INSERT STUDENT I.D. NUMBER (PEN)

STICKER IN THIS SPACE

JANUARY 1995

PROVINCIAL EXAMINATION

MINISTRY OF EDUCATION

MATHEMATICS 12

GENERAL INSTRUCTIONS

1. Insert the stickers with your Student I.D. Number (PEN) in the allotted spaces above. **Under no circumstance is your name or identification, other than your Student I.D. Number, to appear on this paper.**

2. Take the separate Answer Sheet and follow the directions on its front page.

3. Be sure you have an HB pencil and an eraser for completing your Answer Sheet. Follow the directions on the Answer Sheet when answering multiple-choice questions.

4. For each of the written-response questions, write your answer in the space provided.

5. When instructed to open this booklet, **check the numbering of the pages** to ensure that they are numbered in sequence from page one to the last page, which is identified by

   ENDOFEXAMINATION .

6. At the end of the examination, place your Answer Sheet inside the front cover of this booklet and return the booklet and your Answer Sheet to the supervisor.

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FOR OFFICE USE ONLY

INSERT STUDENT I.D. NUMBER (PEN)

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MATHEMATICS 12 JANUARY 1995 PROVINCIAL
(MAP)

1. _____ (3)

2. _____ (3)

3. _____ (3)

4. _____ (2)

5. _____ (2)

6. _____ (4)

7. _____ (3)

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MATHEMATICS 12 PROVINCIAL EXAMINATION

		Value	Suggested Time
1.	This examination consists of **two** parts:		
PART A	50 multiple-choice questions	50	75
PART B	7 written-response questions 2 questions worth **two** marks each, 4 questions worth **three** marks each, and 1 question worth **four** marks.	20	45
	Total:	**70 marks**	**120 minutes**

2. The last **three** pages inside the back cover contain **A Summary of Basic Identities and Formulae**, **Rough Work for Graphing**, and **Rough Work for Multiple-Choice**. These pages may be detached for convenient reference prior to writing this examination.

3. You will not be provided with any additional paper since rough-work space for the written-response questions has been incorporated into the space allowed for answering each question. You may not need all of the space provided to answer each question.

4. An approved scientific calculator is essential for the examination. The calculator must be a hand-held device designed **only** for mathematical computations such as logarithmic and trigonometric functions. It **can be** programmable, but **must not** contain any graphing capabilities. You **must not** bring into the examination room any devices to support calculators such as manuals, printed or electronic cards, printers, memory expansion chips or cards, or keyboards.

5. You are permitted to use rulers, compasses, and protractors.

6. You have **two hours** to complete this examination.

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PART A: MULTIPLE-CHOICE

Value: 50 marks **Suggested Time: 75 minutes**

INSTRUCTIONS: For each question, select the **best** answer and record your choice on the Answer Sheet provided. Using an HB pencil, completely fill in the circle that has the letter corresponding to your answer.

1. Determine the distance between $(-2,\ 3)$ and $(5,\ -1)$.

   A. $\sqrt{33}$
   B. $\sqrt{53}$
   C. $\sqrt{57}$
   D. $\sqrt{65}$

2. Determine the midpoint of the line segment joining the points $(-3,\ 4)$ and $(2,\ -1)$.

   A. $\left(-\frac{5}{2},\ \frac{5}{2}\right)$
   B. $\left(-\frac{1}{2},\ \frac{3}{2}\right)$
   C. $\left(-\frac{1}{2},\ \frac{5}{2}\right)$
   D. $\left(\frac{1}{2},\ \frac{1}{2}\right)$

3. Determine the vertex of the parabola $x = -(y-2)^2 - 3$.

   A. $(-3,\ 2)$
   B. $(-2,\ 3)$
   C. $(2,\ -3)$
   D. $(3,\ -2)$

OVER

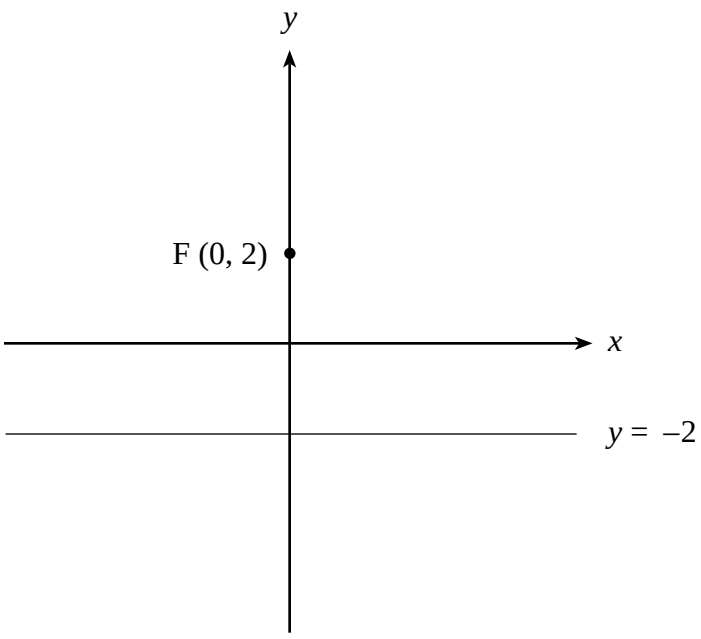
4. Solve: $|x-1|>3$

   A. $x>4$
   B. $-2<x<4$
   C. $x<-4$ or $x>2$
   D. $x<-2$ or $x>4$

5. Determine the vertices of $\frac{(x+2)^2}{4}-\frac{(y-1)^2}{9}=-1$.

   A. (–2, –2) and (–2, 4)
   B. (0, 1) and (–4, 1)
   C. (0, –1) and (4, –1)
   D. (2, 2) and (2, 4)

6. A point P (x, y) moves such that it is always the same distance from the point F (0, 2) as it is from the line defined by $y=-2$. Identify the locus.



