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BRITISH COLUMBIA

Principles of Mathematics 12

JUNE 2001

Course Code = MA

Student Instructions

1. Place the stickers with your Personal Education Number (PEN) in the allotted spaces above. **Under no circumstance is your name or identification, other than your Personal Education Number, to appear on this booklet.**
2. Ensure that in addition to this examination booklet, you have an **Examination Response Form**. Follow the directions on the front of the Response Form.
3. **Disqualification** from the examination will result if you bring books, paper, notes or unauthorized electronic devices into the examination room.
4. When instructed to open this booklet, **check the numbering of the pages** to ensure that they are numbered in sequence from page one to the last page, which is identified by

   END OF EXAMINATION.

5. At the end of the examination, place your Response Form inside the front cover of this booklet and return the booklet and your Response Form to the supervisor.

Question 1:

1. .

(4)

Question 2:

2. .

(4)

Question 3:

3. .

(4)

Question 4a:

4. .

(3)

Question 4b:

5. .

(2)

Question 5:

6. .

(4)

Question 6:

7. .

(4)

Question 7:

8. .

(4)

Question 8:

9. .

(5)

PRINCIPLES OF MATHEMATICS 12

JUNE 2001

COURSE CODE = MA

GENERAL INSTRUCTIONS

1. Aside from an approved calculator, electronic devices, including dictionaries and pagers, are **not** permitted in the examination room.

2. All multiple-choice answers must be entered on the Response Form using an **HB pencil**. Multiple-choice answers entered in this examination booklet will **not** be marked.

3. For each of the written-response questions, write your answer in the space provided in this booklet.

   You will not be provided with any additional paper since rough-work space for the written-response questions has been incorporated into the space allowed for answering each question. You may not need all of the space provided to answer each question.

4. Ensure that you use language and content appropriate to the purpose and audience of this examination. Failure to comply may result in your paper being awarded a zero.

5. This examination is designed to be completed in **two hours**. *Students may, however, take up to 30 minutes of additional time to finish*.

PRINCIPLES OF MATHEMATICS 12 PROVINCIAL EXAMINATION

		Value	Suggested Time
1.	This examination consists of **two** parts:		
	PART A: 44 multiple-choice questions	66	75
	PART B: 8 written-response questions	34	45
	Total:	**100 marks**	**120 minutes**

2. The last **three** pages inside the back cover contain **A Summary of Basic Identities and Formulae**, **Rough Work for Graphing**, and **Rough Work for Multiple-Choice**. These pages may be detached for convenient reference prior to writing this examination.

3. **A graphing calculator is essential for the Principles of Mathematics 12 Provincial Examination.** The calculator must be a hand-held device designed primarily for mathematical computations involving logarithmic and trigonometric functions as well as for graphing functions. Computers, calculators with a QWERTY keyboard or symbolic manipulation abilities, and electronic writing pads will not be allowed. Students must not bring any external devices (peripherals) to support calculators such as manuals, printed or electronic cards, printers, memory expansion chips or cards, CD-ROMs, libraries or external keyboards. Students may have more than one calculator available during the examination, of which one may be a scientific calculator. Calculators may not be shared and must not have the ability to either transmit or receive electronic signals. In addition to an approved calculator, students will be allowed to use rulers, compasses, and protractors during the examination.

   Calculators must not have any information programmed into the memory which would not be acceptable in paper form. Specifically, calculators must not have any built-in notes, definitions, or libraries. There is no requirement to clear memories at the beginning of the examination but the use of calculators with built-in notes is equivalent to the use of notes in paper form. Any student deemed to have cheated on a provincial examination will receive a "0" on that examination and will be permanently disqualified from the Provincial Examination Scholarship Program.

4. If, in a justification, you refer to information produced by the calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem, it is important to sketch the graph, showing its general shape and indicating the appropriate window dimensions.

