

**JANUARY 1994**

## **PROVINCIAL EXAMINATION**

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**MINISTRY OF EDUCATION**

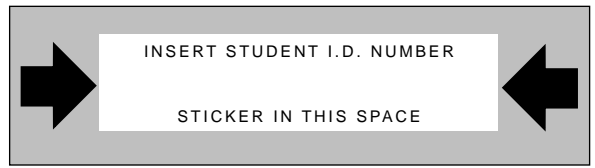
# **MATHEMATICS 12**

### **GENERAL INSTRUCTIONS**

1. Insert the stickers with your Student I.D. Number in the allotted spaces above. **Under no circumstance is your name or identification, other than your Student I.D. Number, to appear on this paper.**
2. Take the separate Answer Sheet and follow the directions on its front page.
3. Be sure you have an HB pencil and an eraser for completing your Answer Sheet. Follow the directions on the Answer Sheet when answering multiple-choice questions.
4. For each of the written-response questions, write your answer in the space provided. When instructed to open this booklet, **check the numbering of the pages** to ensure that they are numbered in sequence from page one to the last page, which is identified by **ENDOFEXAMINATION**.
5. At the end of the examination, place your Answer Sheet inside the front cover of this booklet and return the booklet and your Answer Sheet to the supervisor.

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**FOR OFFICE USE ONLY**



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**MATHEMATICS 12 JANUARY 1994 PROVINCIAL  
(MAP)**

1.  $\frac{\quad}{(2)}$

5.  $\frac{\quad}{(3)}$

2.  $\frac{\quad}{(3)}$

6.  $\frac{\quad}{(3)}$

3.  $\frac{\quad}{(3)}$

7.  $\frac{\quad}{(2)}$

4.  $\frac{\quad}{(4)}$

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## MATHEMATICS 12 PROVINCIAL EXAMINATION

	Value	Suggested Time
1. This examination consists of two parts:		
PART A: 50 multiple-choice questions	50	75
PART B: 7 written-response questions	20	45
2 questions worth <b>two</b> marks each, 4 questions worth <b>three</b> marks each, and 1 question worth <b>four</b> marks.		
	<b>TOTAL</b>	<b>70 marks</b> <b>120 minutes</b>

- The last **three** pages inside the back cover contain “**A Summary of Basic Identities and Formulae**”, “**Rough Work for Graphing**”, and “**Rough Work for Multiple-Choice**”. These pages may be detached for convenient reference prior to writing this examination.
- Students will not to be provided with any additional paper since rough-work space for the written-response questions has been incorporated into the space allowed for answering each question. Students may not need all of the space provided to answer each question.
- An approved scientific calculator is essential for the examination. The calculator **must not** be programmable to process alpha-numeric strings, nor should it be capable of processing user-defined functions. It **must not** have the capacity to accept coefficients from either an equation or a system of equations, thereby producing the roots of that equation or system. The calculator **must not** contain a plotter or printer.
- Students are permitted to use rulers, compasses, and protractors.
- Students have **two hours** to complete this examination.

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## PART A: MULTIPLE-CHOICE QUESTIONS

Value: 50 (one mark per question)

Suggested Time: 75 minutes

**INSTRUCTIONS:** For each question, select the **best** answer and record your choice on the answer sheet provided. Using an HB pencil, completely fill in the circle that has the letter corresponding to your answer.

- Which conic is described by the equation  $4x^2 + 4y^2 - x + y = 0$  ?
  - circle
  - ellipse
  - parabola
  - hyperbola
- What is the length of the minor axis of the ellipse  $\frac{x^2}{9} + \frac{y^2}{16} = 1$  ?
  - 3
  - 4
  - 6
  - 8
- A circle, with centre  $(-2, 3)$ , goes through the point  $(5, -6)$ . Determine the length of the radius to 1 decimal place.
  - 7.6
  - 9.5
  - 11.4
  - 16.0
- What is the equation of the axis of symmetry of the parabola  $y - 2 = \frac{1}{2}(x + 3)^2$  ?
  - $x = -3$
  - $x = 3$
  - $y = -2$
  - $y = 2$

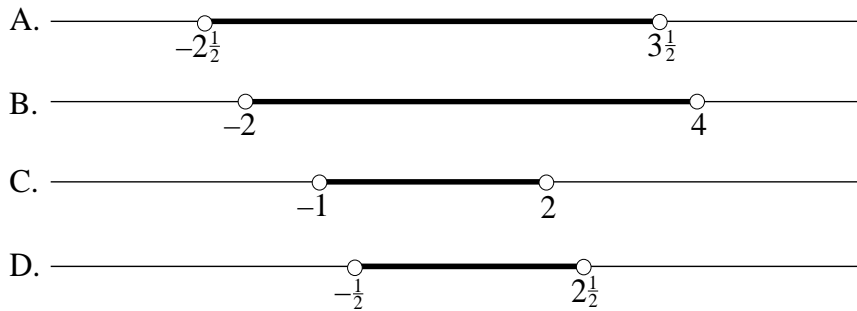
5. Determine the vertex of the parabola given by the equation  $4x - 8 = y^2 + 4y$ .