   A. line
   B. circle
   C. ellipse
   D. parabola

7. Write $9x^2 + y^2 + 36x - 9 = 0$ in standard form.

A. $\dfrac{(x+2)^2}{\frac{13}{9}} + \dfrac{y^2}{13} = 1$

B. $\dfrac{(x+2)^2}{3} + \dfrac{y^2}{27} = 1$

C. $\dfrac{(x+2)^2}{5} + \dfrac{y^2}{45} = 1$

D. $\dfrac{(x+2)^2}{9} + \dfrac{y^2}{27} = 1$

8. Solve the following system for ***x*** only:

$$y = x + 3$$

$$2y = x^2 - 2$$

A. $x = 4$ or $x = -2$

B. $x = -4$ or $x = 2$

C. $x = 1 + \sqrt{6}$ or $x = 1 - \sqrt{6}$

D. $x = -1 + \sqrt{6}$ or $x = -1 - \sqrt{6}$

9. Determine all values for r $(r > 0)$ such that the following system has **exactly** 2 different real solutions:

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

$$x^2 + y^2 = r^2$$

A. $r = 2$

B. $r < 2$

C. $r = 2$ or $r = 3$

D. $2 < r < 3$

OVER

10. Determine the distance between the vertices of the hyperbola $xy = 6$.

A. $2\sqrt{6}$
B. $4\sqrt{6}$
C. $2\sqrt{3}$
D. $4\sqrt{3}$

11. Convert $325°$ to radian measure. (Accurate to 2 decimal places.)

A. 0.96
B. 5.67
C. 6.14
D. 11.34

12. Evaluate: $\csc\frac{5\pi}{3}$ (Accurate to 2 decimal places.)

A. –1.15
B. –0.87
C. 0.19
D. 2.00

13. Determine the amplitude of the graph of $y = -\frac{3}{5}\sin\frac{1}{3}x$.

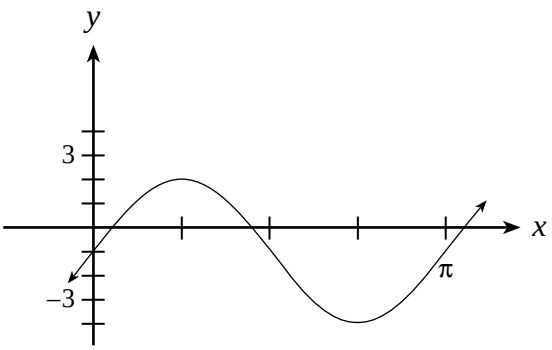
A. $-\frac{3}{5}$
B. $\frac{1}{3}$
C. $\frac{3}{5}$
D. $\frac{2\pi}{3}$

14. Solve: $\sec x = 3.45 \quad (0 \le x < 2\pi)$ (Accurate to 2 decimal places.)

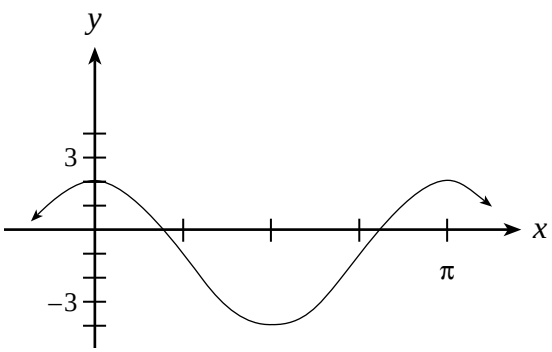
A. 0.29, 2.85
B. 0.29, 5.99
C. 1.28, 1.86
D. 1.28, 5.01

15. Which of the following represents the graph of $y = 3\sin 2x - 1$?

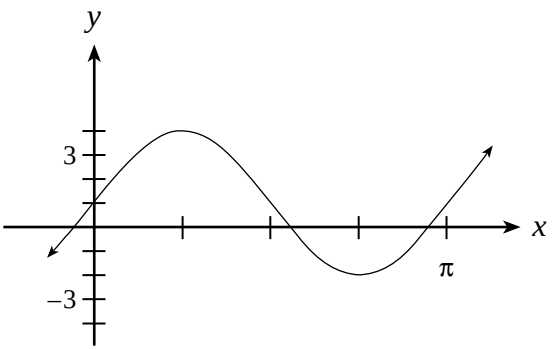
A.



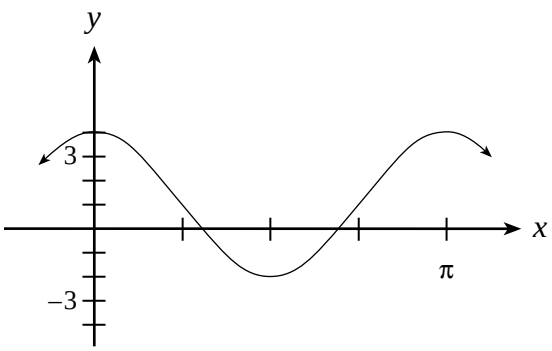
B.



C.



D.



