic is described by $3x^2 + 3y = 12$?

   A. circle
   B. ellipse
   C. parabola
   D. hyperbola

2. Determine the midpoint of the line segment joining the points $\text{A}(-8,\ -6)$ and $\text{B}(4,\ -3)$.

   A. $(-2,\ -4)$

   B. $(-4,\ -9)$

   C. $\left(-6,\ -\frac{3}{2}\right)$

   D. $\left(-2,\ -\frac{9}{2}\right)$

3. A circle with centre $(-1,\ -2)$ passes through the point $(-3,\ 1)$. What is the length of the radius?

   A. $\sqrt{13}$
   B. $\sqrt{17}$
   C. 13
   D. 17

4. What is the domain of the relation $(x-1)^2 + (y+3)^2 = 25$?

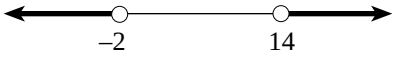
   A. $-4 \le x \le 6$
   B. $-6 \le x \le 4$
   C. $-24 \le x \le 26$
   D. $-26 \le x \le 24$

OVER

5. Graph the solution of $\left| \frac{x}{2} + 3 \right| > 4$.

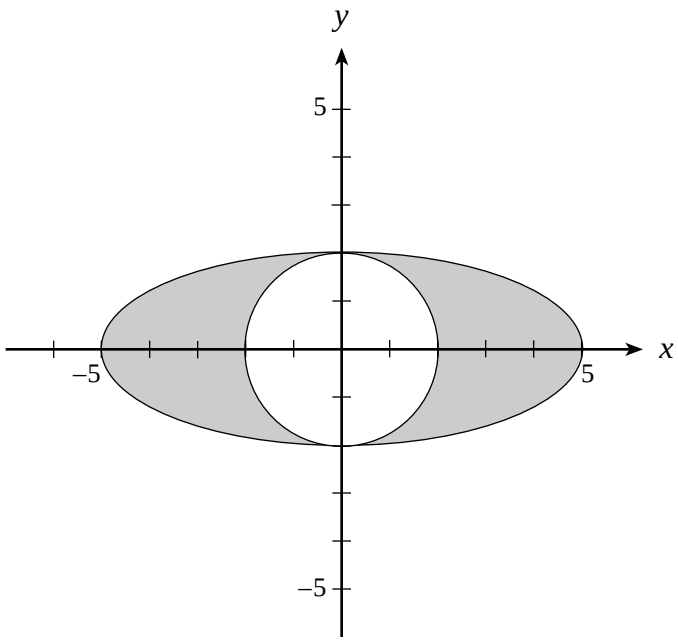
A. 

B. 

C. 

D. 

6. Which system describes the shaded region in the diagram below?



A. $x^2 + y^2 \leq 4$
$4x^2 + 25y^2 \leq 100$

B. $x^2 + y^2 \geq 4$
$4x^2 + 25y^2 \geq 100$

C. $x^2 + y^2 \leq 4$
$4x^2 + 25y^2 \geq 100$

D. $x^2 + y^2 \geq 4$
$4x^2 + 25y^2 \leq 100$

7. A point P moves such that it is always the same distance from the point (0, 1) as it is from the line defined by $y = -1$. Determine an equation of the locus.