- A.  $(-1, 2)$
- B.  $(5, 2)$
- C.  $(1, -2)$
- D.  $(0, -4)$

6. A point P moves such that it is always equidistant from 2 fixed points. Identify the locus.

- A. line
- B. circle
- C. ellipse
- D. parabola

7. Select the graph of the solution of  $|2x - 1| < 3$ .



8. A rectangular hyperbola with centre  $(2, 1)$  has one vertex at  $(2, 7)$ . What is its equation?

- A.  $\frac{(x-2)^2}{36} - \frac{(y-1)^2}{36} = 1$
- B.  $\frac{(x-2)^2}{36} - \frac{(y-1)^2}{36} = -1$
- C.  $\frac{(x-2)^2}{49} - \frac{(y-1)^2}{49} = 1$
- D.  $\frac{(x-2)^2}{49} - \frac{(y-1)^2}{49} = -1$



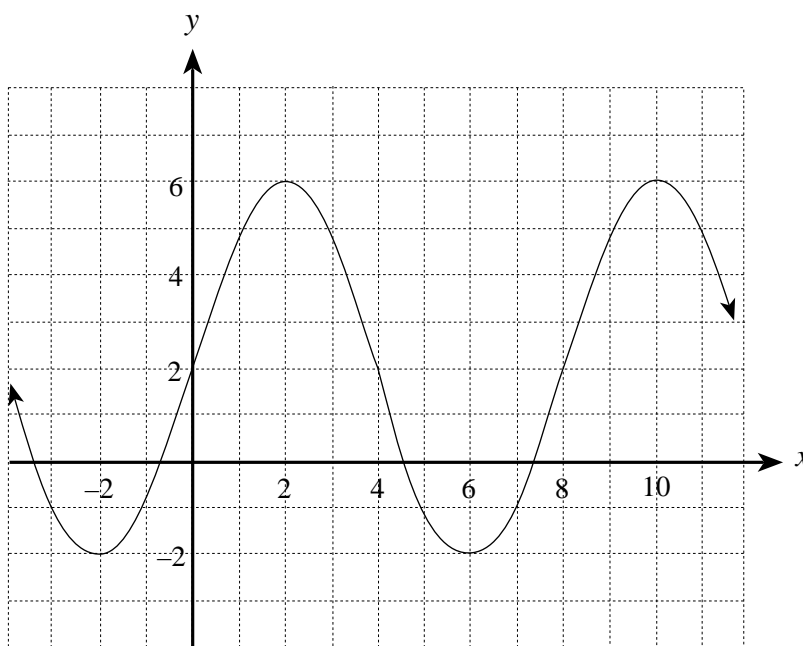
9. Determine the measure of the acute angle formed by the intersection of the asymptotes of the hyperbola  $\frac{x^2}{36} - \frac{y^2}{16} = 1$ . (accurate to 1 decimal place)

- A.  $47.9^\circ$
- B.  $56.3^\circ$
- C.  $66.7^\circ$
- D.  $67.4^\circ$

10. A line segment AB has a midpoint M. If A has coordinates (7, 4) and B lies on the line  $x + 3 = 0$  and M lies on the line  $y + 1 = 0$ , determine the y-coordinate of B.

- A. -6
- B. -2
- C. 2
- D. 6

11. Determine the amplitude of the function whose graph is shown.



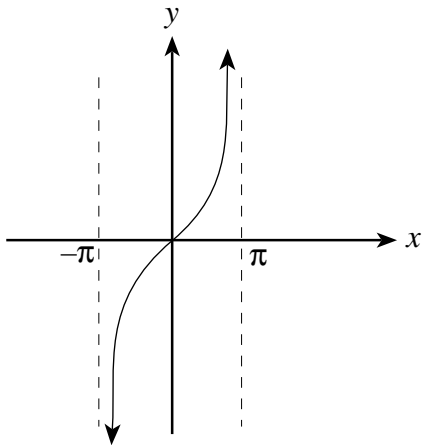
- A. 2
- B. 4
- C. 6
- D. 8

12. Evaluate to 2 decimal places:  $\sec 0.73$

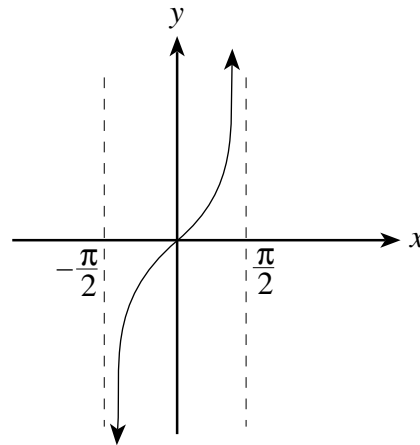
- A. 0.20
- B. 0.75
- C. 1.34
- D. 1.50

13. Which graph shows one period of  $y = \tan x$  ?

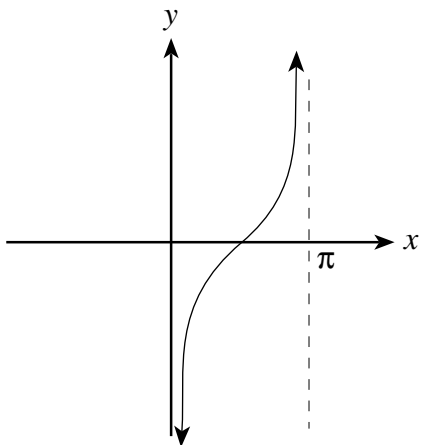
A.



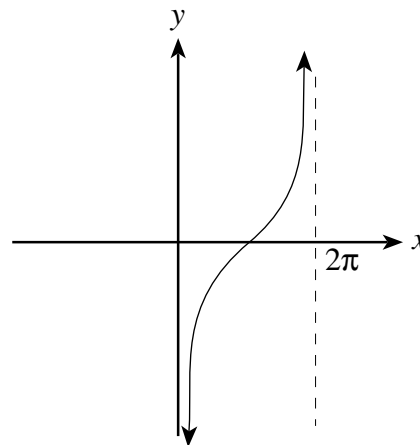
B.



