

JUNE 1994

PROVINCIAL EXAMINATION

MINISTRY OF EDUCATION

MATHEMATICS 12

GENERAL INSTRUCTIONS

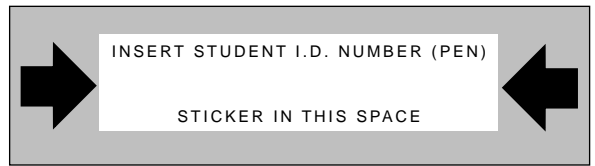
1. Insert the stickers with your Student I.D. Number (PEN) in the allotted spaces above. **Under no circumstance is your name or identification, other than your Student I.D. Number, to appear on this paper.**
2. Take the separate Answer Sheet and follow the directions on its front page.
3. Be sure you have an HB pencil and an eraser for completing your Answer Sheet. Follow the directions on the Answer Sheet when answering multiple-choice questions.
4. For each of the written-response questions, write your answer in the space provided. When instructed to open this booklet, **check the numbering of the pages** to ensure that they are numbered in sequence from page one to the last page, which is identified by

ENDOFEXAMINATION .

5. At the end of the examination, place your Answer Sheet inside the front cover of this booklet and return the booklet and your Answer Sheet to the supervisor.

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FOR OFFICE USE ONLY



_____ - _____

**MATHEMATICS 12 JUNE 1994 PROVINCIAL
(MAP)**

1. $\frac{\quad}{(3)}$

5. $\frac{\quad}{(4)}$

2. $\frac{\quad}{(2)}$

6. $\frac{\quad}{(3)}$

3. $\frac{\quad}{(2)}$

7. $\frac{\quad}{(3)}$

4. $\frac{\quad}{(3)}$

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MATHEMATICS 12 PROVINCIAL EXAMINATION

	Value	Suggested Time
1. This examination consists of two parts:		
PART A: 50 multiple-choice questions	50	75
PART B: 7 written-response questions	20	45
2 questions worth two marks each, 4 questions worth three marks each, and 1 question worth four marks.		
	TOTAL	70 marks 120 minutes

- The last **three** pages inside the back cover contain “**A Summary of Basic Identities and Formulae**”, “**Rough Work for Graphing**”, and “**Rough Work for Multiple-Choice**”. These pages may be detached for convenient reference prior to writing this examination.
- Students will not be provided with any additional paper since rough-work space for the written-response questions has been incorporated into the space allowed for answering each question. Students may not need all of the space provided to answer each question.
- An approved scientific calculator is essential for the examination. The calculator **must not** be programmable to process alpha-numeric strings, nor should it be capable of processing user-defined functions. It **must not** have the capacity to accept coefficients from either an equation or a system of equations, thereby producing the roots of that equation or system. The calculator **must not** contain a plotter or printer.
- Students are permitted to use rulers, compasses, and protractors.
- Students have **two hours** to complete this examination.

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PART A: MULTIPLE-CHOICE QUESTIONS

Value: 50 (one mark per question)

Suggested Time: 75 minutes

INSTRUCTIONS: For each question, select the **best** answer and record your choice on the answer sheet provided. Using an HB pencil, completely fill in the circle that has the letter corresponding to your answer.

1. Which expression will calculate the distance between (x_1, y_1) and (x_2, y_2) ?

A. $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

B. $\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$

C. $\sqrt{(x_2 + x_1)^2 - (y_2 + y_1)^2}$

D. $\sqrt{(x_2 + x_1)^2 + (y_2 + y_1)^2}$

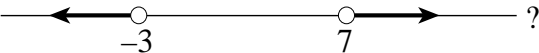
2. Determine the centre of the circle with a diameter whose endpoints are $(-2, 7)$ and $(-4, -5)$.