A. $\sqrt{(x-0)^2+(y-1)^2} = \sqrt{(x-x)^2+(y+1)^2}$

B. $\sqrt{(x+0)^2+(y+1)^2} = \sqrt{(x+x)^2+(y-1)^2}$

C. $\sqrt{(x-1)^2+(y-0)^2} = \sqrt{(x-1)^2+(y-y)^2}$

D. $\sqrt{(x+1)^2+(y+0)^2} = \sqrt{(x+1)^2+(y+y)^2}$

8. At what point(s) will the graph of $x^2 - y^2 = 16$ intersect the graph of $x^2 + 4x + y^2 = 0$?

A. $(-4,\ 0)$

B. $(-4,\ 0)\ ,\ (4,\ 0)$

C. $(-4,\ 0)\ ,\ (4,\ 0)\ ,\ \left(2,\ 2\sqrt{3}\right)$

D. $(-4,\ 0)\ ,\ (4,\ 0)\ ,\ \left(2,\ 2\sqrt{3}\right)\ ,\ \left(2,\ -2\sqrt{3}\right)$

9. Determine an equation for the ellipse that has vertices at (2, 2) and (–10, 2) and is tangent to the line $y = 5$.

A. $\dfrac{(x+4)^2}{36} + (y-2)^2 = 1$

B. $\dfrac{(x+4)^2}{36} + \dfrac{(y-2)^2}{9} = 1$

C. $\dfrac{(x-2)^2}{9} + \dfrac{(y+4)^2}{36} = 1$

D. $(x-2)^2 + \dfrac{(y+4)^2}{36} = 1$

10. Determine the number of points of intersection of $y = ax^2 + b$ and $y = 5x^2$, if $a > 5$ and $b < 0$.

A. 0
B. 1
C. 2
D. 4

OVER

11. Convert 1 radian to degrees.

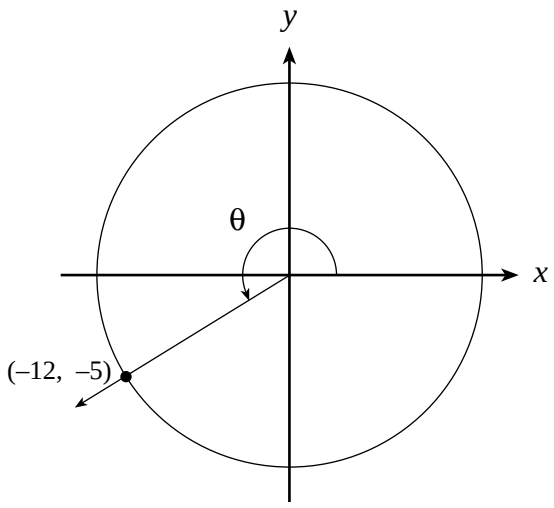
A. $\frac{\pi}{360}^\circ$

B. $\frac{\pi}{180}^\circ$

C. $\frac{180}{\pi}^\circ$

D. $\frac{360}{\pi}^\circ$

12. Given standard position angle θ as shown in the diagram, determine $\sec\theta$.



A. $-\frac{13}{5}$

B. $-\frac{13}{12}$

C. $\frac{13}{12}$

D. $\frac{13}{5}$

13. Determine the range of the function $y = 2\cos(3\theta - \pi) + 1$.

A. $-3 \le y \le 1$
B. $-2 \le y \le 2$
C. $-1 \le y \le 3$
D. all real numbers

14. Which expression is equivalent to $\csc\theta \cot\theta \sec\theta \sin\theta$?

A. $\sec\theta$

B. $\csc\theta$

C. $\dfrac{\sin\theta}{\cos^2\theta}$

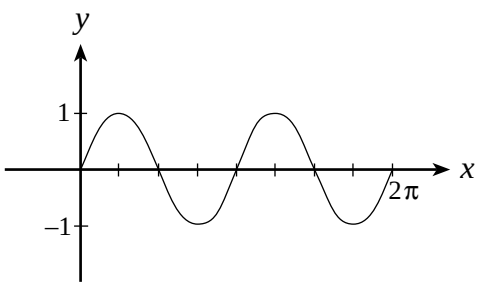
D. $\dfrac{\cos\theta}{\sin^2\theta}$

15. Solve: $\cot\theta = 8$, $0 \le \theta < \dfrac{\pi}{2}$ (Accurate to 2 decimal places.)

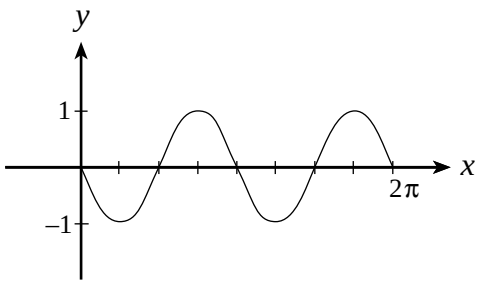
A. 0.12
B. 0.69
C. 1.45
D. no solution

16. Which of the following is the graph of $y = -\sin 2x$, for $0 \le x \le 2\pi$?

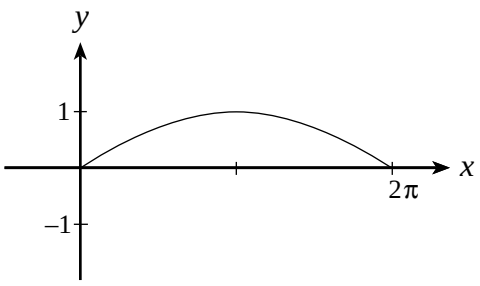
A.



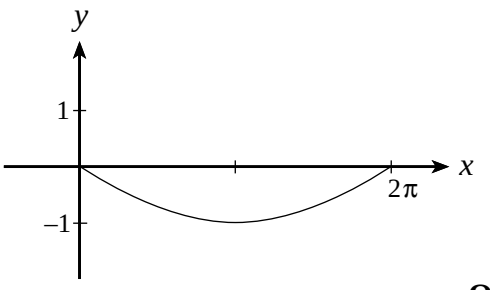
B.



C.



D.