C.



D.



14. Convert  $x$  radians to degrees.

- A.  $\frac{\pi^\circ}{180x}$
- B.  $\frac{\pi x^\circ}{180}$
- C.  $\frac{180^\circ}{\pi x}$
- D.  $\frac{180x^\circ}{\pi}$

15. Simplify:  $\cos\left(\frac{3\pi}{2} + \theta\right)$

- A.  $\sin \theta$
- B.  $-\sin \theta$
- C.  $\cos \theta$
- D.  $-\cos \theta$

16. Determine the phase shift of the function  $f(x) = \sin\left(2x - \frac{\pi}{6}\right) + \frac{\pi}{2}$ .

- A.  $\frac{\pi}{12}$
- B.  $\frac{\pi}{6}$
- C.  $\frac{\pi}{3}$
- D.  $\frac{\pi}{2}$

17. Which expression is equivalent to  $(\sin^2 \theta - \cos^2 \theta)^2 - \sin^2 2\theta$  ?

- A.  $-2\sin^2 2\theta$
- B.  $2\sin^2 2\theta$
- C.  $-\cos 4\theta$
- D.  $\cos 4\theta$

18. Determine the exponential form of  $a = \log_b c$ .

- A.  $a = b^c$
- B.  $b = a^c$
- C.  $c = a^b$
- D.  $c = b^a$

19. Evaluate:  $\log_2 \sqrt{8}$

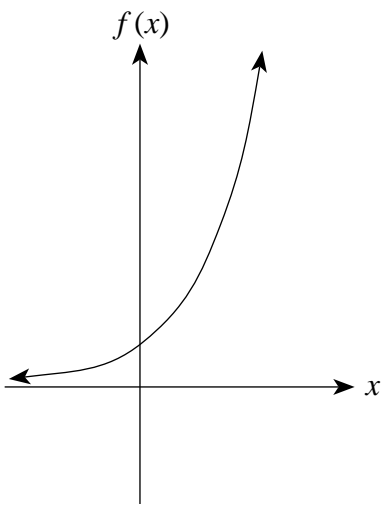
- A.  $\sqrt{2}$
- B.  $\frac{3}{2}$
- C. 3
- D. 8

20. Which expression is equivalent to  $\log_5 30$ ?

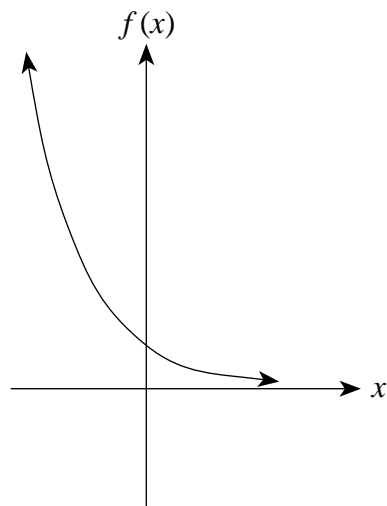
- A.  $\log 6$
- B.  $\log 30 - \log 5$
- C.  $\frac{\log 5}{\log 30}$
- D.  $\frac{\log_2 30}{\log_2 5}$

21. Which graph best represents the function  $f(x) = 2^{\pm x}$ ?

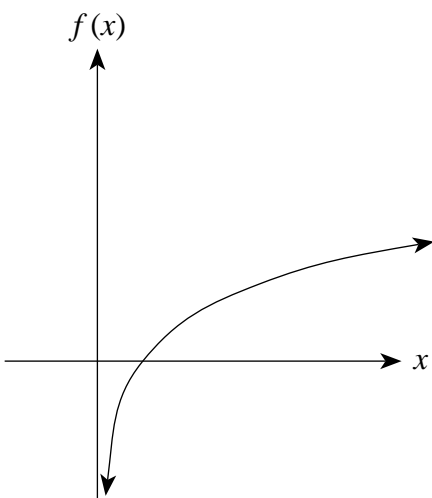
A.



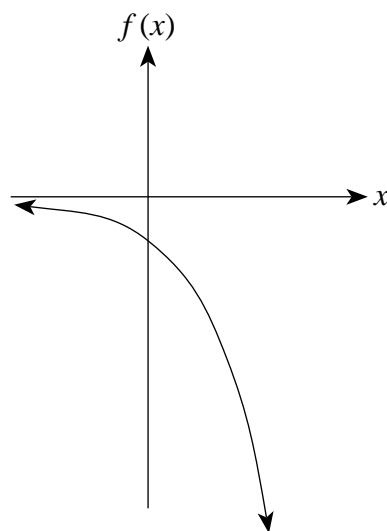
B.



C.



D.