5. When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.

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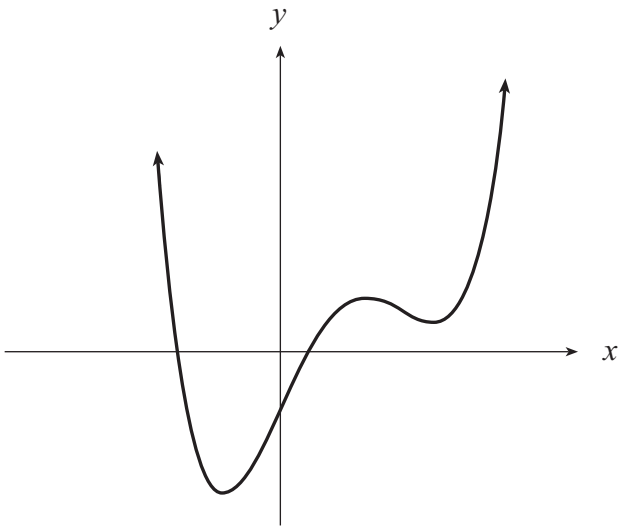
PART A: MULTIPLE CHOICE

Value: 66 marks **Suggested Time: 75 minutes**

INSTRUCTIONS: For each question, select the **best** answer and record your choice on the Response Form provided. Using an HB pencil, completely fill in the circle that has the letter corresponding to your answer.

1. Determine the number of real zeros of the function graphed below.



A. 1
B. 2
C. 3
D. 4

2. Find the quotient when $2x^3 - 3x^2 + 2x - 8$ is divided by $x + 1$.

A. $x^2 - 2x$
B. $x^2 - 4x + 6$
C. $2x^2 - x + 1$
D. $2x^2 - 5x + 7$

OVER

3. The polynomial equation $mx^3 + 7x^2 - 3x + n = 0$, where m and n are integers, has a root of $\frac{4}{9}$. According to the Rational Root Theorem, which of the following could be a value for m ?

A. 2
B. 4
C. 6
D. 18

4. Solve: $x^3 < x$

A. $x < 0,\ x > 1$
B. $-1 < x < 1$
C. $-1 < x < 0,\ x > 1$
D. $x < -1,\ 0 < x < 1$

5. Solve: $x^3 + 2x^2 - 104x + 192 = 30$

A. $1.65,\ 8.24$
B. $2.37,\ 7.73$
C. $-12.11,\ 2.37,\ 7.73$
D. $-11.89,\ 1.65,\ 8.24$

6. The points $(-2,\ 0)$, $(0,\ 5)$ and $(2,\ -4)$ are on the graph of a third degree polynomial function, $y = p(x)$. If $p(x)$ is divided by $x - 2$, determine the remainder.