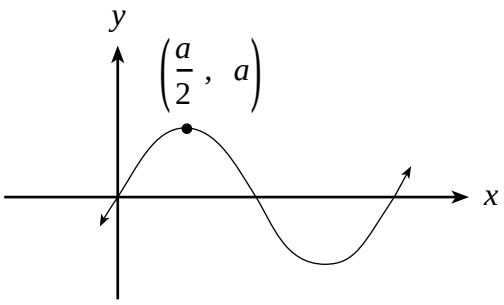


OVER

17. Solve: $4\tan^2 x = -\tan x,\ 0 \le x < 2\pi$ (Accurate to 2 decimal places.)

A. 0.24, 3.39
B. 2.90, 6.04
C. 0.00, 0.24, 3.14, 3.39
D. 0.00, 2.90, 3.14, 6.04

18. Determine the equation of the following sine curve.



A. $y = a\sin\frac{\pi}{a}x$

B. $y = a\sin\frac{a}{\pi}x$

C. $y = \frac{a}{2}\sin\frac{\pi}{a}x$

D. $y = \frac{a}{2}\sin\frac{2a}{\pi}x$

19. If $8^{14} = x$, then

A. $\log_x 8 = 14$
B. $\log_x 14 = 8$
C. $\log_{14} 8 = x$
D. $\log_8 x = 14$

20. Evaluate: $\log_{\sqrt{5}} 125$

A. $\frac{3}{2}$
B. 3
C. 5
D. 6

21. Solve: $5^{x+1} = 9$ (Accurate to 3 decimal places.)

A. 0.268
B. 0.255
C. 0.365
D. 2.365

22. The graph of the exponential function $y = b^x$ contains the point $(-2,\ 9)$. Determine the value of b.

A. $-\frac{9}{2}$

B. -3

C. $\frac{1}{3}$

D. 3

23. Given $f(x) = 10^{x+2}$, determine $f^{-1}(x)$, the inverse of $f(x)$.

A. $f^{-1}(x) = 10^{-x-2}$
B. $f^{-1}(x) = (x+2)^{10}$
C. $f^{-1}(x) = \log(x+2)$
D. $f^{-1}(x) = -2 + \log x$

24. Solve: $7^{2\log_7 x + \log_7 x} = 27$

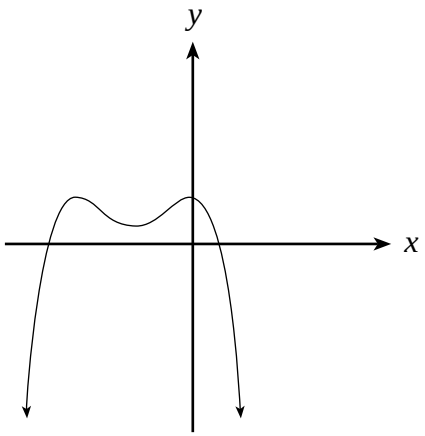
A. 2
B. 3
C. 5
D. 7

25. Determine the point where the asymptotes of the graphs $y = b^{x-1} + 2$ and $y = \log_b(x-1) + 2$ intersect.

A. (1, 1)
B. (1, 2)
C. (2, 1)
D. (2, 2)

OVER

26. Determine the minimum degree of the polynomial function whose graph is shown below.



A. 2
B. 3
C. 4
D. 6

27. How many **real roots** are there for the polynomial equation $(x-2)\left(x^2-3\right)\left(x^2+4\right)=0$?

A. 1
B. 3
C. 4
D. 5

28. Which polynomial has $x-2$ as a factor?

A. x^2+4
B. x^2-2x+1
C. x^3-3x^2+4
D. x^3-3x^2+3x-1

29. Solve: $x^3+2x^2-x-2=0$

A. –2, –1, 1
B. –2, –1, 2
C. –2, 1, 2
D. –1, 1, 2

30. What numbers should replace p and q in the incomplete synthetic division shown below?

$$\begin{array}{r|cccc} \square & 2 & p & q & 2 \\ & & \square & \square & \square \\ \hline & 2 & -7 & 4 & -2 \end{array}$$

A. $p = -5,\ q = -3$
B. $p = -5,\ q = 3$
C. $p = 5,\ q = -3$
D. $p = 5,\ q = 3$

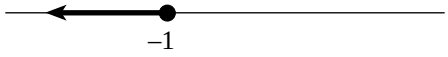
31. Determine the coefficient of x in the quotient when $2x^4 - 7x^3 + 9x^2 + 2x - 8$ is divided by $x^2 - 3x + 4$.

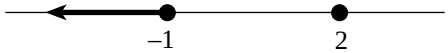
A. –13
B. –1
C. 1
D. 13

32. The function $P(x) = x^3 + 3x - 3$ **must** have at least one real zero between which two values of x?

A. –2 and –1
B. –1 and 0
C. 0 and 1
D. 1 and 2

33. Graph the solution: $(x+1)(x-2)^2(x+3)^3 \le 0$

A.