22. Express  $\frac{1}{2} \log a - \log b - \log c$  as a single logarithm.

A.  $\log\left(\frac{\sqrt{a}}{bc}\right)$

B.  $\log\left(\frac{c\sqrt{a}}{b}\right)$

C.  $\log\left(\frac{1}{2} a - b - c\right)$

D.  $\log\left(\frac{a}{2bc}\right)$

23. The inverse relation of  $y = \log 2x$  is given by which one of the following?

A.  $y = \frac{10^x}{2}$

B.  $y = 5^x$

C.  $y = 10^{2x}$

D.  $y = \frac{1}{\log 2x}$

24. Solve for  $x$ :  $\log_2[\log_x(\log_3 9)] = -1$

A. 2

B. 3

C. 4

D. 5

25. According to the Rational Root Theorem, what are the possible rational roots of  $2x^4 + 3x^2 - 7x + 3 = 0$ ?

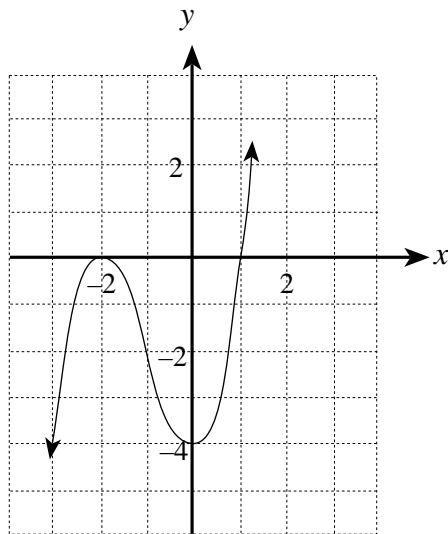
A.  $\pm 1, \pm 3$

B.  $\pm 1, \pm 2, \pm \frac{1}{3}, \pm \frac{2}{3}$

C.  $\pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}$

D.  $\pm 1, \pm 2, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}$

26. Which equation could represent the following graph?



- A.  $y = (x + 2)^2 (x - 1)$
- B.  $y = (x + 2)^2 (x + 1)$
- C.  $y = (x - 2)^2 (x - 1)$
- D.  $y = (x - 2)^2 (x + 1)$

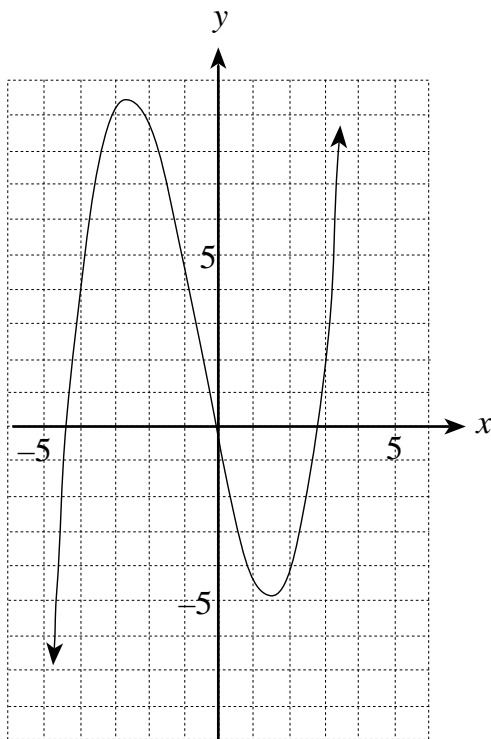
27. Determine the quotient when  $x^3 - 2x^2 - 9$  is divided by  $x - 3$ .

- A.  $x^2 + 5x + 15$
- B.  $x^2 + x - 6$
- C.  $x^2 - 5x + 6$
- D.  $x^2 + x + 3$

28. What value of  $k$  would make  $x + 2$  a factor of  $2x^3 - 5x^2 - 2kx + 8$  ?

- A.  $-7$
- B.  $-1$
- C.  $1$
- D.  $7$

29. From the graph of  $y = f(x)$  below, find the approximate solutions to  $f(x) = 2$ .



- A. -4.2
- B. 8.6
- C. -4.5, 0.3, 2.5
- D. -4.2, -0.5, 3.0

30. Determine the graph of the solution set of the inequality  $x(x-1)^n(x+2)^m \leq 0$ , if  $n$  is an even positive integer and  $m$  is an odd positive integer.

- A.
- B.
- C.
- D.

31. Given that  $t_n = (-3)(5)^n$  for a geometric sequence, determine the common ratio  $r$ .

- A. -5
- B. -3
- C. 3
- D. 5

**OVER**

32. Find the 91st term of the arithmetic sequence 2, 5, 8, ... .
- A. 266
  - B. 269
  - C. 272
  - D. 275
33. Determine a single geometric mean between 9 and 25.
- A.  $\sqrt{34}$
  - B. 15
  - C. 16
  - D. 17
34. Determine the number of terms in the geometric sequence  $2^{-3}$ ,  $2^{-2}$ ,  $2^{-1}$ , ... ,  $2^{11}$ .
- A. 13
  - B. 14
  - C. 15
  - D. 16
35. Evaluate:  $\sum_{k=1}^{100} 3k$
- A. 5 050
  - B. 5 150
  - C. 15 150
  - D. 30 300
36. If the sum of an infinite geometric series is 9 and the first term is 6, determine the common ratio.
- A.  $-\frac{1}{3}$
  - B.  $\frac{1}{3}$
  - C.  $\frac{2}{3}$
  - D.  $\frac{3}{2}$



37. Determine the sum of  $n$  terms of the sequence 3, 7, 11, ... ,  $4n - 1$ .

- A.  $2n^2 + n$
- B.  $2n^2 + 5n$
- C.  $4n^2 + n$
- D.  $4n - 1$

38. Solve for  $x$ :  $\sum_{j=3}^5 (j - 6)x = 72$

- A.  $-6$
- B.  $-12$
- C.  $-14.4$
- D.  $-72$

39. What is the derivative of the function  $f(x) = 5x^2 - 3x - 1$  ?

- A.  $7x - 3$
- B.  $10x - 4$
- C.  $10x - 3$
- D.  $25x - 3$

40. Evaluate:  $\lim_{x \rightarrow 5} \frac{x^2 - 2x - 15}{x^2 - 7x + 10}$

- A. 0
- B. 1
- C.  $\frac{8}{3}$
- D. limit does not exist (no finite limit)

41. If  $(a, b)$  is a critical point of a polynomial function  $y = f(x)$ , then which one of the following **must** be true?