A. -4
B. 0
C. 4
D. 5

7. Which expression represents the distance between points (a, b) and (c, d) ?

A. $\sqrt{(a-b)^2+(c-d)^2}$

B. $\sqrt{(a-c)^2+(b-d)^2}$

C. $\sqrt{(a-d)^2+(b-c)^2}$

D. $\sqrt{(a+b)^2+(c+d)^2}$

8. Which absolute value inequality describes the solution shown?

A. $|x-3|<7$

B. $|x+3|<7$

C. $|x-7|<3$

D. $|x+7|<3$

9. Determine the length of the transverse axis of the hyperbola $\frac{x^2}{9}-\frac{y^2}{16}=1$.

A. 3

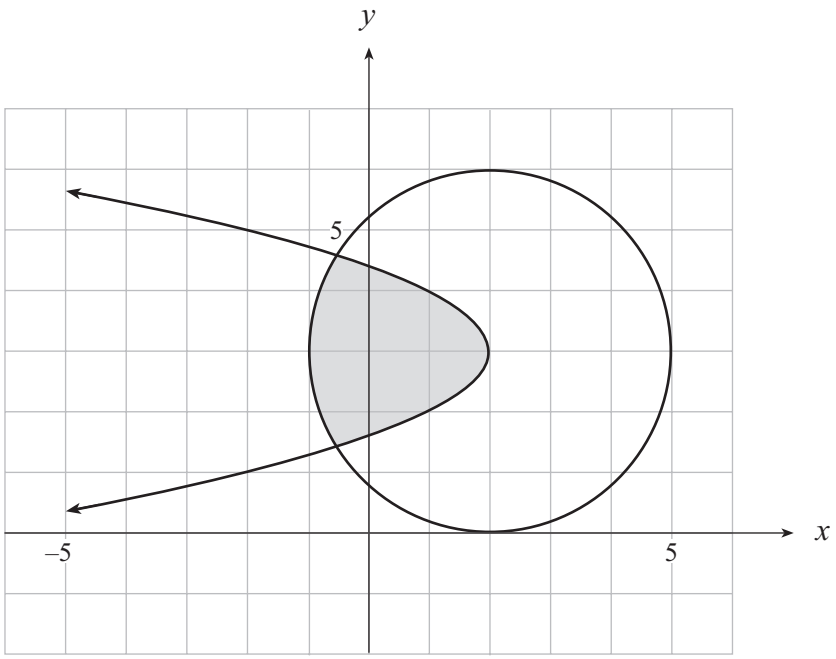
B. 4

C. 6

D. 8

OVER

10. Which system has the shaded portion shown below as its solution?



A. $(x-2)^2+(y-3)^2 \le 9$
$x \le -(y-3)^2+2$

B. $(x-2)^2+(y-3)^2 \le 9$
$x \ge -(y-3)^2+2$

C. $(x-2)^2+(y-3)^2 \le 9$
$x \le -(y-2)^2+3$

D. $(x-2)^2+(y-3)^2 \le 9$
$x \ge -(y-2)^2+3$

11. Change the following equation to standard form.

$$5x^2 + y^2 - 20x - 10 = 0$$

A. $\dfrac{(x-2)^2}{2} + \dfrac{y^2}{10} = 0$

B. $\dfrac{(x-2)^2}{6} + \dfrac{y^2}{30} = 1$

C. $\dfrac{(x-2)^2}{\frac{14}{5}} + \dfrac{y^2}{14} = 1$

D. $\dfrac{(x-2)^2}{4} + \dfrac{y^2}{20} = 1$

12. A point $\mathrm{P}(x,\ y)$ moves such that the slope of the line through P and $\mathrm{A}(-2,\ 0)$ is always twice the slope of the line through P and $\mathrm{B}(2,\ 0)$. Determine an equation of this locus.

A. $\dfrac{y-0}{x+2} = 2\left(\dfrac{y-0}{x-2}\right)$

B. $\dfrac{y-0}{x-2} = 2\left(\dfrac{y-0}{x+2}\right)$

C. $\dfrac{y-0}{x+2} = 2 + \dfrac{y-0}{x-2}$

D. $\dfrac{y-0}{x-2} = 2 + \dfrac{y-0}{x+2}$

13. A line segment AB has midpoint $\mathrm{M}(m,\ 0)$. If A has coordinates $(0,\ a)$, determine the coordinates of B.

A. $\left(\dfrac{m}{2},\ \dfrac{a}{2}\right)$

B. $(2m,\ -a)$

C. $(2m,\ a)$

D. $(2m,\ 2a)$

OVER

14. Determine all values for m such that the following system has no real solutions.

$$\frac{x^2}{16} - \frac{y^2}{4} = -1$$
$$y = mx$$

A. $-\frac{1}{2} \le m \le \frac{1}{2}$

B. $m \ge \frac{1}{2}$ or $m \le -\frac{1}{2}$

C. $-2 \le m \le 2$

D. $m \ge 2$ or $m \le -2$

15. Change $b^a = c$ to logarithmic form.

A. $\log_b a = c$

B. $\log_c b = a$

C. $\log_b c = a$

D. $\log_c a = b$

16. In which line would the graph of a function $f(x)$ be reflected to obtain the graph of $f^{-1}(x)$, the inverse of $f(x)$?