B.


C.


D.


OVER

34. Which of the following is a geometric sequence?

A. 2, 4, 6
B. 8, 12, 18
C. 10, 30, 60
D. 12, 8, 4

35. Determine the fourth term of the sequence defined by $t_n = \frac{(-1)^n}{n+2}$.

A. $-\frac{2}{3}$
B. $-\frac{1}{6}$
C. $\frac{1}{6}$
D. $\frac{2}{3}$

36. Determine the number of terms in the series $\sum_{k=5}^{67}(k+1)^2$.

A. 61
B. 62
C. 63
D. 67

37. Determine two geometric means between -12 and $\frac{3}{2}$.

A. $-\frac{15}{2}$, -3
B. -6, -3
C. 6, -3
D. $\frac{15}{2}$, -3

38. The sequence $-14,\ -6,\ 2,\ \ldots,\ t_n$ is an arithmetic sequence. If t_n is the largest four-digit term in the sequence, find n.

A. 1 251
B. 1 252
C. 1 253
D. 1 254

39. Given that r is any real number, determine $\frac{d}{dx}\left(x^r\right)$.

A. rx^{r-1}
B. rx^{r+1}
C. $(r-1)x^r$
D. $(r+1)x^r$

40. Evaluate: $\lim\limits_{x\to\infty} \frac{3x-2}{9x+8}$

A. $\frac{1}{17}$
B. $\frac{1}{4}$
C. $\frac{1}{3}$
D. $\frac{1}{2}$

41. Evaluate: $\lim\limits_{x\to -2} \frac{x-2}{x^2-4}$

A. $-\frac{1}{4}$
B. 0
C. $\frac{1}{2}$
D. limit does not exist

OVER

42. Find the slope of the tangent to $y = x^3 - 2x^2 + 6$ at $(2,\ 6)$.

A. 4
B. 6
C. 10
D. 20

43. A particle moves along the x-axis according to the function $x(t) = t^2 - 4t + 3$, where x (metres) is the position of the particle at time t (seconds). At what time t does the particle have a velocity of 6 m/s ?

A. 1
B. 2
C. 5
D. 8

44. Give all values of x where the function $f(x) = x^3 - 3x + 4$ is increasing.

A. $x > 1$
B. $x < -1$
C. $-1 < x < 1$
D. $x < -1$ or $x > 1$

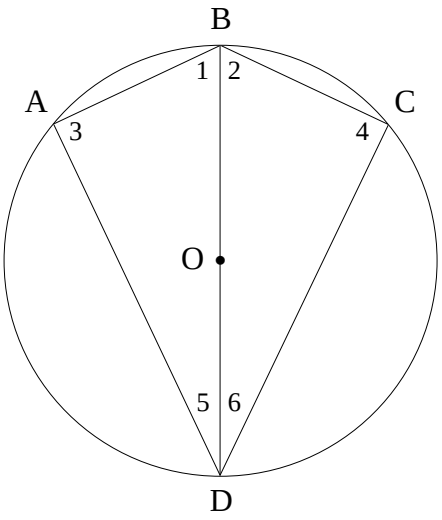
45. At what point on the curve $y = x^2 - 4$ is the tangent parallel to the line $6x + y = 4$?

A. (–3, 22)
B. (–3, 5)
C. (3, –14)
D. (3, 5)

Use the following diagram and proof to answer question 46.

Given: BD bisects $\angle ABC$
O is the centre

Prove: AB = CB



Proof

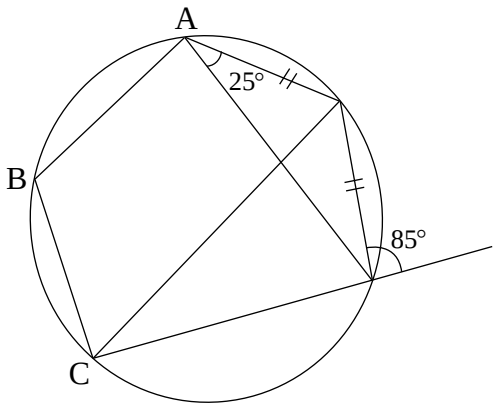
	Statement	Reason
	BD bisects $\angle ABC$	given
(a)	$\angle 1 = \angle 2$	definition of $\angle$ bisector
(b)	$\angle 3 = 90°,\ \angle 4 = 90°$	inscribed $\angle$ in semicircle
	$\angle 3 = \angle 4$	both = 90°
(c)	$\angle 5 = \angle 6$	3rd $\angle$s of Δs are =
(d)	BD = BD	same side
	AB = BC	chords opposite = inscribed $\angle$s are =