- A.  $f(a) > 0$
- B.  $f(a) = 0$
- C.  $f'(a) > 0$
- D.  $f'(a) = 0$

42. Which of the following represents the limit of the sequence  $1, \frac{4}{3}, \frac{3}{2}, \frac{8}{5}, \dots$ ?

A.  $\lim_{n \rightarrow 0} \frac{2n}{n+1}$

B.  $\lim_{n \rightarrow \infty} \frac{2n}{n+1}$

C.  $\lim_{n \rightarrow 0} \frac{n+2}{3n}$

D.  $\lim_{n \rightarrow \infty} \frac{n+2}{3n}$

43. Given  $y = \frac{1}{x^3}$ , determine  $\frac{dy}{dx}$ .

A.  $\frac{-3}{x^2}$

B.  $\frac{-3}{x^4}$

C.  $\frac{1}{3x^2}$

D.  $\frac{1}{3x^4}$

44. Determine the slope of the line tangent to the graph of  $y = x^3 - x^2$  at the point where  $x = 2$ .

A. 2

B. 4

C. 8

D. 10

45. The position of an object moving in a straight path is given by  $x(t) = kt^2 + 12t$ , where  $x$  is in metres and  $t$  is in seconds. Find the value of  $k$  if the velocity of the object is 4 m/s when  $t = 2$  seconds.

A. -12

B. -6

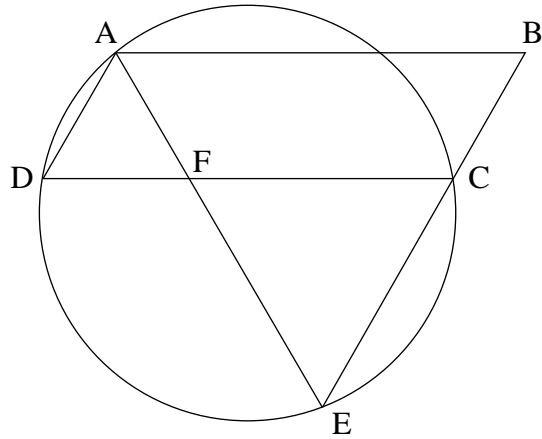
C. -3

D. -2

Use the following diagram and proof to answer questions 46 and 47.

Given: ABCD is a parallelogram

Prove:  $\triangle ABE$  is isosceles



Proof	
Statement	Reason
ABCD is a parallelogram	given
(a) $\angle D = \angle B$	opp $\angle$ s of parallelogram are =
(b) $AD = BC$	opp sides of parallelogram are =
(c) $\angle D = \angle E$	insc $\angle$ s on same arc AC are =
(d) $\angle B = \angle E$	both = $\angle D$
$\triangle ABE$ is isosceles	2 $\angle$ s are =

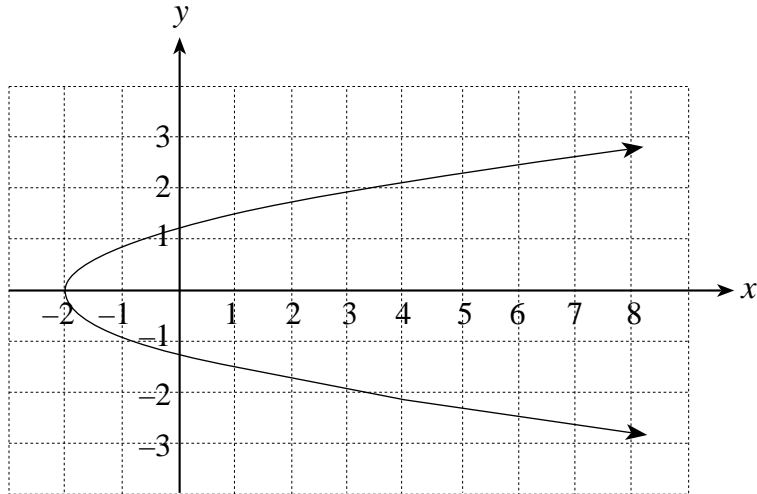
46. Which line is **not** necessary in the given proof?

- A. a
- B. b
- C. c
- D. d

47. If  $\angle DCB = 125^\circ$ , determine the measure of  $\angle AFC$ . (Diagram is not drawn to scale.)

- A.  $110^\circ$
- B.  $115^\circ$
- C.  $120^\circ$
- D.  $125^\circ$

48. Determine the number of lines that pass through the point  $(-1, 2)$  and are tangent to the parabola graphed.



- A. 0  
 B. 1  
 C. 2  
 D. 3
49. What restrictions for  $x$  exist for the equation  $\log_x x + \log_x(2 - x) = 5$  ?
- A.  $x > 0, x \neq 1$   
 B.  $x < 2$   
 C.  $0 < x < 2$   
 D.  $0 < x < 2, x \neq 1$
50. Describe  $y = |x - 3|$  as a piecewise function.