A. $(-3, 6)$

B. $(-3, 2)$

C. $(-3, 1)$

D. $(-1, 6)$

3. Which absolute value inequality has the solution  ?

A. $|x - 2| > 5$

B. $|x + 2| > 5$

C. $|x - 5| > 2$

D. $|x + 5| > 2$

4. What are the slopes of the asymptotes of the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$?

A. $\pm \frac{4}{9}$

B. $\pm \frac{9}{4}$

C. $\pm \frac{2}{3}$

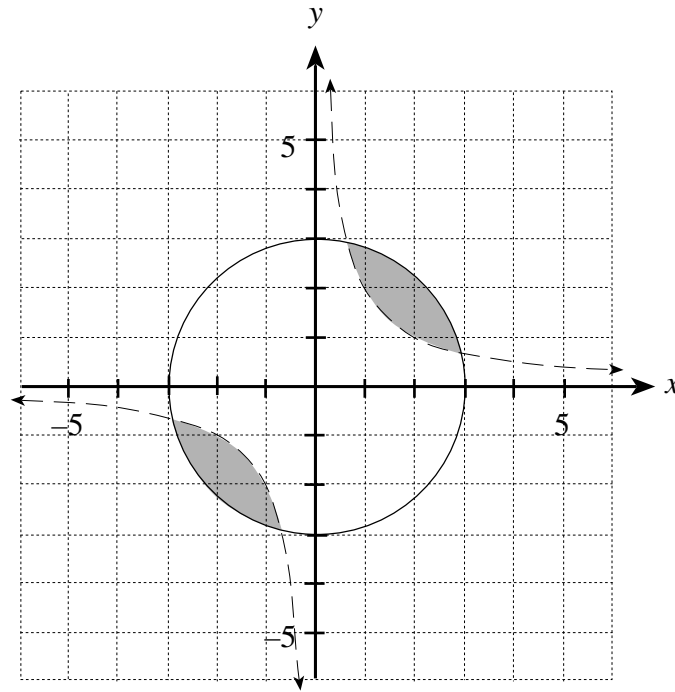
D. $\pm \frac{3}{2}$

5. What is the equation of the line that contains the vertices of the hyperbola

$$\frac{(x-1)^2}{16} - \frac{(y+2)^2}{25} = 1?$$

- A. $x = 1$
- B. $x = -1$
- C. $y = 2$
- D. $y = -2$

6. Which system has the shaded portion shown below as its solution?



- A. $x^2 + y^2 \leq 9$
 $xy > 2$
- B. $x^2 + y^2 \geq 9$
 $xy < 2$
- C. $x^2 + y^2 \geq 9$
 $xy > 2$
- D. $x^2 + y^2 \leq 9$
 $xy < 2$

7. How many real solutions are there for the following system?

$$\frac{x^2}{4} + \frac{y^2}{25} = 1$$

$$x = -y^2 + 2$$

- A. 1
- B. 2
- C. 3
- D. 4

8. Every point on a conic is equidistant from the point $(5, -6)$ and the conic contains the point $(1, 9)$. Determine the equation of this conic.

- A. $(x - 5)^2 + (y + 6)^2 = 241$
- B. $(x + 5)^2 + (y - 6)^2 = 241$
- C. $(x - 5)^2 + (y + 6)^2 = 25$
- D. $(x + 5)^2 + (y - 6)^2 = 25$

9. If the lengths of the major and minor axes of an ellipse are $2a$ and $2b$ respectively, then the area of the ellipse is given by $A = \pi ab$. Determine the area of the ellipse $\frac{x^2}{16} + \frac{y^2}{36} = 1$, accurate to 1 decimal place.

- A. 75.4
- B. 78.5
- C. 150.8
- D. 301.6

10. A circle with centre $(0, 0)$ is tangent to the line $x + y = 16$. Determine the equation of this circle.

- A. $x^2 + y^2 = 32$
- B. $x^2 + y^2 = 64$
- C. $x^2 + y^2 = 128$
- D. $x^2 + y^2 = 144$

11. θ is an angle in standard position with $\sin \theta > 0$ and $\tan \theta < 0$. In which quadrant does the terminal arm of θ lie?
- A. I
B. II
C. III
D. IV
12. Simplify: $\sin^2 x - \cos^2 x$
- A. $-\cos 2x$
B. $\cos 2x$
C. -1
D. 1
13. Evaluate: $\csc 2.4$ (accurate to 2 decimal places)
- A. -1.36
B. -0.74
C. 0.68
D. 1.48
14. Solve: $\cos x = -0.4566$, where $0 \leq x < 2\pi$ (accurate to 2 decimal places)
- A. $2.04, 4.24$
B. $2.67, 5.81$
C. $3.62, 5.81$
D. $4.24, 5.19$
15. For what value of x is the following expression undefined?