46. Which line (or group of lines) is **not** necessary in the given proof?

A. a
B. b
C. c
D. d

OVER

47. Determine the measure of $\angle ABC$.



A. 85°
B. 95°
C. 110°
D. 115°

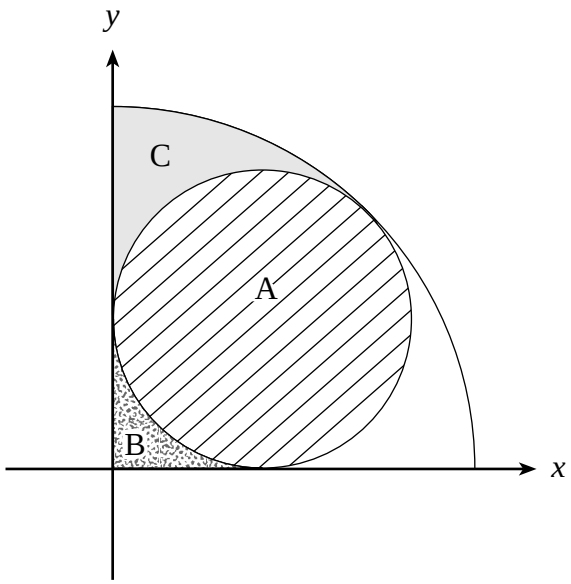
48. Determine the coordinates of the points on the parabola $y = x^2$ which, with the vertex, are the vertices of an equilateral triangle.

A. $(1,\ 1)\ ,\ (-1,\ 1)$
B. $(2,\ 4)\ ,\ (-2,\ 4)$
C. $\left(\sqrt{2},\ 2\right)\ ,\ \left(-\sqrt{2},\ 2\right)$
D. $\left(\sqrt{3},\ 3\right)\ ,\ \left(-\sqrt{3},\ 3\right)$

49. A Pythagorean triple is represented by $n,\ x,\ x+2,$ where n is an even number greater than 2. If $x+2$ represents the largest value, express x in terms of n.

A. $x = \dfrac{n^2-4}{2}$
B. $x = \dfrac{n^2-4}{4}$
C. $x = \dfrac{n^2+4}{2}$
D. $x = \dfrac{n^2+4}{4}$

50. A circle is inscribed in the quadrant I sector of circle $x^2 + y^2 = 36$. If A and B represent the areas of the indicated regions, determine an expression for the area of region C.



A. $\frac{9\pi - A - B}{2}$ units^2

B. $9\pi - A - B$ units^2

C. $\frac{36\pi - A - B}{2}$ units^2

D. $36\pi - A - B$ units^2

This is the end of the multiple-choice section.
Answer the remaining questions directly in this examination booklet.

OVER

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PART B: WRITTEN-RESPONSE

Value: 20 marks **Suggested Time: 45 minutes**

INSTRUCTIONS:	Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided. **Full marks will NOT be given for the final answer only.**

OVER

1. Solve: $\log(x+11)+\log x=\log(x+1)+\log 6$ **(3 marks)**

ANSWER:	Score for Question 1: 1. _____ (3)

OVER

2. The equation of a hyperbola is $9x^2 - 4y^2 - 8y + 32 = 0$.

a) Change the equation to standard form. **(2 marks)**

ANSWER:	Score for Question 2a: 2. _____ (2)

b) Give the equations of the asymptotes of the hyperbola. **(1 mark)**

ANSWER:	Score for Question 2b: 3. _____ (1)

OVER

3. Given $f(x) = x^2 - 3x$, use the **definition of the derivative** to show that $f'(x) = 2x - 3$.

(3 marks)

Score for Question 3:
4. _____ (3)

OVER

4. Prove the identity. **(2 marks)**

$$\frac{\cos\theta + \sin\theta\tan\theta}{\sin\theta\sec\theta} = \csc\theta$$

Left side	Right side

Score for Question 4:
5. _____ (2)

OVER

5. The sum of the first 100 terms of an arithmetic series is $-11\,200$. The sum of the first 200 terms is 17 600. Determine the first term a and the common difference d of the series. **(3 marks)**

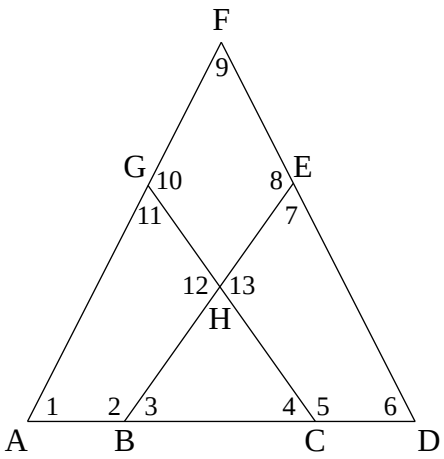
ANSWER:	Score for Question 5:
a =	
d =	6. _____ (3)