- A.  $y = \begin{cases} x - 3 & x \geq 0 \\ -x - 3 & x < 0 \end{cases}$   
 B.  $y = \begin{cases} x - 3 & x \geq 3 \\ -x + 3 & x < 3 \end{cases}$   
 C.  $y = \begin{cases} x + 3 & x \geq 3 \\ -x - 3 & x < 3 \end{cases}$   
 D.  $y = \begin{cases} -x + 3 & x \geq 3 \\ x - 3 & x < 3 \end{cases}$

**This is the end of the multiple-choice section.  
 Answer the remaining questions directly in this examination booklet.**

## **PART B: WRITTEN-RESPONSE QUESTIONS**

**Value: 20 marks total**

**Suggested Time: 45 minutes**

**INSTRUCTIONS:** Rough-work space has been incorporated into the space allowed for answering each question. Students may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

**Full marks will NOT be given for the final answer only.**

1. Solve:  $2x^3 - x^2 - 8x + 4 = 0$

**(2 marks)**

ANSWER:

Score for  
Question 1:

1.           
(2)

**OVER**

2. Solve for  $x$ :  $2 \tan^2 x - 5 \tan x - 3 = 0$ , where  $0 \leq x < 2\pi$  (accurate to 2 decimal places)  
**(3 marks)**



ANSWER:

Score for  
Question 2:

2.  $\frac{\quad}{(3)}$

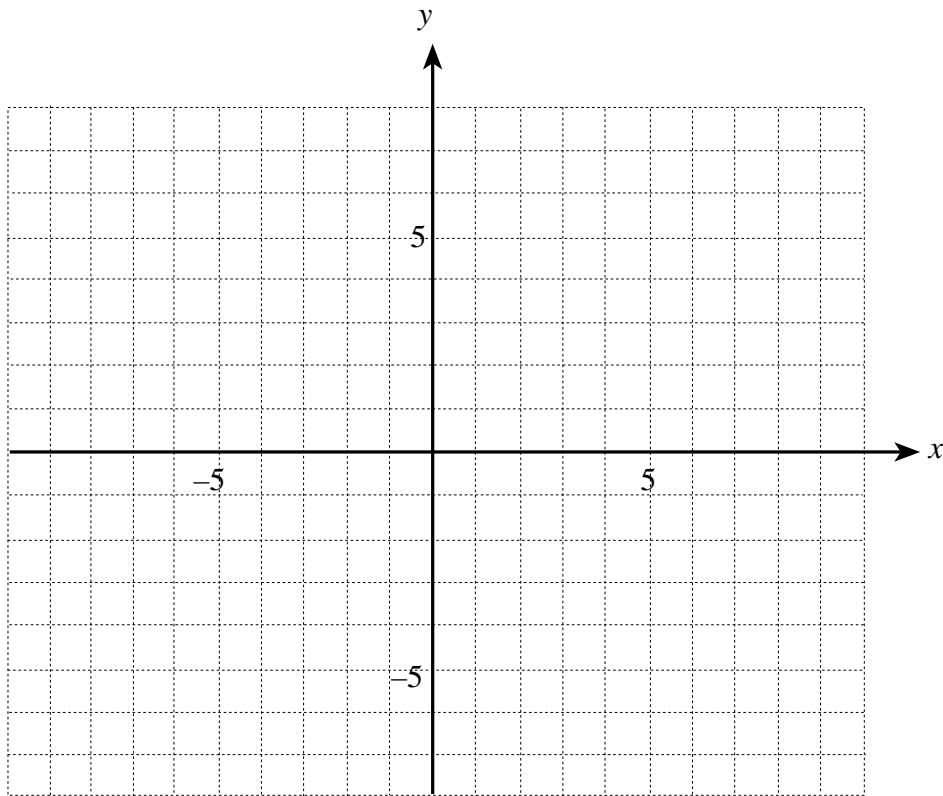
**OVER**

3. Graph the following system of inequalities:

**(3 marks)**

$$(x - 3)^2 + y^2 > 36$$

$$x^2 - y^2 \leq 9$$



ANSWER:

Score for  
Question 3:

3.           
(3)

**OVER**



Score for  
Question 4:

4.           
(4)

**OVER**

5. A city has a population of 15 000 and the population decreases by 8% per year. How many years will it take for the population to become 5 000? (accurate to 1 decimal place) **(3 marks)**

ANSWER:

Score for  
Question 5:

5.           
(3)

**OVER**

6. Determine all values of  $x$  for which  $\sqrt{4-x^2}$  is an integer.

**(3 marks)**



ANSWER:

Score for  
Question 6:

6.           
(3)

**OVER**

7. Given  $f(x) = x^2 + 5x$ , use the **definition of the derivative** to show that  $f'(x) = 2x + 5$ .  
(2 marks)

Score for  
Question 7:

7.  $\frac{\quad}{(2)}$

**END OF EXAMINATION**

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## A SUMMARY OF BASIC IDENTITIES AND FORMULAE

### Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

### Reciprocal and Quotient Identities

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

### Addition Identities

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

### Double-Angle Identities

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - 2 \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