$$\frac{\sin x}{1 + \cos x}, \text{ where } 0 \leq x < 2\pi$$

- A. 0
B. $\frac{\pi}{2}$
C. π
D. $\frac{3\pi}{2}$

16. In the interval $0 \leq x < 2\pi$, determine the equations of all asymptotes for the function $y = \tan x$.

A. $x = 0$

B. $x = \frac{\pi}{2}$

C. $x = 0, x = \pi$

D. $x = \frac{\pi}{2}, x = \frac{3\pi}{2}$

17. Simplify: $\frac{\sqrt{\sec^2 x - 1}}{\sqrt{\csc^2 x - 1}}$

A. $\tan^2 x$

B. $\cot^2 x$

C. $\tan^4 x$

D. $\cot^4 x$

18. Determine the value of $\sec \theta$ if $\cot \theta = -a$, where $a > 0$ and $\sin \theta < 0$.

A. $\frac{\sqrt{a^2 + 1}}{a}$

B. $-\frac{\sqrt{a^2 + 1}}{a}$

C. $\frac{a+1}{a}$

D. $-\frac{a+1}{a}$

19. Determine the logarithmic form of $4^x = 3$.

A. $\log_3 x = 4$

B. $\log_3 4 = x$

C. $\log_x 4 = 3$

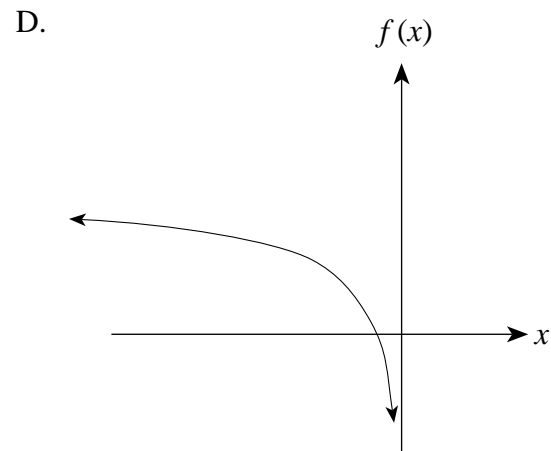
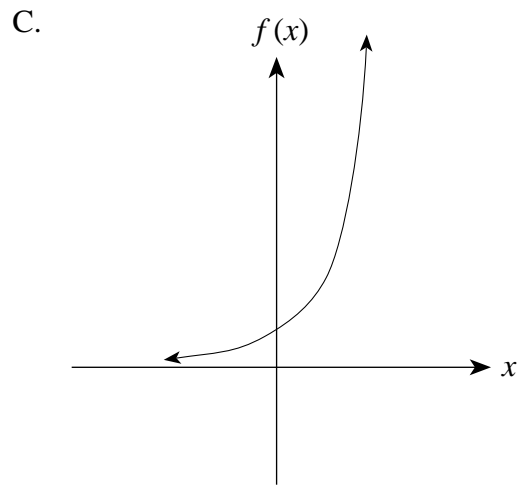
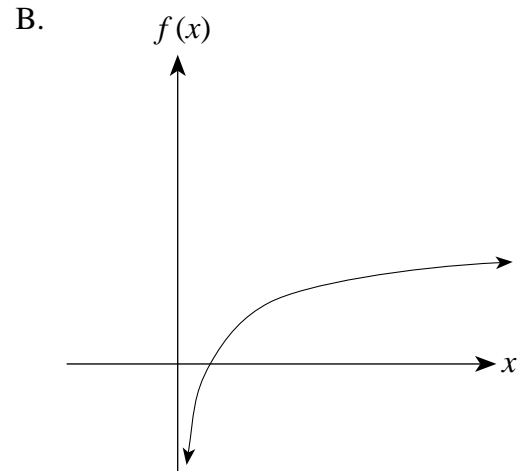
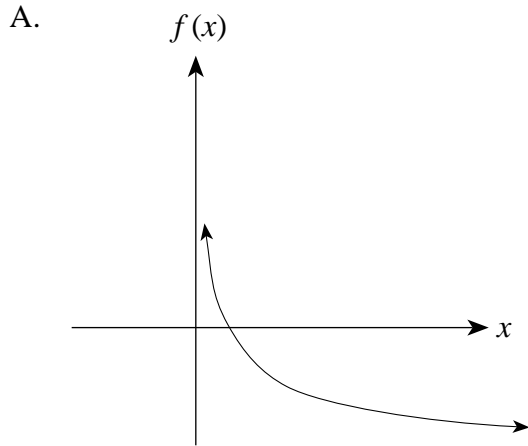
D. $\log_4 3 = x$

OVER

20. What is the value of x if $\log_5 x = 4$?

- A. $\sqrt[5]{4}$
- B. $\sqrt[4]{5}$
- C. 5^4
- D. 4^5

21. Which graph **best** represents the function $f(x) = \log_3 x$?



22. Find the inverse of the function $y = \frac{2x-1}{3}$.

A. $y = \frac{3}{2x-1}$

B. $y = \frac{3x+1}{2}$

C. $y = \frac{3}{2x+1}$

D. $y = \frac{3x-1}{2}$

23. Evaluate: $-16 \log_2 \left(\frac{1}{8} \right)$

A. -4

B. $-\frac{1}{4}$

C. 0

D. 48

24. If $\log_5 x = 4.26$, what is the value of $\log_5 25x^2$?

A. 2.66

B. 3.80

C. 8.26

D. 10.52

25. Simplify: $(\log_x y)(\log_y x)$

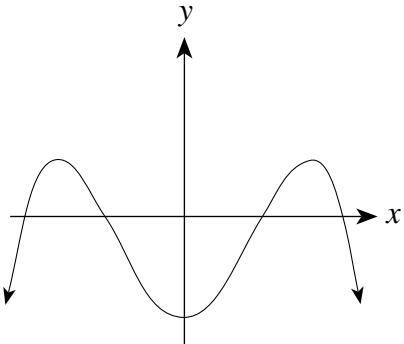
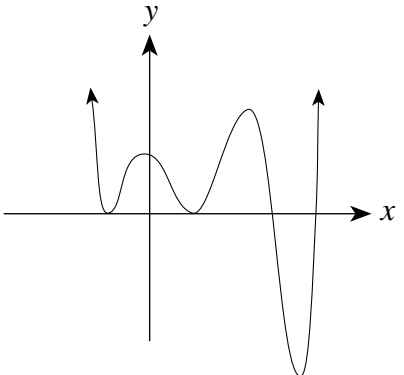
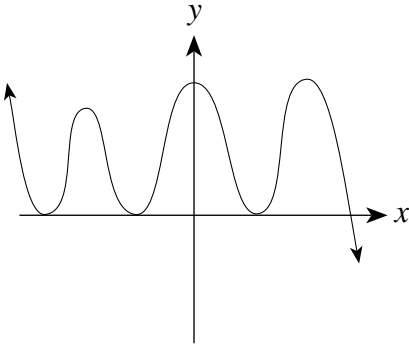
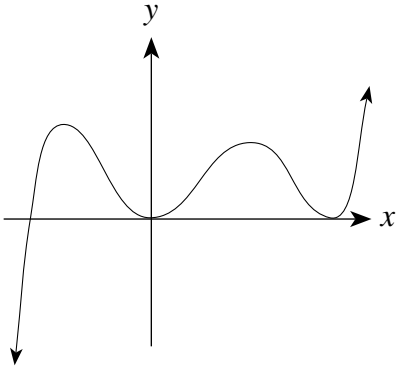
A. 0