OVER

6. Complete the proof. **(4 marks)**

Given: AF = DF
$\angle 2 = \angle 5$

Prove: $\angle 10 = \angle 8$



Proof

Statement	Reason

Score for Question 6:
7. _____ (4)

OVER

7. Find a polynomial equation of lowest degree with integral coefficients such that one root of $f(x) = 0$ is $\sqrt{2} + \sqrt{3}$. **(2 marks)**

ANSWER:	Score for Question 7: 8. _____ (2)

END OF EXAMINATION

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A SUMMARY OF BASIC IDENTITIES AND FORMULAE

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$ $\qquad 1 + \tan^2\theta = \sec^2\theta$ $\qquad 1 + \cot^2\theta = \csc^2\theta$

Reciprocal and Quotient Identities

$\sec\theta = \dfrac{1}{\cos\theta}$ $\qquad \csc\theta = \dfrac{1}{\sin\theta}$ $\qquad \cot\theta = \dfrac{1}{\tan\theta}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\qquad \cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Addition Identities

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\qquad \tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\qquad \tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

Double-Angle Identities

$\cos 2\theta = \cos^2\theta - \sin^2\theta$ $\qquad \sin 2\theta = 2\sin\theta\cos\theta$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$ $\qquad \tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

Formulae

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\qquad t_n = a + (n-1)d$ $\qquad t_n = ar^{n-1}$

$\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$ $\qquad S_n = \dfrac{n}{2}\left[2a + (n-1)d\right]$ $\qquad S_n = \dfrac{a\left(1 - r^n\right)}{1 - r}$

$c^2 = a^2 + b^2 - 2ab\cos C$ $\qquad S_n = \dfrac{n}{2}(a + \ell)$ $\qquad S_n = \dfrac{a - \ell r}{1 - r}$

$S = \dfrac{a}{1 - r}$

You may detach this page for convenient reference.
Exercise care when tearing along perforations.

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ROUGH WORK FOR GRAPHING

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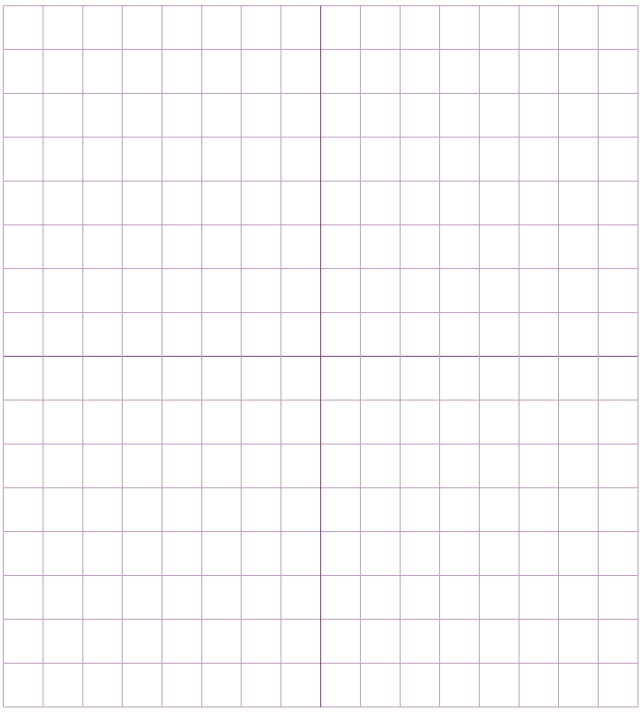