16. How many solutions does $\sin^2 x = \frac{1}{3}$ have over the interval $0 \le x < 2\pi$?

A. 1
B. 2
C. 3
D. 4

17. Which expression is equivalent to $\frac{2\tan\theta\cot 2\theta}{1+\tan\theta}$?

A. $1 - \tan\theta$

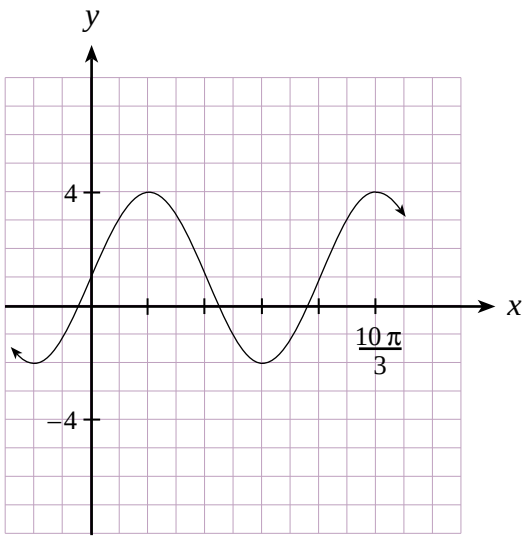
B. $\frac{1}{1-\tan\theta}$

C. $\frac{1}{1+\tan\theta}$

D. $\frac{1}{\tan\theta}$

OVER

18. The function $y = a\cos b(x - c) + d$ is graphed below. Determine b.



A. $\frac{3}{5}$

B. $\frac{3}{4}$

C. $\frac{8\pi}{3}$

D. $\frac{10\pi}{3}$

19. Evaluate: $\log_7 15$ (Accurate to 2 decimal places.)

A. 0.17
B. 0.33
C. 1.18
D. 1.39

20. If the graph of $y = a^x$ goes through the point (3, 216), determine a.

A. $\frac{1}{6}$

B. 4.89

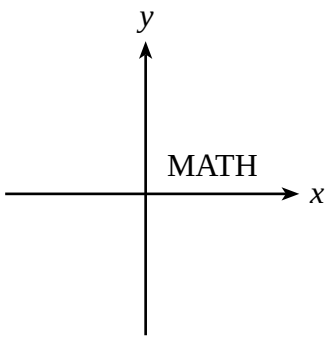
C. 6

D. 72

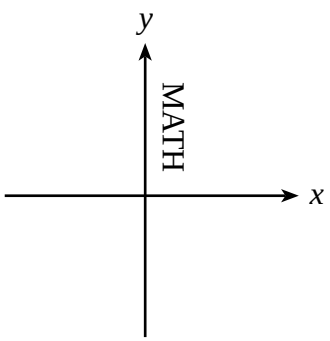
21. Determine the domain of the function $y = \log_3(x-2)$.

A. all real numbers
B. $x > 0$
C. $x < 2$
D. $x > 2$

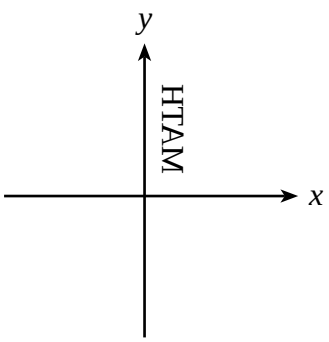
22. Which graph **best** represents the inverse relation of the graph shown?



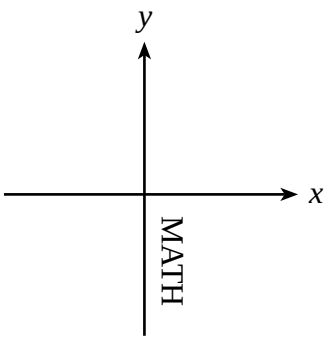
A.



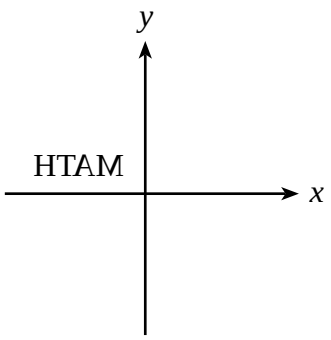
B.



C.



D.



OVER

23. Determine an expression for x, if $\log x = \log a - 2\log b + \frac{1}{4}\log c$.

A. $\dfrac{a}{b^2 \sqrt[4]{c}}$

B. $\dfrac{a \sqrt[4]{c}}{b^2}$

C. $\dfrac{a}{c^4 \sqrt{b}}$

D. $\dfrac{ac^4}{\sqrt{b}}$

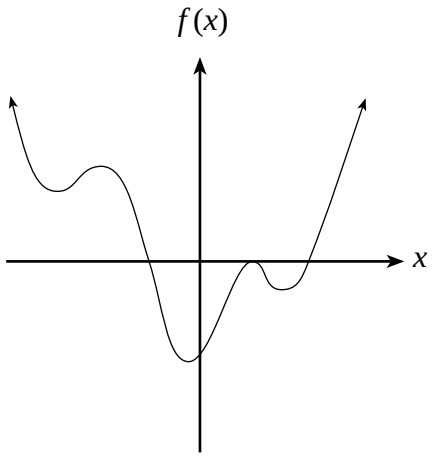
24. A radioactive material decays according to the formula $A = A_o 10^{-kt}$, where A is the final amount, A_o is the initial amount and t is time in years. Find k, if 500 grams of this material decays to 450 grams in 10 years. (Accurate to 4 decimal places.)

A. –16.9897
B. – 0.9000
C. 0.0046
D. 1.1065

25. If $a = b^{c \log_b d}$, then which of the following **must** be true?

A. $a = cd$
B. $a = b^c$
C. $a = d^c$
D. $a = dc$

26. Determine the minimum degree of the polynomial function graphed below.



A. 3
B. 4
C. 5
D. 6

27. If $p(x) = x^3 - 3x^2 + kx + 1$, determine k if $p(3) = -5$.

A. –12
B. –2
C. 4
D. 16

28. Determine all real roots of the equation $\left(x^2 - 4\right)\left(x^2 + 9\right)(x - 5)^2 = 0$.

A. $2,\ 3,\ 5$
B. $\pm 2,\ 5$
C. $\pm 2,\ \pm 3,\ 5$
D. $\pm 2,\ \pm 3,\ \pm 5$

29. The polynomial equation $x^3 - ax^2 + bx - c = 0$, where a, b and c are integers, has 6 as one of its roots. According to the Rational Root Theorem, which of the following could be a value of c ?