B. 1

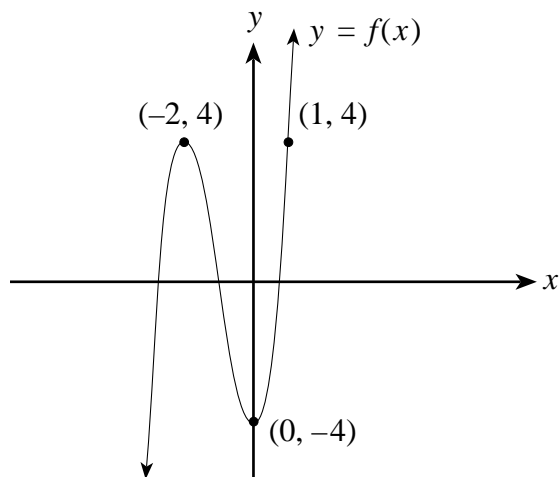
C. $xy^{(x+y)}$

D. $\log_{xy} (x+y)$

OVER

26. According to the Rational Root Theorem, which one of the following is a **possible** root of the equation $8x^4 + 19x^3 - 13x^2 + 7x - 3 = 0$?
- A. 2
 B. 3
 C. 4
 D. 8
27. Which graph could represent a polynomial function of degree 5?
- A. 
- B. 
- C. 
- D. 
28. When $4x^2 + 2kx - 5$ is divided by $x + 2$ the remainder is 3. What is the value of k ?
- A. -6
 B. -2
 C. 2
 D. $\frac{11}{4}$
29. Solve: $x^3 - 2x^2 - 5x + 6 = 0$
- A. 1, 2, -3
 B. 1, -2, 3
 C. -1, 2, -3
 D. -1, -2, 3

30. Determine the remainder when $p(x) = 4x^3 - 6x^2 + 4x - 3$ is divided by $2x - 1$.
- A. -7
 B. -4
 C. -3
 D. -2
31. Determine a polynomial equation that has roots of $\sqrt{3}$, $-\sqrt{3}$ and 2 .
- A. $x^3 - 2x^2 - 3x + 6 = 0$
 B. $x^3 + 2x^2 - 3x - 6 = 0$
 C. $x^3 - 2x^2 - 9x + 18 = 0$
 D. $x^3 + 2x^2 - 9x - 18 = 0$
32. Which polynomial inequality has the solution $-3 < x < -2$ or $x > 1$?
- A. $(x + 3)(x + 2)(x - 1) < 0$
 B. $(x + 3)(x + 2)(x - 1) > 0$
 C. $(x - 3)(x - 2)(x + 1) < 0$
 D. $(x - 3)(x - 2)(x + 1) > 0$
33. The graph of a cubic polynomial function, $y = f(x)$, is shown. Determine the equation of $y = f(x) - 4$.



- A. $y = (x + 2)^2(x + 1)$
 B. $y = (x + 2)^2(x - 1)$
 C. $y = 2(x + 2)^2(x + 1)$
 D. $y = 2(x + 2)^2(x - 1)$

34. Determine the common ratio for the geometric sequence $6, -2, \frac{2}{3}$.

A. -3

B. $-\frac{1}{3}$

C. $\frac{1}{3}$

D. 3

35. Determine the number of terms in the arithmetic sequence $-3, -1, 1, \dots, 101$.

A. 50

B. 52

C. 53

D. 98

36. Determine the fourth term in the expansion of $\sum_{k=2}^{25} (3k-1)$.

A. 8

B. 11

C. 14

D. 17

37. Determine the values of x ($x \neq 0$) such that the following infinite geometric series has a finite sum.

$$1 + \frac{1}{4}x + \frac{1}{16}x^2 + \frac{1}{64}x^3 + \dots$$

A. $x < \frac{1}{4}$

B. $x > 4$

C. $-4 < x < 4$

D. $-\frac{1}{4} < x < \frac{1}{4}$

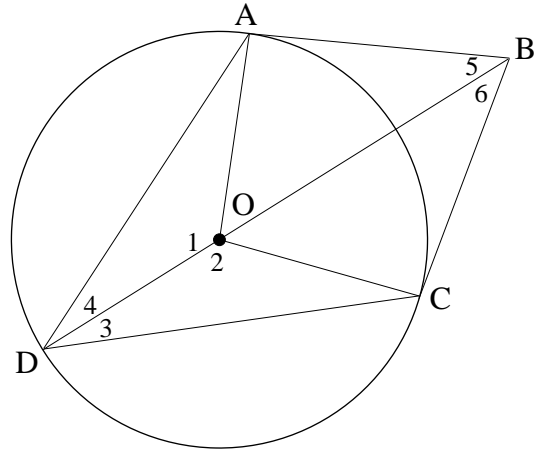
38. Determine the fourth term of the sequence defined by the formula $t_n = 2t_{n-1} - 3t_{n-2} + 1$, where $t_1 = 2a$ and $t_2 = 3a - 1$.
- A. -1
 - B. $-9a + 2$
 - C. $-5a + 4$
 - D. $21a - 13$
39. Given $y = -6x^2 + 11x - 10$, determine y' .
- A. $-12x + 11$
 - B. $-12x + 1$
 - C. $-4x + 12$
 - D. $36x + 11$
40. Evaluate: $\lim_{x \rightarrow \infty} \frac{6x^2 + 5x - 7}{4x^2 - 8x + 3}$
- A. $-\frac{7}{3}$
 - B. 0
 - C. $\frac{3}{2}$
 - D. limit does not exist (no finite limit)
41. Given $f(x) = \frac{5}{x^2}$, determine $f'(x)$.
- A. $\frac{-10}{x}$
 - B. $\frac{-10}{x^3}$
 - C. $\frac{3}{x^3}$
 - D. $\frac{5}{2x}$