A. $y = 0$

B. $x = 0$

C. $y = -x$

D. $y = x$

17. Express $2\log k + \log 5 - \log p$ as a single logarithm.

A. $\log \frac{5k^2}{p}$

B. $\log \frac{10k}{p}$

C. $\log\left(k^2 + 5 - p\right)$

D. $\log(2k + 5 - p)$

18. Solve: $\log_2 x + \log_2(x-2) = 3$

A. 2.5

B. 3

C. 4

D. $1+\sqrt{7}$

19. The population of a type of bacteria triples every 20 hours. In how many hours will a population of 30 become a population of 1 000 ?

A. 63.84
B. 101.18
C. 106.83
D. 169.32

20. Determine the domain of the function $y = \log_{2x-3}(x)$.

A. $x > 0,\ x \neq 1$

B. $x > 0,\ x \neq 2$

C. $x > \frac{3}{2},\ x \neq 1$

D. $x > \frac{3}{2},\ x \neq 2$

21. Solve for x: $\log_a a^{2x} = \log_{b^2} b^{3x-3}$

A. -3

B. $-\frac{3}{2}$

C. $\frac{3}{2}$

D. 3

OVER

22. What is the single positive geometric mean between 1 and 4 ?

A. 1.5
B. 2
C. 2.5
D. 3

23. Which of the following sequences is arithmetic?

A. $\sqrt{2},\ 2,\ 2\sqrt{2},\ 4$
B. $\sqrt{2},\ 2\sqrt{2},\ 4\sqrt{2},\ 6\sqrt{2}$
C. $\sqrt{2},\ 2\sqrt{2},\ 3\sqrt{2},\ 4\sqrt{2}$
D. $\sqrt{2},\ \sqrt{3},\ \sqrt{4},\ \sqrt{5}$

24. Determine the 100^{th} term of the arithmetic sequence: $15,\ 12,\ 9,\ \ldots$

A. −282
B. −285
C. 312
D. 315

25. Determine the sum of the infinite geometric series: $24 - 12 + 6 - 3 + \ldots$

A. 8
B. 16
C. 48
D. no finite sum

26. Determine the sum of the first 15 terms of the geometric series: $3 + 9 + 27 + \ldots$

A. 7 174 452
B. 14 348 907
C. 21 523 359
D. 43 046 721

27. How many terms are there in the series: $\sum_{k=5}^{n}(2k+3)$?

A. n
B. $n-4$
C. $n-5$
D. $n-6$

28. Determine an expression for S_n if $t_n = 4n+2$.

A. $2n^2$
B. $2n^2+4$
C. n^2+5n
D. $2n^2+4n$

29. Convert 2.1 radians to degrees.

A. $60.16°$
B. $120.32°$
C. $126.35°$
D. $240.64°$

30. Which expression below is equivalent to $2\cot\frac{\pi}{5}$?

A. $2\tan\frac{5}{\pi}$

B. $\frac{1}{2\tan\frac{\pi}{5}}$

C. $\frac{2}{\tan\frac{5}{\pi}}$

D. $\frac{2}{\tan\frac{\pi}{5}}$

OVER

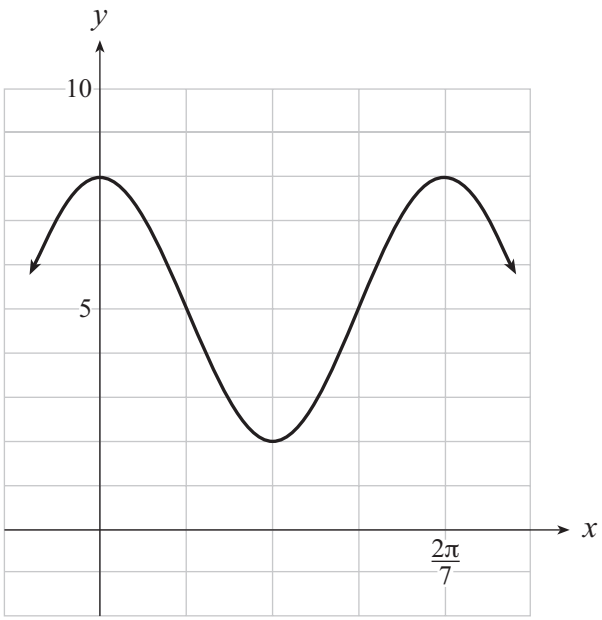
31. Simplify: $\dfrac{2\cos\theta}{\sin 2\theta}$