A. 2
B. 3
C. 9
D. 18

OVER

30. Determine the quotient and remainder: $\left(t^4+3t^3+5t^2+21t-14\right) \div \left(t^2+3t-2\right)$

A. quotient: t^2+7 remainder: 0
B. quotient: t^2+7 remainder: -28
C. quotient: t^2+3 remainder: $12t-8$
D. quotient: t^2+3 remainder: $30t-20$

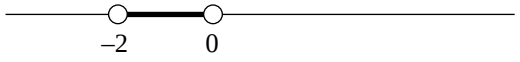
31. Find the remainder when $x^{39}-3x^{15}-2x+1$ is divided by $x-1$.

A. -3
B. -1
C. 1
D. 5

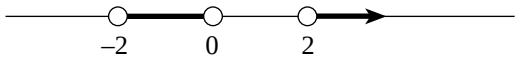
32. Determine all real solutions for $x^3-2x^2-5x+6=0$.

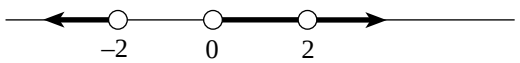
A. $-1,\ 3,\ -2$
B. $-1,\ -3,\ 2$
C. $1,\ 3,\ -2$
D. $1,\ -3,\ 2$

33. Solve the inequality: $x(x-2)\left(x^2-4\right)<0$

A. 

B. 

C. 

D. 

34. Which of the following sequences is geometric, for all positive values of a ?

A. $a,\ 2a,\ 3a,\ 4a$

B. $a,\ a^2,\ a^4,\ a^8$

C. $a,\ a^2,\ a^3,\ a^4$

D. $\log a,\ \log a^2,\ \log a^3$

35. Determine the 80th term in the arithmetic sequence $20,\ 11,\ 2,\ \ldots$

A. -849

B. -731

C. -700

D. -691

36. The sum of the first four terms of a geometric series with a first term of 5 is given by $S_4 = \dfrac{5\left(1-3^4\right)}{1-3}$. Determine the common ratio r.

A. -3

B. 3

C. 4

D. 5

37. Write the first 3 terms of the sequence defined by: $t_1 = -2, \quad t_n = 3t_{n-1} - 2, \quad n > 1$

A. $-2,\ -9,\ -30$

B. $-2,\ -9,\ -29$

C. $-2,\ -8,\ -27$

D. $-2,\ -8,\ -26$

38. Evaluate: $\displaystyle\sum_{k=3}^{\infty} 12\left(-\frac{2}{3}\right)^{k-1}$

A. $\dfrac{36}{5}$

B. $\dfrac{16}{5}$

C. 16

D. 36

OVER

39. For the polynomial function $y = f(x)$, the expression $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ represents:

A. the minimum value of $f(x)$.
B. the maximum value of $f(x)$.
C. the slope of a secant line of $f(x)$.
D. the slope of a tangent line of $f(x)$.

40. Find $\frac{dy}{dx}$, if $y = -3x^2 + 6x$.

A. $-6x + 6$
B. $9x + 6$
C. $-x + 7$
D. $-6x$

41. Evaluate: $\lim_{n \to \infty} \frac{n^2 + 2n - 3}{5 - 3n + 6n^2}$

A. $-\frac{3}{5}$
B. 0
C. $\frac{1}{6}$
D. limit does not exist (no finite limit)

42. Evaluate: $\lim_{x \to 2} \frac{x+6}{x-2}$

A. 0
B. 1
C. 8
D. limit does not exist

43. Determine the minimum value of the function $y = 3x^2 - 24x - 7$.

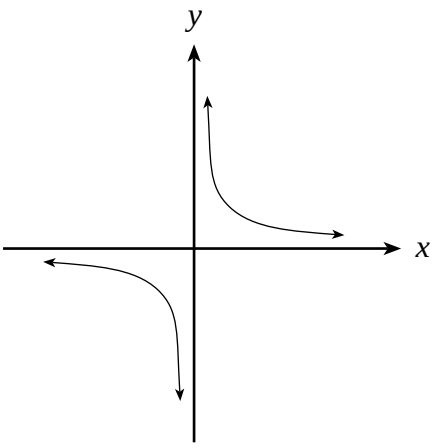
A. –79
B. –55
C. 4
D. there is no minimum value

44. A particle moves along the *x*-axis according to the position function $x(t) = 2t^3 - 6t + 1$, where x is in metres and t is in seconds. For what values of t is the particle moving to the right?

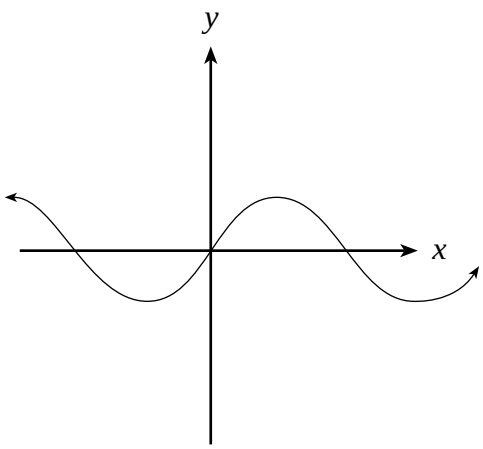
A. $-1 < t < 1$
B. $t < -1$ or $t > 1$
C. all values of t
D. no values of t

45. For which of the following functions $y = f(x)$ is $f'(x)$ **always** less than 0 ?

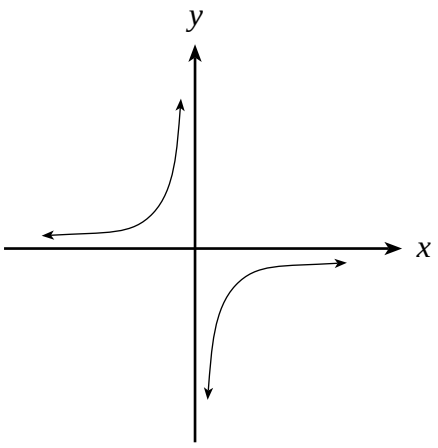
A.