42. If the function $y = f(x)$ is decreasing for all values of x , then which one of the following **must** be true?
- A. $f'(x) < 0$
 - B. $f'(x) > 0$
 - C. $f(x) < 0$
 - D. $f(x) > 0$
43. Which of the following represents the slope of the tangent to the function $f(x) = \sqrt{x}$?
- A. $\lim_{x \rightarrow 0} \frac{\sqrt{x+h} + \sqrt{x}}{h}$
 - B. $\lim_{h \rightarrow 0} \frac{\sqrt{x+h} + \sqrt{x}}{h}$
 - C. $\lim_{x \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$
 - D. $\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$
44. Find the minimum value of the function $f(x) = 2x^2 - 12x + 25$.
- A. 0
 - B. 3
 - C. 7
 - D. 25
45. For the curve $y = x^k$ ($k \neq 0$), the slope of the tangent is equal to $16k$ when $x=2$. Determine the value of k .
- A. 3
 - B. 4
 - C. 5
 - D. 8

Use the following diagram and proof to answer questions 46 and 47.

Given: D, O, B are collinear
 O is the centre
 BA and BC are tangents
 $\angle 1 = \angle 2$

Prove: $\triangle ADB \cong \triangle CDB$



Statement	Proof	Reason
D, O, B are collinear		given
BA, BC are tangents		given
(a) BA = BC		tangents from an external point are =
$\angle 1 = \angle 2$		given
O is the centre		given
(b) AD = DC		chords opposite = central \angle s are =
(c) DB = DB		same side
(d) OA = OC		radii are =
$\triangle ADB \cong \triangle CDB$		SSS

46. Which line is **not** necessary in the given proof?

- A. a
- B. b
- C. c
- D. d

47. If $\angle 5 = 40^\circ$, determine the measure of $\angle 4$. (diagram is not to scale)

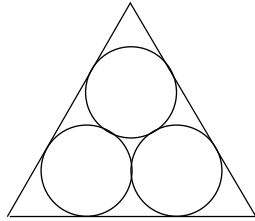
- A. 20°
- B. 25°
- C. 30°
- D. 35°

OVER

48. How many diagonals can be drawn in a convex polygon with 6 sides?

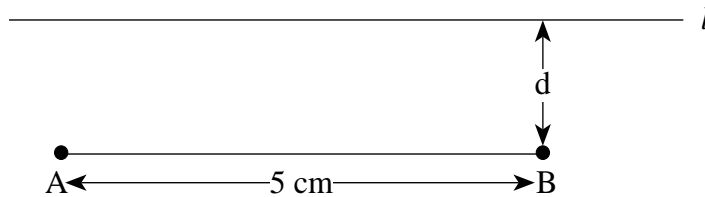
- A. 8
- B. 9
- C. 10
- D. 11

49. Three circles, each with a radius of 4, are all tangent to each other. Determine the perimeter of the equilateral triangle, where each side is tangent to 2 of the circles. (accurate to 2 decimal places)



- A. 41.57
- B. 53.57
- C. 65.57
- D. 74.57

50. Line segment $AB = 5$ cm and AB is parallel to line l . Determine the distance d so that **only 1** isosceles triangle can be drawn using AB as one side with the other vertex on line l .



- A. $d > 0$
- B. $0 < d < 5$
- C. $d = 5$
- D. $d > 5$

This is the end of the multiple-choice section.
Answer the remaining questions directly in this examination booklet.

PART B: WRITTEN-RESPONSE QUESTIONS

Value: 20 marks total

Suggested Time: 45 minutes

INSTRUCTIONS: Rough-work space has been incorporated into the space allowed for answering each question. Students may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

Full marks will NOT be given for the final answer only.

1. Write in standard form: $9x^2 + y^2 - 54x + 4y + 49 = 0$

(3 marks)

ANSWER:

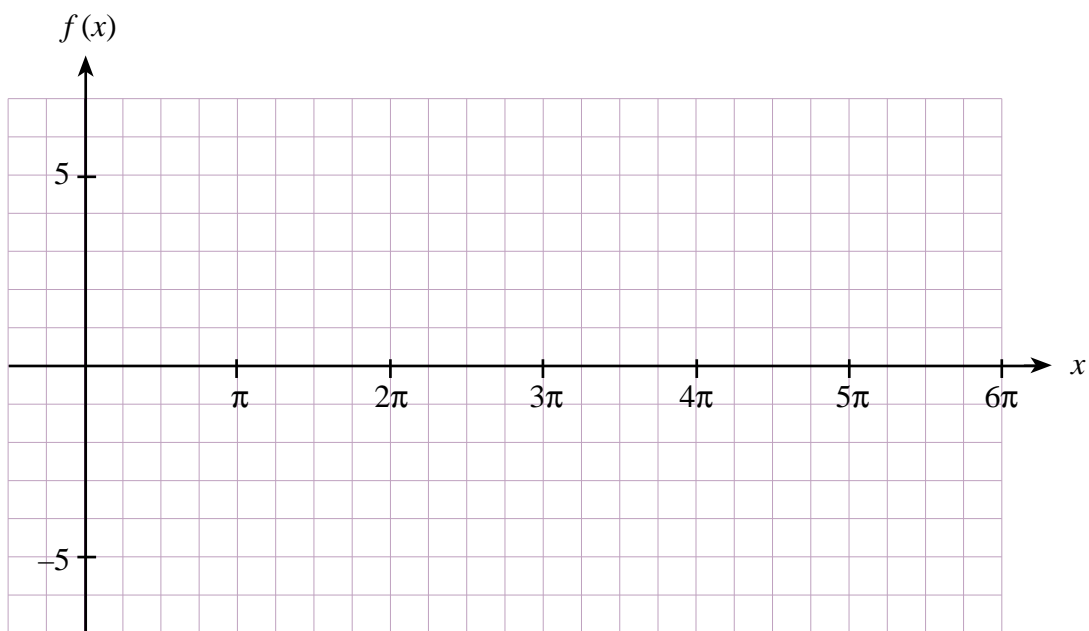
Score for
Question 1:

1.
(3)

OVER

2. Graph at least one period of $f(x) = 3 \cos \frac{1}{2}x + 1$ on the grid provided.

(2 marks)



Score for
Question 2:

2.
(2)

OVER

3. Determine the value(s) of k for which the graph of the relation $(2+k)x^2 + (1-k^2)y^2 + x - 2y = 17$ represents a parabola. **(2 marks)**

ANSWER:

Score for
Question 3:

3. $\frac{\quad}{(2)}$

OVER

4. Solve for x : $\log_5(2x + 1) = 1 - \log_5(x + 2)$

(3 marks)

ANSWER:

Score for
Question 4:

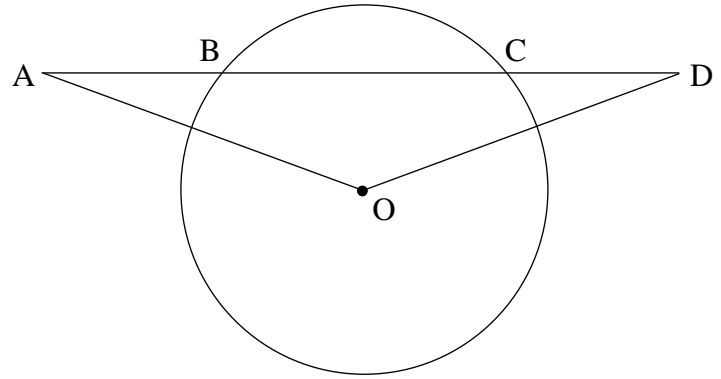
4.
(3)

OVER

5. Complete the proof.

(4 marks)

Given: O is the centre
 A, B, C, D are collinear
 $AB = CD$



Prove: $OA = OD$

Note: Students are encouraged to use numbers to label angles.

Proof

Statement	Reason

Score for
Question 5:

5. $\frac{\quad}{(4)}$

OVER

6. A college student was offered two different summer jobs. Job A would last 17 weeks and pay \$225 per week, with weekly raises of \$10. Job B would last 4 months and pay \$1 100 per month, with monthly raises of 10% of the previous month's salary. How much more would the college student earn by accepting Job A? **(3 marks)**

ANSWER:

Score for
Question 6:

6.
(3)

OVER

7. Two numbers have the property that the sum of twice one number and three times the second number is 40. Find the two numbers such that their product is a maximum. **(3 marks)**

ANSWER:

Score for
Question 7:

7.
(3)

END OF EXAMINATION

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A SUMMARY OF BASIC IDENTITIES AND FORMULAE

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Reciprocal and Quotient Identities

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Addition Identities

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

Double-Angle Identities

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - 2 \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Formulae

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t_n = a + (n-1)d$$

$$t_n = ar^{n-1}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$S_n = \frac{n}{2}(a + \ell)$$