A. $\sin\theta$
B. $\cot\theta$
C. $\sec\theta$
D. $\csc\theta$

32. Solve: $\tan\theta = -1.25\,, \quad 0 \le \theta < 2\pi$

A. $0.90\,,\ 4.04$
B. $2.25\,,\ 5.39$
C. $2.25\,,\ 6.15$
D. $3.01\,,\ 6.15$

33. Determine an equation of the cosine function graphed below.



A. $y = 3\cos 7x + 2$
B. $y = 3\cos 7x + 5$
C. $y = 6\cos 7x + 5$
D. $y = 8\cos 7x + 2$

34. Solve: $\sec\theta + \cot\theta = 2\,, \quad 0 \le \theta < 2\pi$

A. 0.64
B. 0.93
C. 3.46, 5.13
D. 4.29, 5.97

35. A and B are complementary angles. If $\sin A = \frac{3}{5}$, find the value of $\sec B$.

A. $\frac{3}{5}$

B. $\frac{4}{5}$

C. $\frac{5}{4}$

D. $\frac{5}{3}$

36. Use a graphing calculator to determine the period of $f(x) = \sin\frac{x}{2} + \cos\frac{x}{2}$.

A. 2π

B. $\frac{\pi}{2}$

C. 4π

D. $\frac{9\pi}{2}$

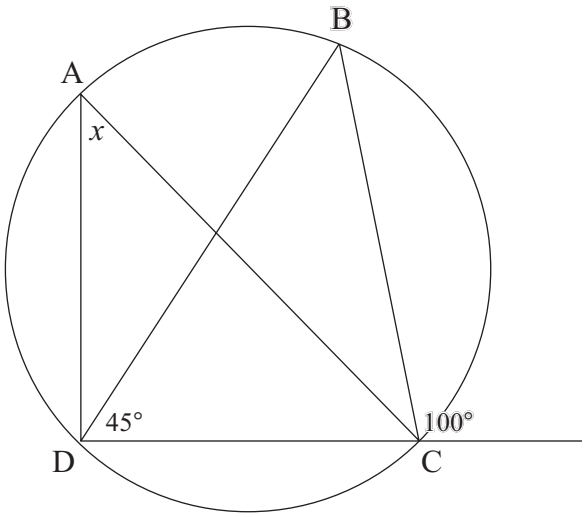
37. State all restrictions for $\frac{\csc\theta - 1}{\csc\theta + 1}$.

A. $\sin\theta \neq 0$
B. $\sin\theta \neq -1$
C. $\sin\theta \neq 0\,,\ \sin\theta \neq -1$
D. $\sin\theta \neq 0\,,\ \sin\theta \neq \pm 1$

OVER

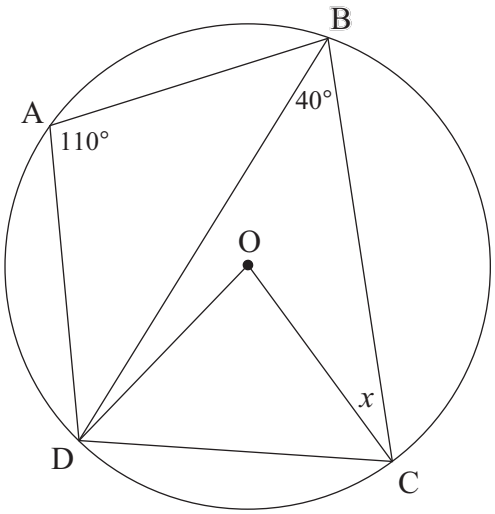
For questions 38 to 41, diagrams are not drawn to scale.