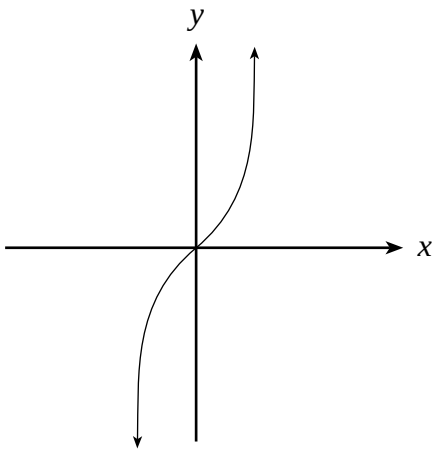
B.



C.



D.

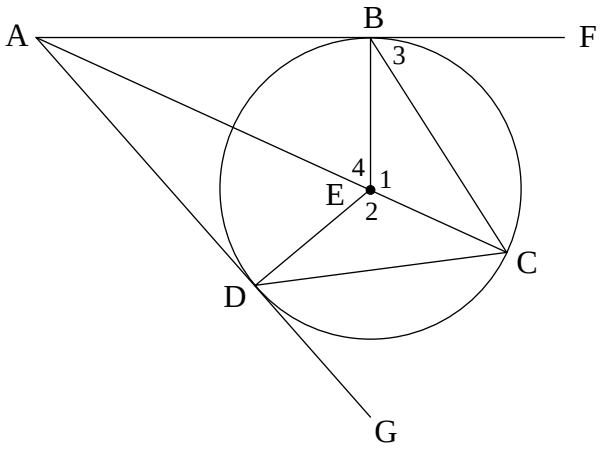


OVER

Use the following diagram and proof to answer questions 46 and 47.

Given: E is the centre
A, E, C are collinear
AF and AG are tangents
$\angle 1 = \angle 2$

Prove: $\Delta BCA \cong \Delta DCA$



Proof

	Statement	Reason
	AF and AG are tangents	given
(a)	AB = AD	tangents from an external point are =
	$\angle 1 = \angle 2$	given
(b)	BC = DC	chords with = central $\angle$s are =
(c)	BE = DE	radii
(d)	AC = AC	same side
	$\Delta BCA \cong \Delta DCA$	SSS

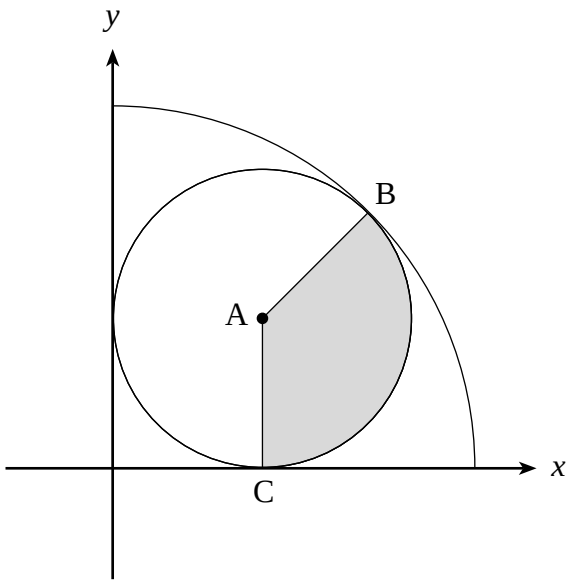
46. Which line is **not** necessary in the given proof?

A. a
B. b
C. c
D. d

47. If $\angle 3 = 58°$, determine the measure of $\angle 4$. (Diagram is not to scale.)

A. 58°
B. 62°
C. 64°
D. 66°

48. A circle with centre A is inscribed in the quadrant I sector of the circle $x^2 + y^2 = 64$. The inscribed circle has an area of 34.50 units^2. B and C are points of tangency. Determine the area of the shaded region. (Accurate to 2 decimal places.)



A. 10.35 units^2

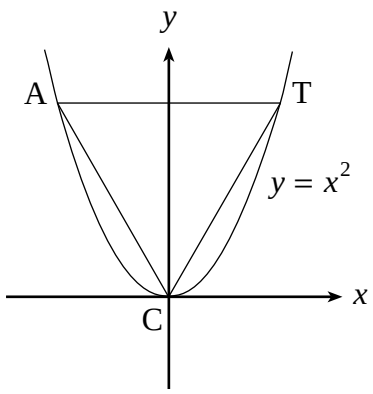
B. 11.50 units^2

C. 12.94 units^2

D. 14.78 units^2

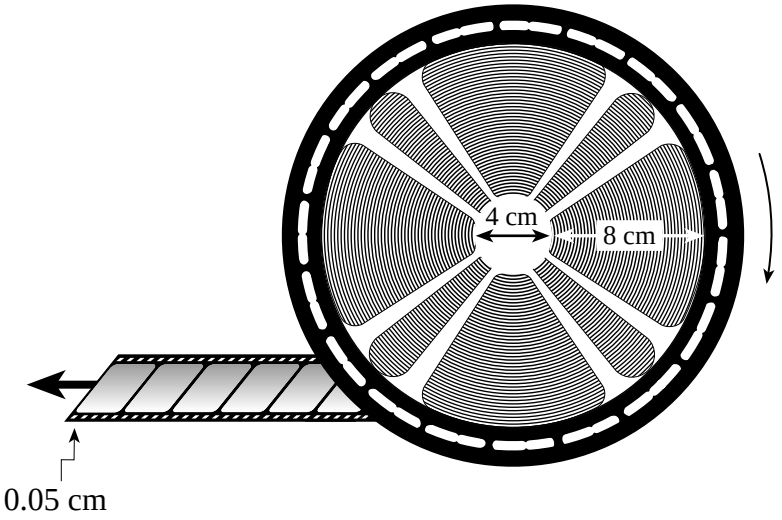
OVER

49. Triangle ATC is an equilateral triangle with vertices A, C, and T on the parabola $y = x^2$. Determine the x-coordinate of point T.