$$S_n = \frac{a - \ell r}{1-r}$$

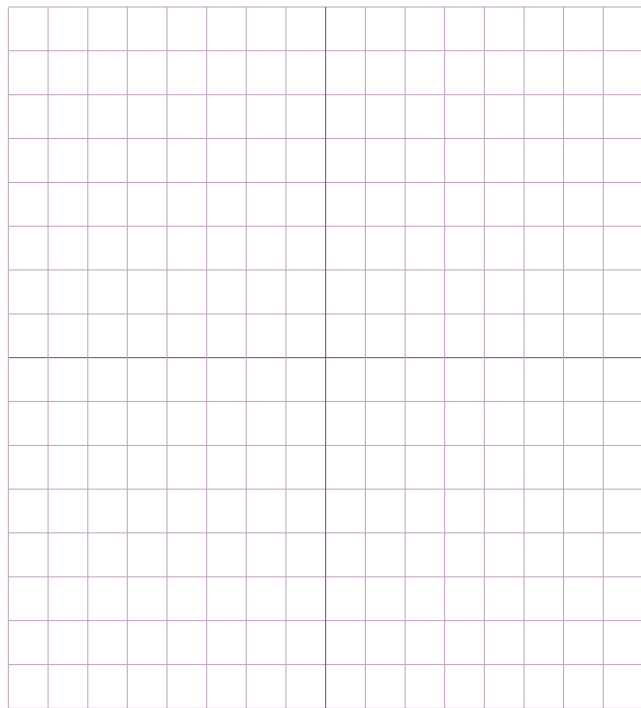
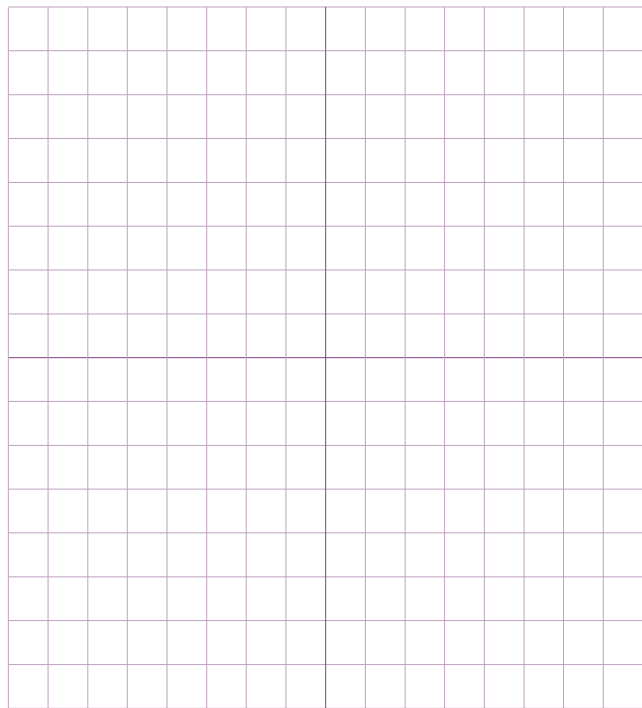
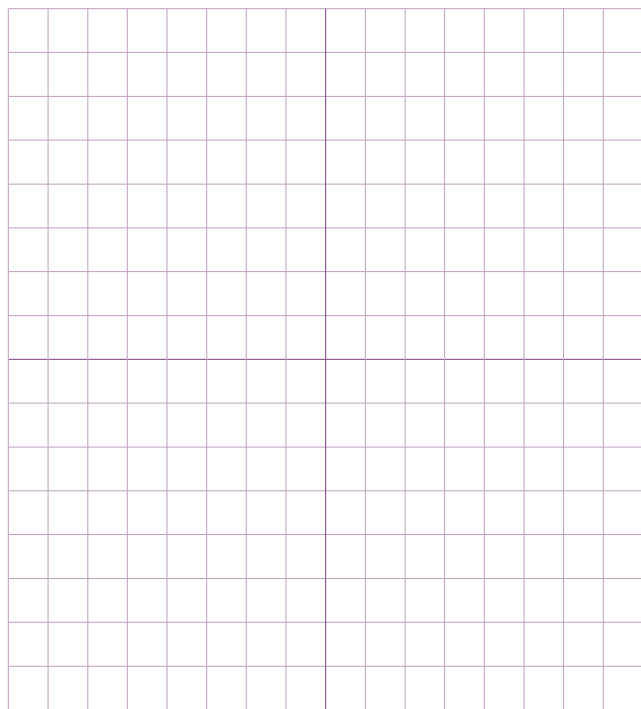
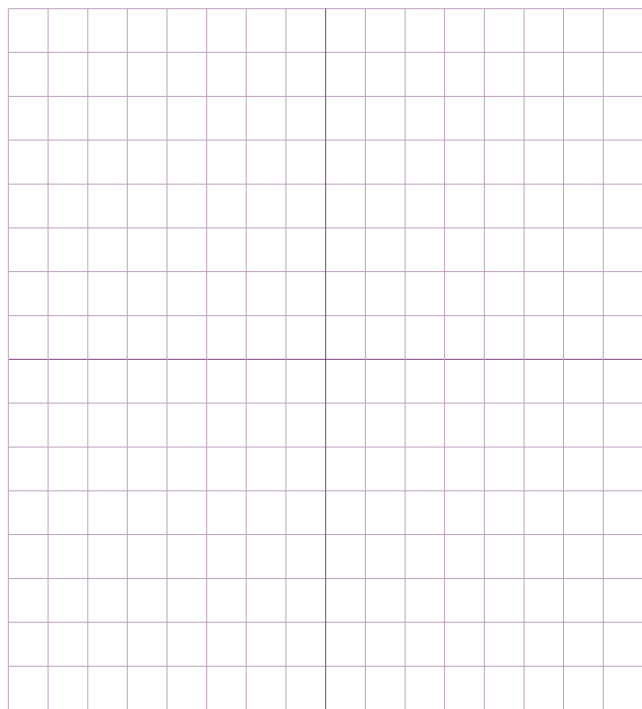
$$S = \frac{a}{1-r}$$