### Formulae

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t_n = a + (n-1)d$$

$$t_n = ar^{n-1}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$S_n = \frac{n}{2}(a + \ell)$$

$$S_n = \frac{a - \ell r}{1-r}$$

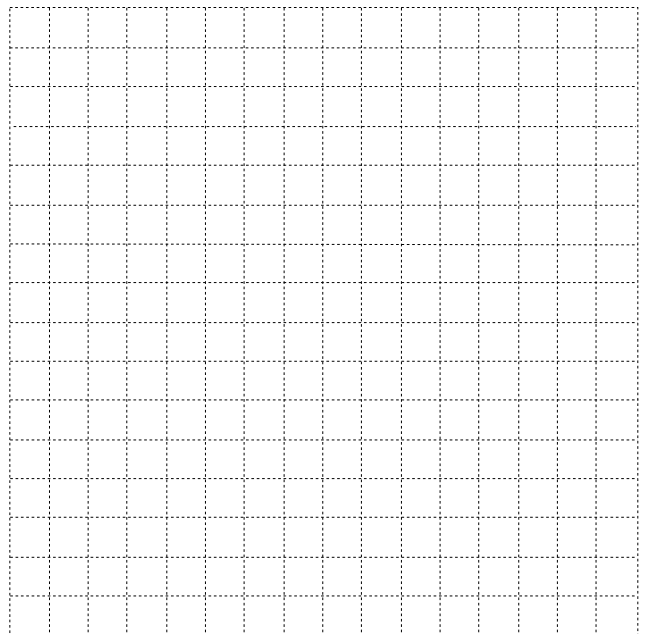
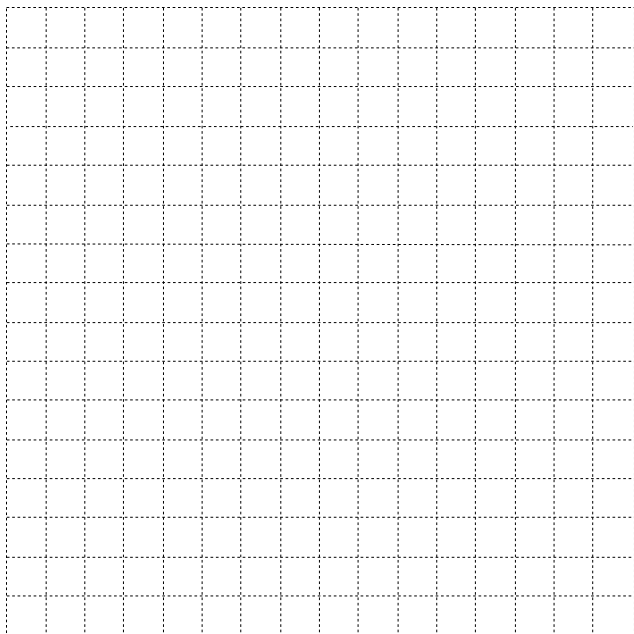
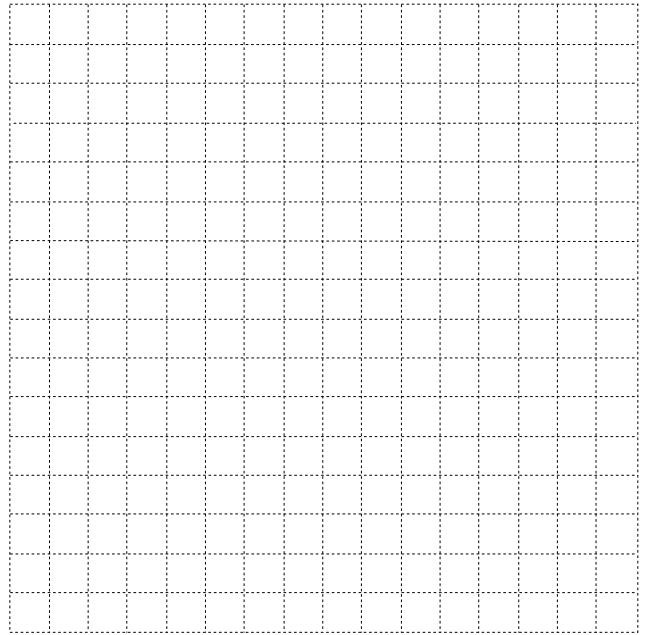
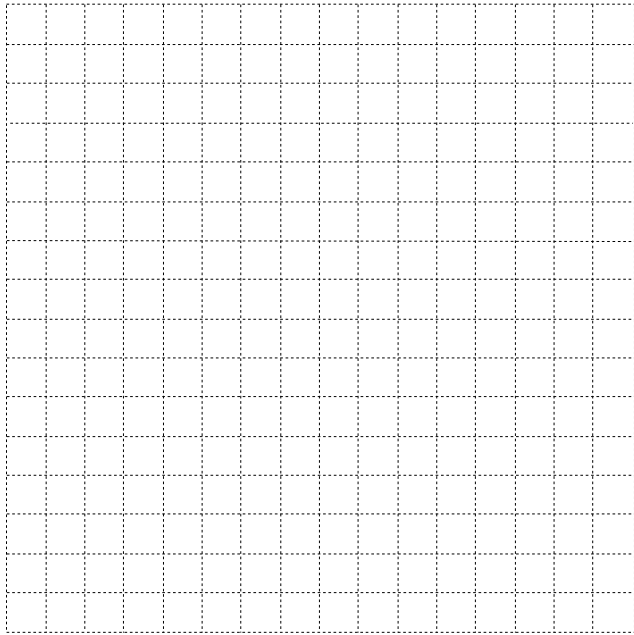
$$S = \frac{a}{1-r}$$

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Exercise care when tearing along perforations.**