A. 2
B. $\sqrt{2}$
C. $\sqrt{3}$
D. 3

50. The diameter of the centre hub of a film projector reel is 4 cm. The total thickness of the film on the reel is 8 cm and the ribbon of film is 0.05 cm thick. What is the total length of the film? (Accurate to the nearest cm.)



A. 4 021
B. 5 278
C. 6 032
D. 6 883

This is the end of the multiple-choice section.
Answer the remaining questions directly in this examination booklet.

PART B: WRITTEN-RESPONSE

Value: 20 marks **Suggested Time: 45 minutes**

INSTRUCTIONS:	Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided. **Full marks will NOT be given for the final answer only.**

OVER

1. Solve: $\log(10-3x)-2\log x=0$ **(3marks)**

ANSWER:	Score for Question 1: 1. _____ (3)

OVER

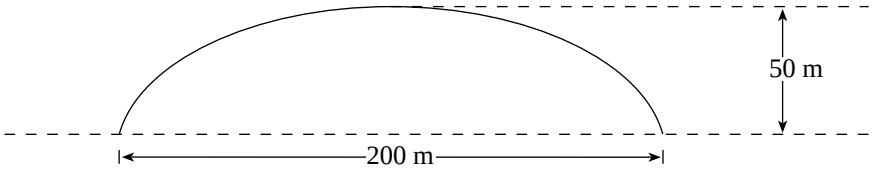
2. The first three terms of an arithmetic sequence are $x+4$, x^2+5 and $x+30$. Determine the values of the first three terms of all such sequences. **(3marks)**

ANSWER:	Score for Question 2: 2. _____ (3)

OVER

3. A sports stadium has a semi-elliptical dome for its roof. If its maximum height is 50 m and its span is 200 m, how high is the dome at a point 72 m from the centre? (Accurate to 1 decimal place.)

(3marks)



ANSWER:	Score for Question 3: 3. _____ (3)

OVER

4. Prove the identity. **(2 marks)**

$$\frac{\sec\theta - \cos\theta}{\tan\theta} = \sin\theta$$

Left side	Right side

Score for Question 4:
4. _____ (2)

OVER

5. Three numbers a, b and c exist such that $a+b=-4$, $a+c=25$ and $b+c=5$.
Determine the value of 'a'. **(2marks)**

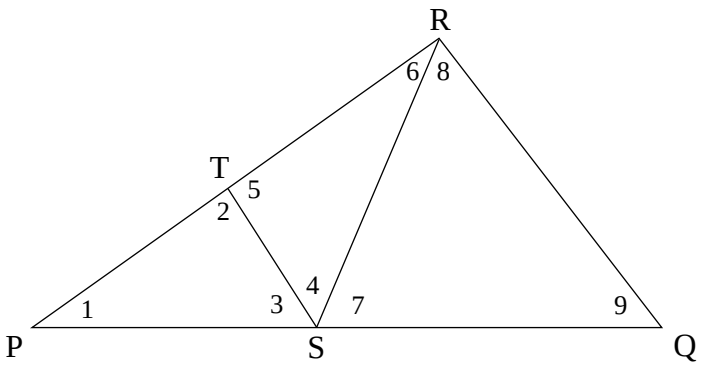
ANSWER:	Score for Question 5: 5. _____ (2)

OVER

6. Complete the proof. **(4 marks)**

Given: TS || RQ
TS bisects ∠PSR

Prove: RS = QS



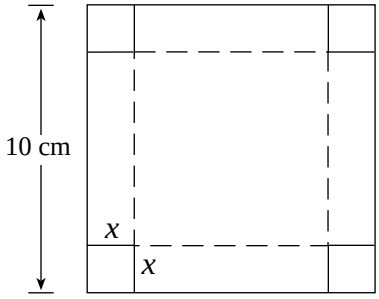
Proof

Statement	Reason

Score for Question 6:
6. _____ (4)

OVER

7. A square piece of cardboard 10 cm by 10 cm will have equal squares with sides of length x cm cut from each corner. The sides will then be folded up to create a box with no top. Determine the value of x that will give the box a maximum volume. **(3 marks)**



ANSWER:	Score for Question 7: 7. _____ (3)

END OF EXAMINATION

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A SUMMARY OF BASIC IDENTITIES AND FORMULAE

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$ $\qquad 1 + \tan^2\theta = \sec^2\theta$ $\qquad 1 + \cot^2\theta = \csc^2\theta$

Reciprocal and Quotient Identities

$\sec\theta = \dfrac{1}{\cos\theta}$ $\qquad \csc\theta = \dfrac{1}{\sin\theta}$ $\qquad \cot\theta = \dfrac{1}{\tan\theta}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\qquad \cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Addition Identities

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\qquad \tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\qquad \tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

Double-Angle Identities

$\cos 2\theta = \cos^2\theta - \sin^2\theta$ $\qquad \sin 2\theta = 2\sin\theta\cos\theta$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$ $\qquad \tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

Formulae

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\qquad t_n = a + (n-1)d$ $\qquad t_n = ar^{n-1}$

$\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$ $\qquad S_n = \dfrac{n}{2}\left[2a + (n-1)d\right]$ $\qquad S_n = \dfrac{a\left(1 - r^n\right)}{1 - r}$

$c^2 = a^2 + b^2 - 2ab\cos C$ $\qquad S_n = \dfrac{n}{2}(a + \ell)$ $\qquad S_n = \dfrac{a - \ell r}{1 - r}$