38. In the diagram below, determine the measure of $\angle x$.



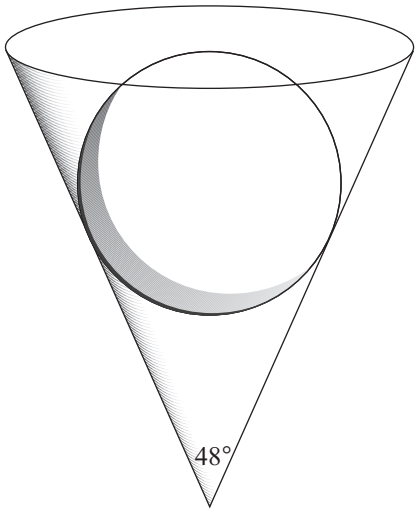
A. 45°
B. 50°
C. 55°
D. 60°

39. If O is the centre of the circle in the diagram below, determine the measure of $\angle x$.



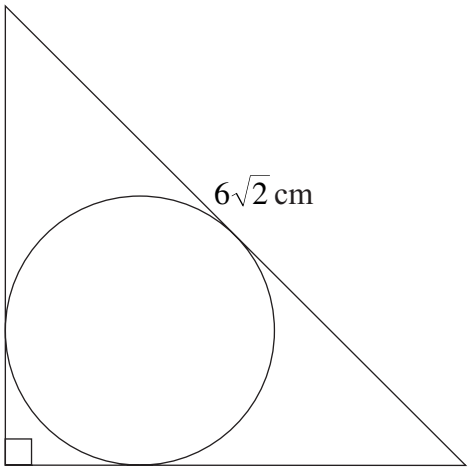
A. 10°
B. 20°
C. 30°
D. 40°

40. The vertical angle of a cone is $48°$, as shown in the diagram. A sphere of radius 8 cm is dropped into the cone. Determine the shortest distance from the sphere to the vertex of the cone.



A. 8.76 cm
B. 9.97 cm
C. 11.67 cm
D. 11.96 cm

41. An isosceles right triangle has a hypotenuse of $6\sqrt{2}$ cm, as shown in the diagram. Find the radius of the circle inscribed in the triangle.



A. 1.62 cm
B. 1.76 cm
C. 3 cm
D. 4.24 cm

OVER

42. Male bees, called drones, hatch from unfertilized eggs; therefore they have a mother but no father. Fertilized eggs hatch into female bees. A female bee, therefore, has both a mother and a father. Determine the number of ancestors that a male bee has in the 7^{th} generation back.

A. 13
B. 20
C. 21
D. 34

43. Determine the product of all positive divisors of 108.

A. $\left(2^8\right)\left(3^{12}\right)$
B. $\left(2^{10}\right)\left(3^{15}\right)$
C. $\left(2^{12}\right)\left(3^{18}\right)$
D. $\left(2^{14}\right)\left(3^{21}\right)$

44. Simplify: $\log_{\frac{1}{x}} \frac{1}{y} - \log_{\frac{1}{x}} y - \log_x \frac{1}{y}$

A. $\log_x y^3$
B. $\log_x y$
C. $-\log_x y$
D. $\log_x y - 2$

This is the end of the multiple-choice section.
Answer the remaining questions directly in this examination booklet.

PART B: WRITTEN RESPONSE

Value: 34 marks **Suggested Time: 45 minutes**

INSTRUCTIONS: Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

If, in a justification, you refer to information produced by the calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem, it is important to sketch the graph, showing its general shape and indicating the appropriate window dimensions.

When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.

Full marks will NOT be given for the final answer only.

OVER

1. A cubic polynomial function has a double zero at -2 and a single zero at 3. If this function passes through the point $(4, -24)$, determine an equation of the function. Answer may be left in factored form. **(4 marks)**

ANSWER:

OVER

2. Solve the following system algebraically. Express answers as ordered pairs. **(4 marks)**

$$3x^2 - 2y^2 = 38$$
$$x^2 + y^2 = 21$$

ANSWER:

OVER

3. Prove: **(4 marks)**

$$\frac{\sin\theta\cos\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\tan\theta}$$