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**ROUGH WORK FOR GRAPHING**

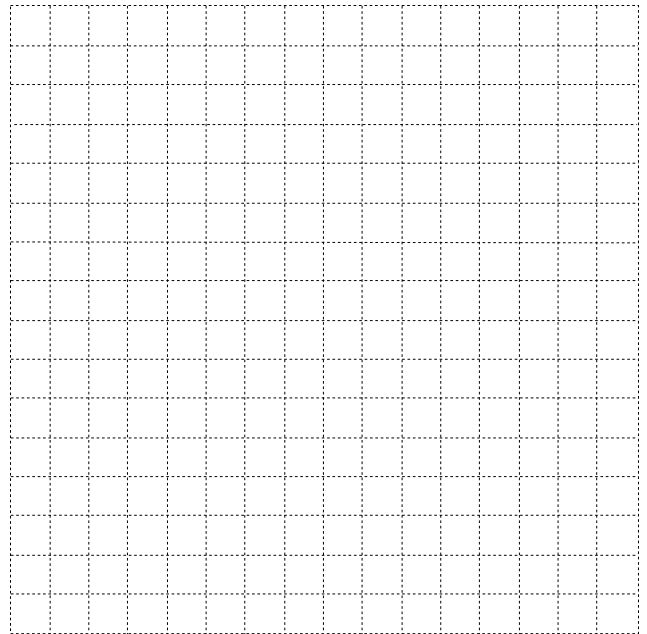
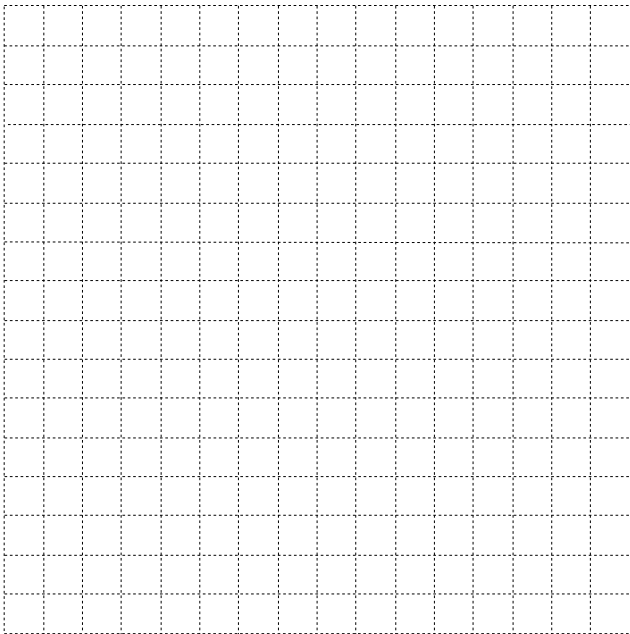
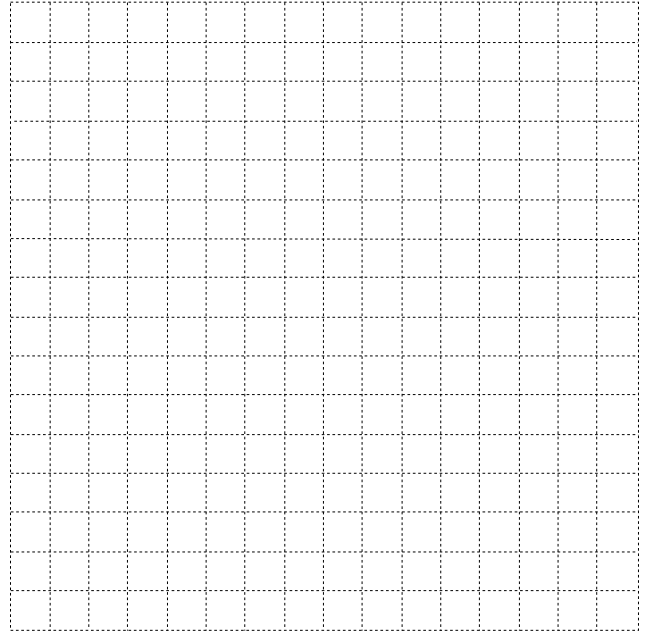
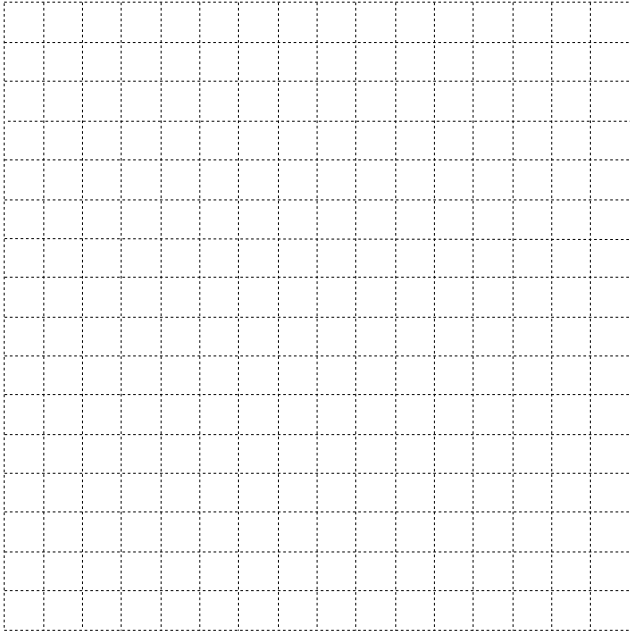
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**ROUGH WORK FOR GRAPHING**

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**ROUGH WORK FOR MULTIPLE-CHOICE**

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## ROUGH WORK FOR MULTIPLE-CHOICE