$S = \dfrac{a}{1 - r}$

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ROUGH WORK FOR GRAPHING

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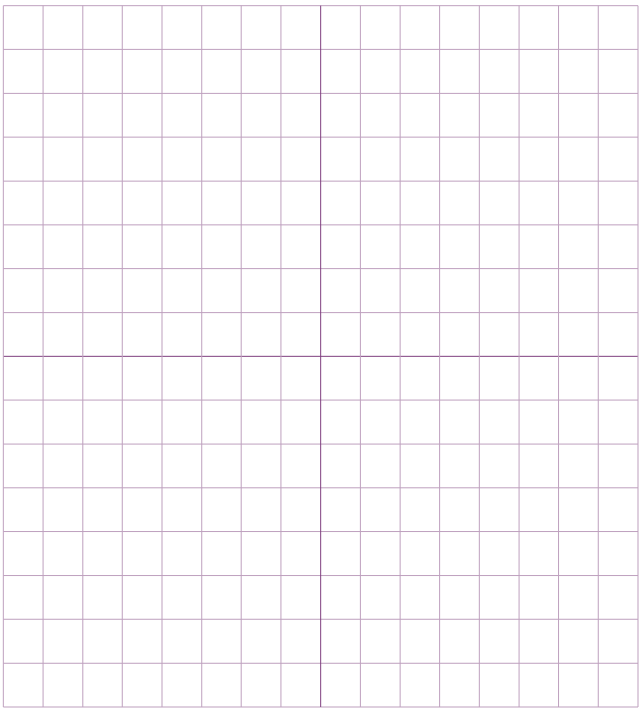