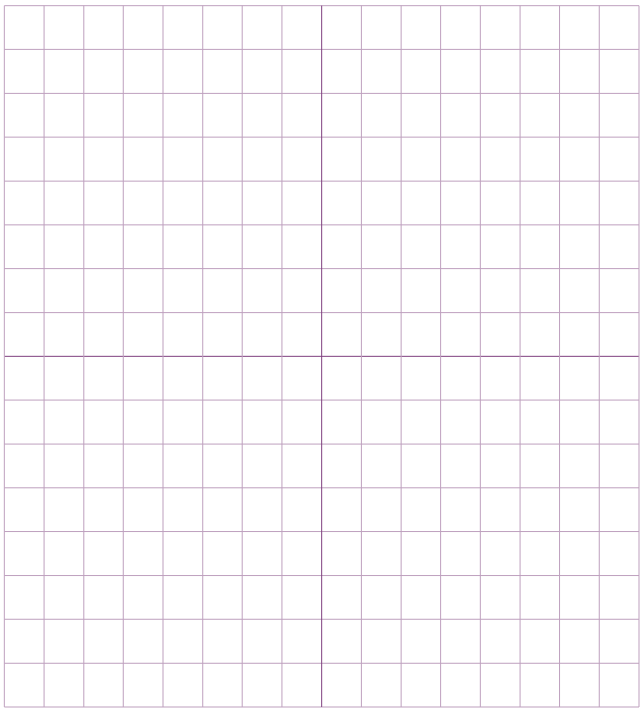

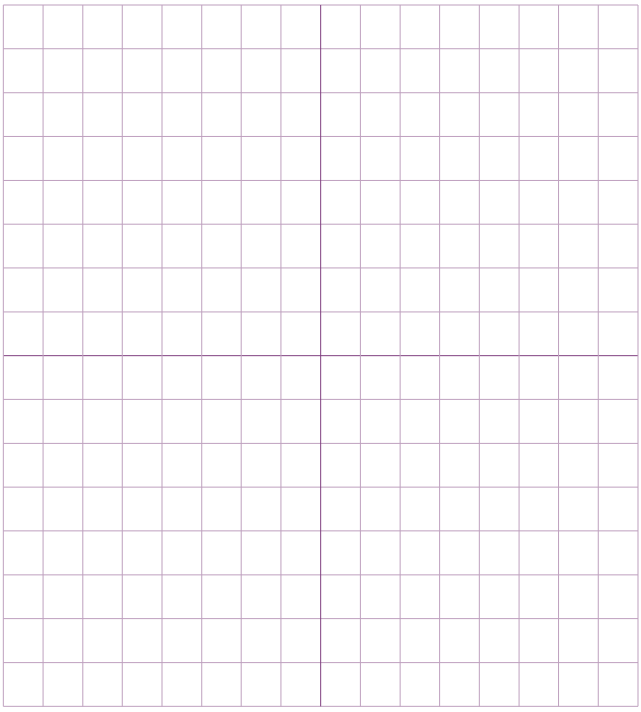

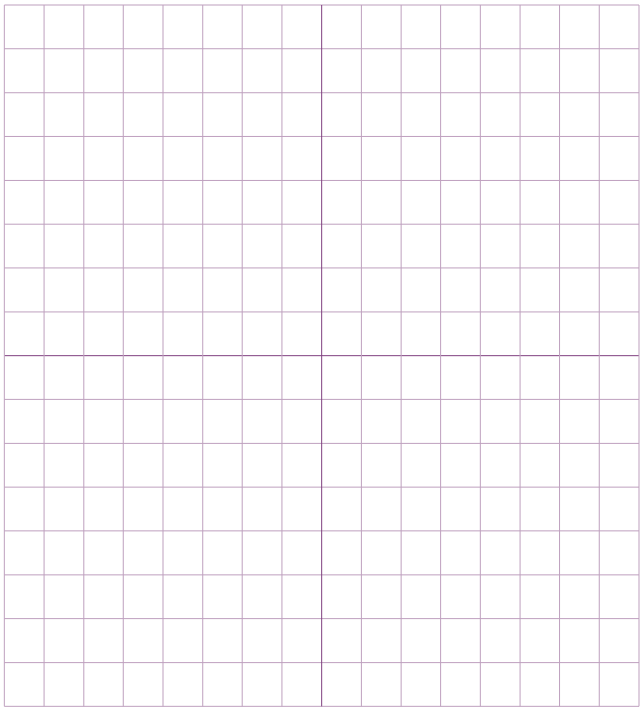

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ROUGH WORK FOR GRAPHING

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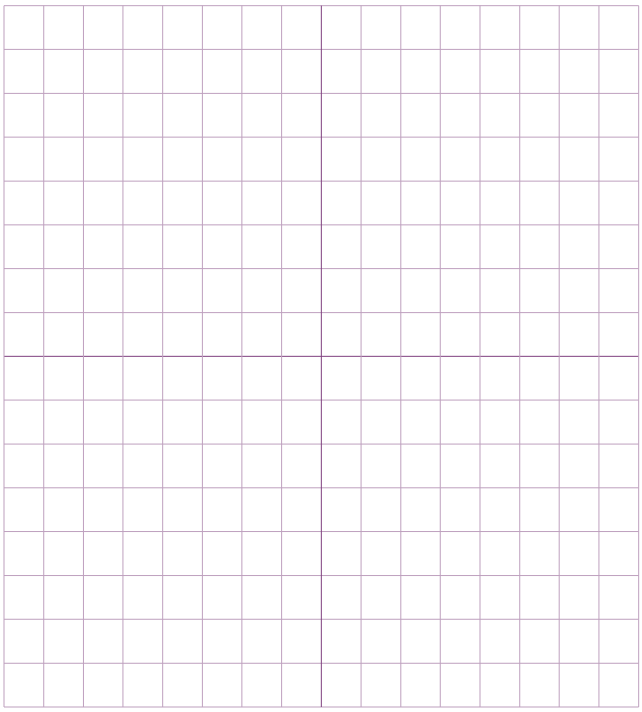