**You may detach this page for convenient reference.
Exercise care when tearing along perforations.**

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ROUGH WORK FOR GRAPHING

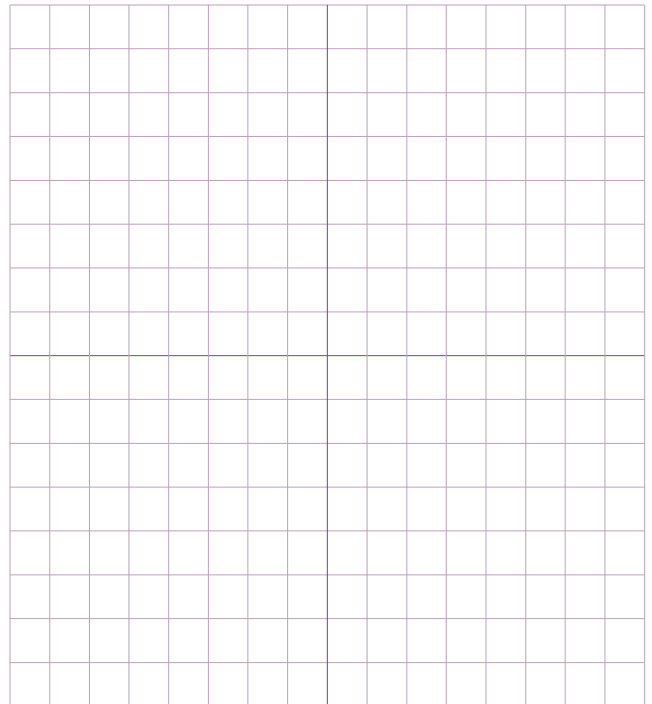
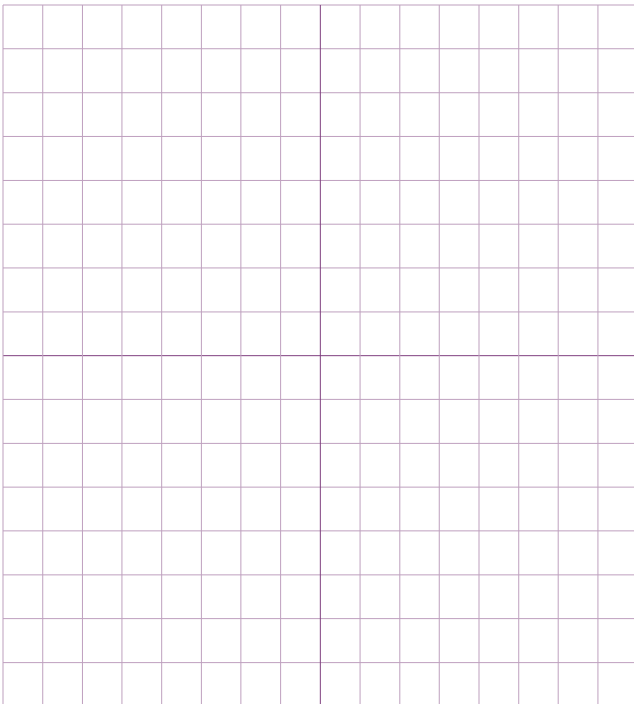
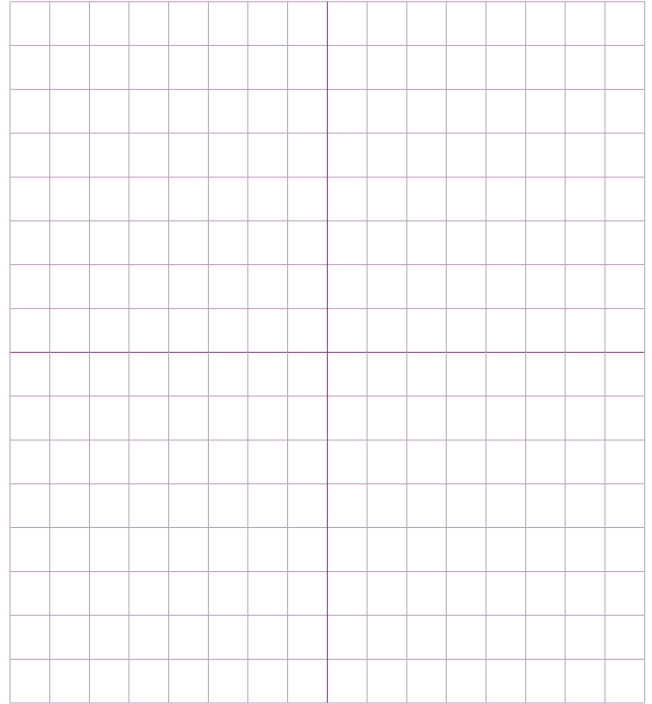
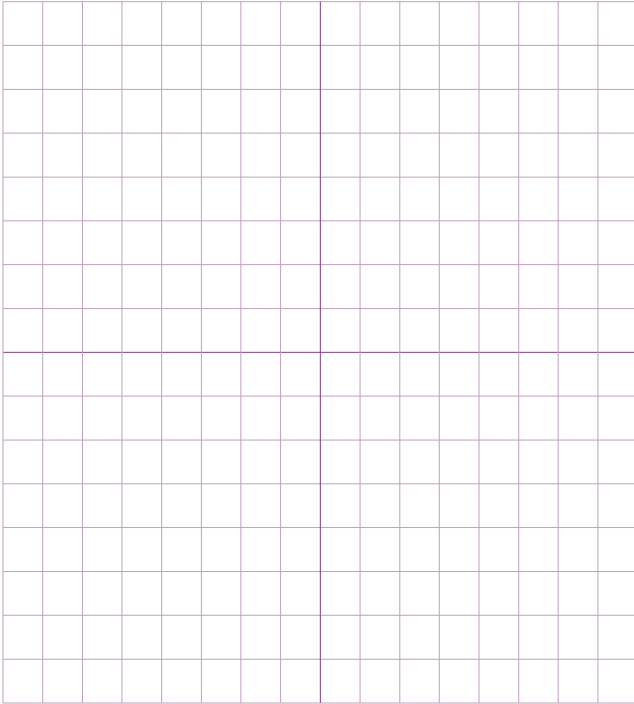
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ROUGH WORK FOR GRAPHING

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ROUGH WORK FOR MULTIPLE-CHOICE

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ROUGH WORK FOR MULTIPLE-CHOICE