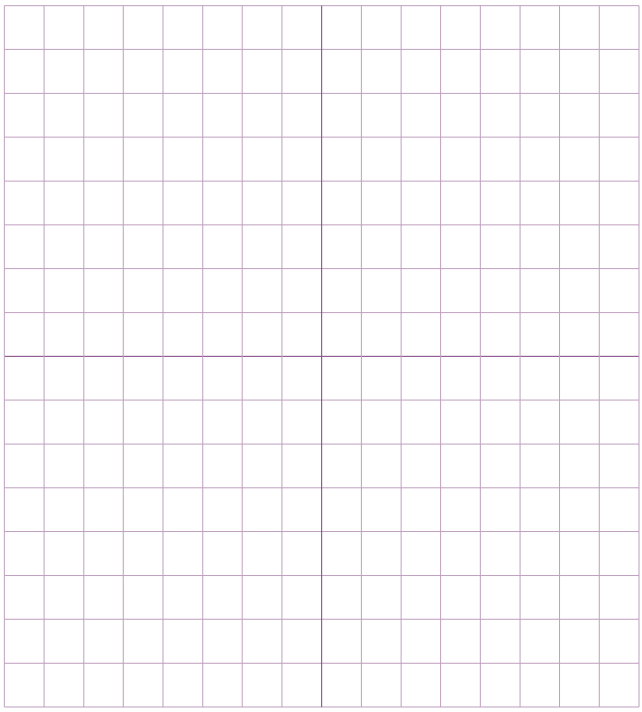

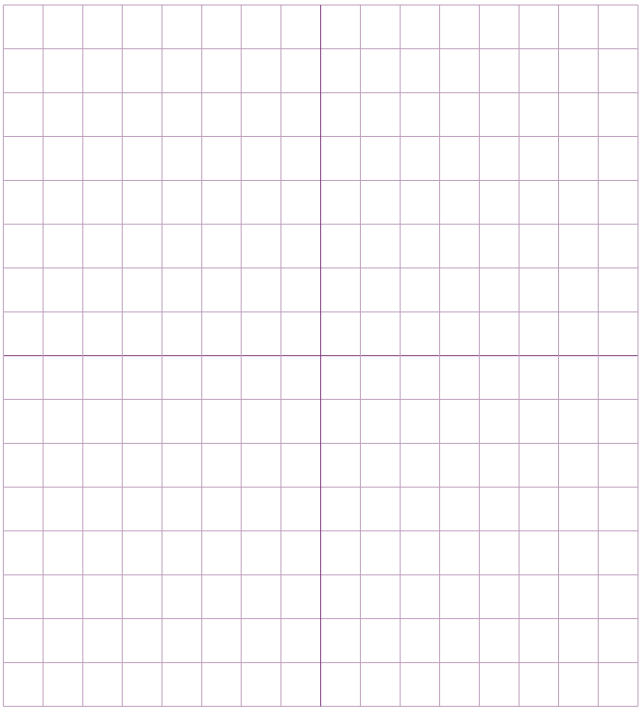

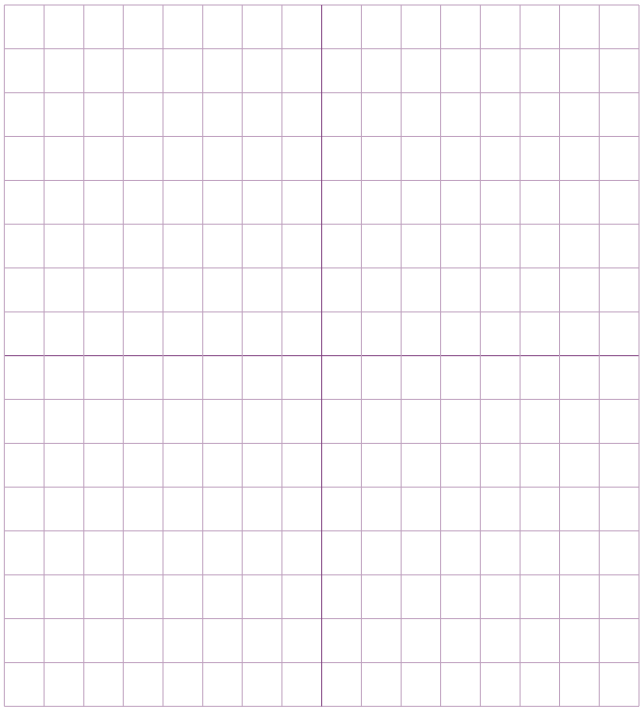

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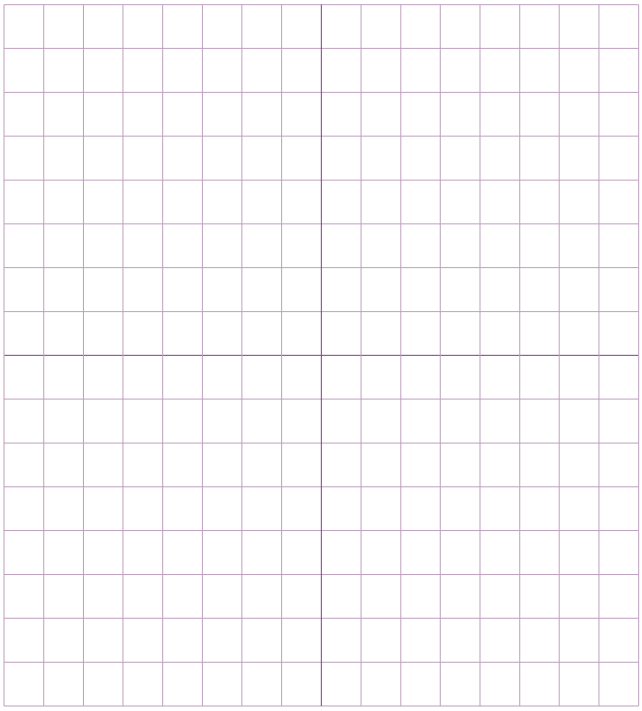

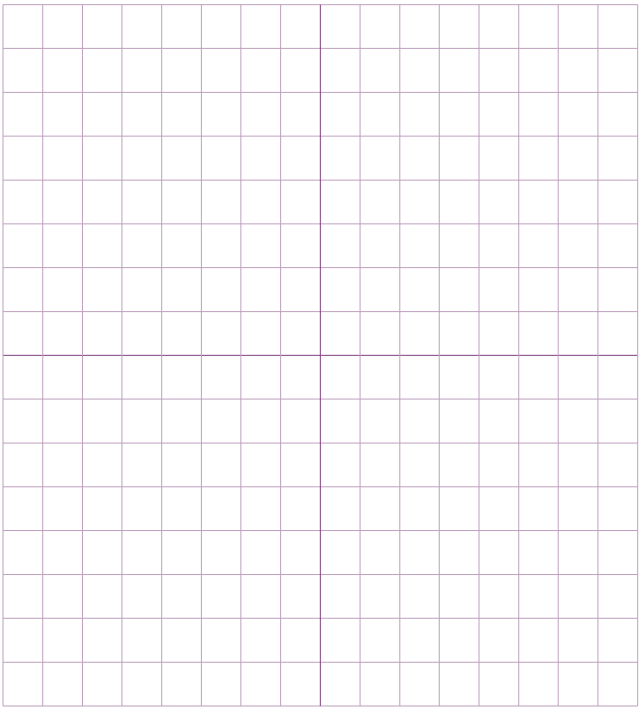

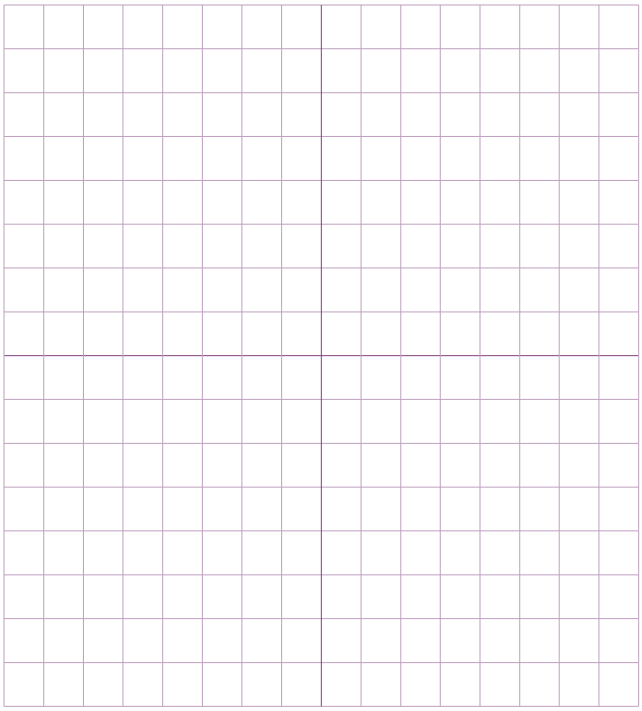

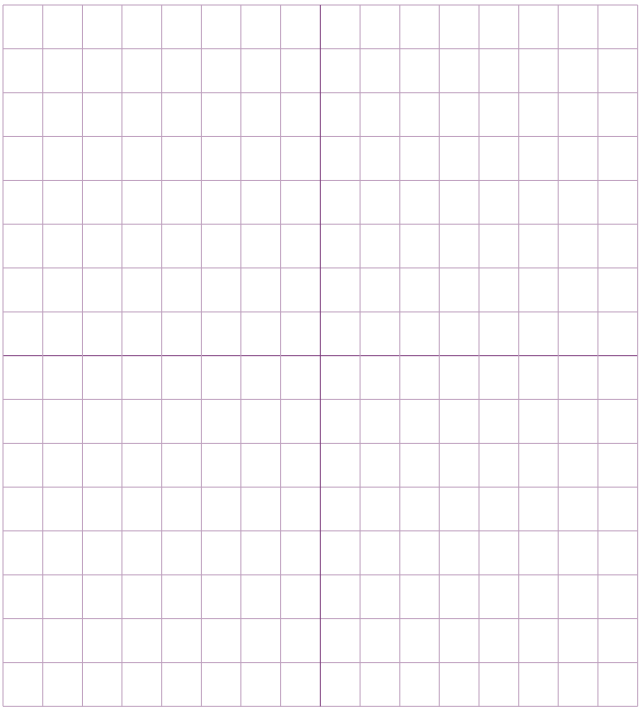

ROUGH WORK FOR MULTIPLE-CHOICE

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ROUGH WORK FOR MULTIPLE-CHOICE