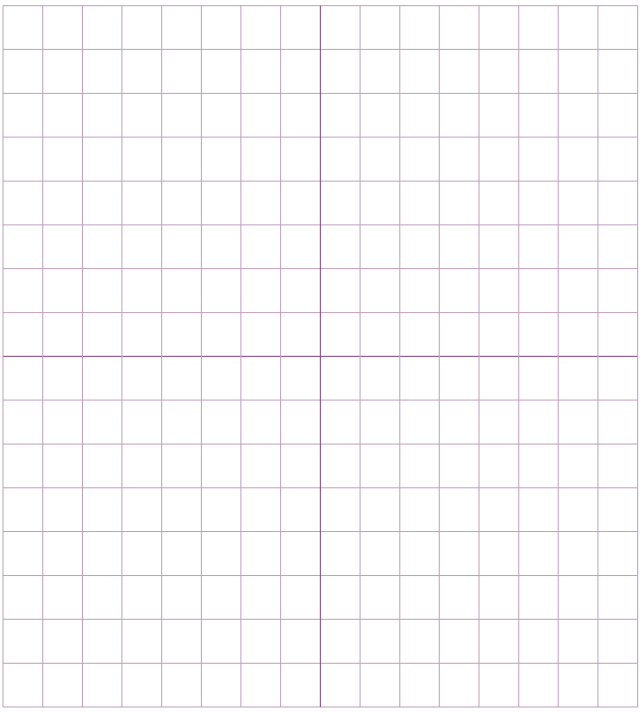

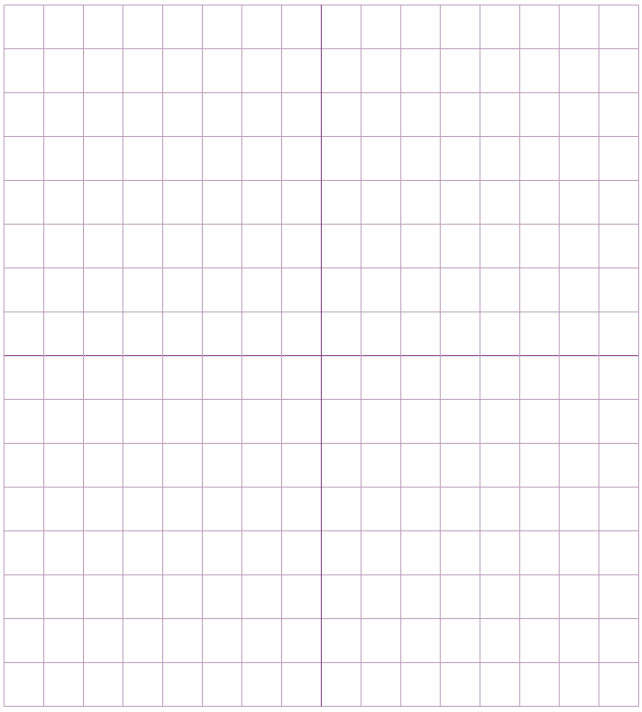

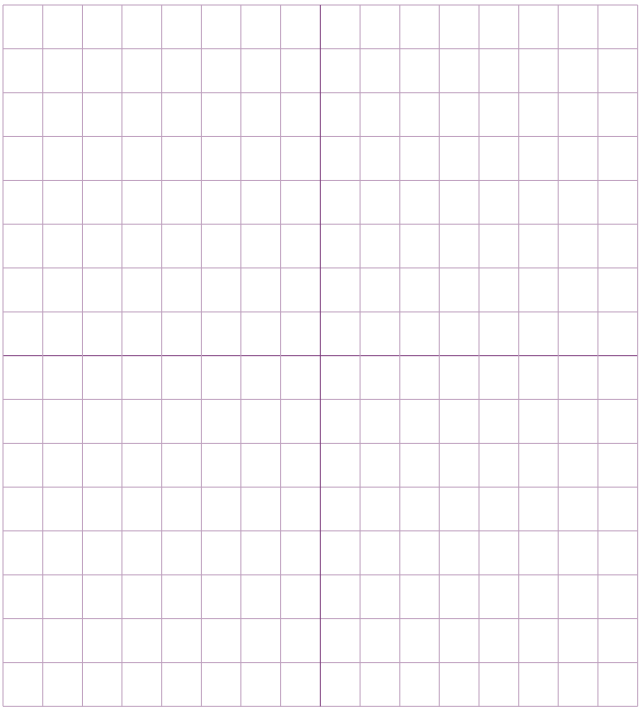

ROUGH WORK FOR MULTIPLE-CHOICE

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ROUGH WORK FOR MULTIPLE-CHOICE