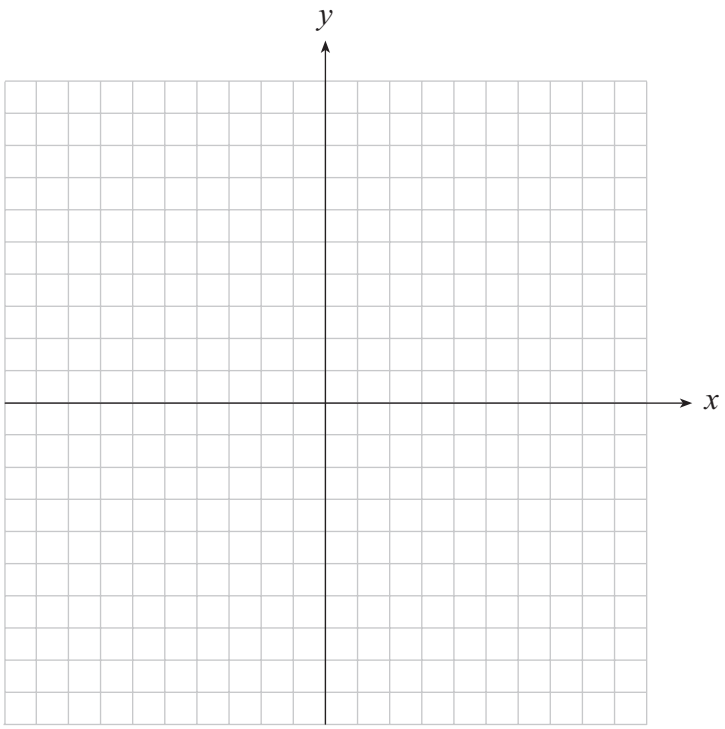
LEFT SIDE	RIGHT SIDE

OVER

Note: this question has two parts, a) and b).
A grid is provided for rough work only.

4. An ellipse which has vertices at $(-2,\ 2)$ and $(8,\ 2)$ is tangent to the x-axis.

   a) Determine an equation of this ellipse. **(3 marks)**



ANSWER:

b) If $(6, y)$ is a point on the ellipse, determine all possible values for y. **(2 marks)**

ANSWER:

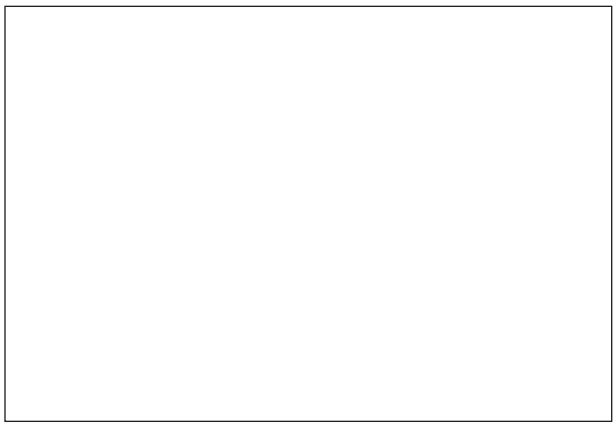
OVER

5. Solve the following system using a graphing calculator. **(4 marks)**

$$y = 2^{x-9} - 3$$

$$y = \log_2(x + 2)$$

Sketch the graph in the viewing window below. State the function(s) that you entered to obtain your graph and your solution. Indicate the dimensions of the viewing window that will show enough of the graph so that recognizable characteristics of the function(s) and all intersection points are visible.



$Y_1 =$

$Y_2 =$

$Y_3 =$

$Y_4 =$

[,] [,]

$x_{\min}$ $x_{\max}$ $y_{\min}$ $y_{\max}$

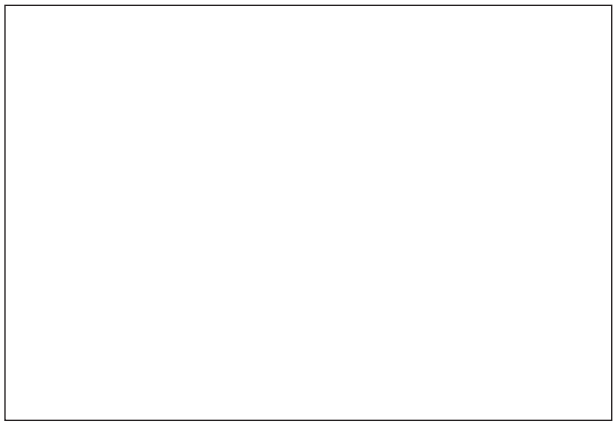
ANSWER:

OVER

6. Solve the following equation using a graphing calculator. **(4 marks)**

$$1.2\left|x-1\right| = \left|x+2\right|$$

Sketch the graph in the viewing window below and indicate appropriate window dimensions. State the function(s) used in your graph. Ensure that the relative maximum and relative minimum points of the function(s) are visible within the viewing window.



$Y_1 =$

$Y_2 =$

$Y_3 =$

$Y_4 =$

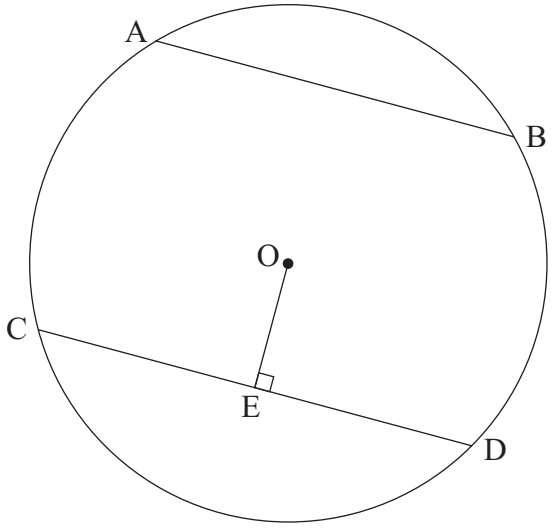
[,] [,]

x_{min} x_{max} y_{min} y_{max}

ANSWER:

OVER

7. A circle with centre O has parallel chords AB and CD. If AB = 12 cm, CD = 16 cm, OE = 5 cm and $OE \perp CD$, determine the distance between the chords. **(4 marks)**



ANSWER:

OVER

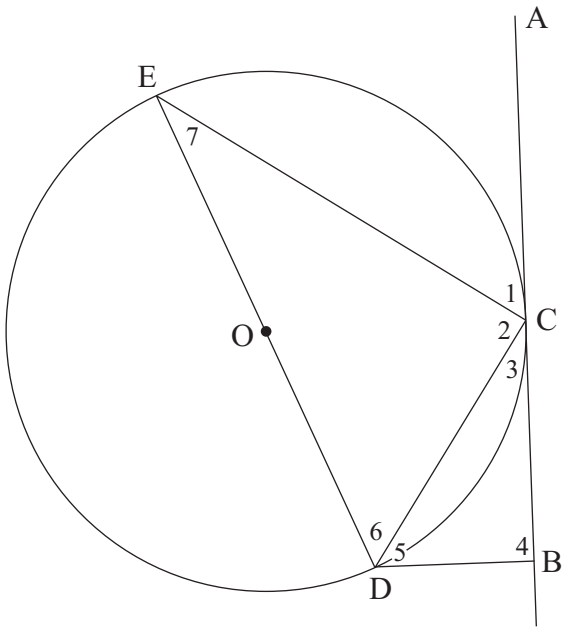
Students must choose one or the other method of proof.

8. Complete the proof. **(5 marks)**

Diagram clarification: O is the centre of the circle

Given: AB is tangent to the circle at C
DC bisects $\angle EDB$

Prove: $DB \perp AB$



Paragraph proof method:

Two-column proof method:

STATEMENT	REASON

END OF EXAMINATION

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A Summary of Basic Identities and Formulae

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$ $\quad$ $1 + \tan^2\theta = \sec^2\theta$ $\quad$ $1 + \cot^2\theta = \csc^2\theta$

Reciprocal and Quotient Identities

$\sec\theta = \dfrac{1}{\cos\theta}$ $\quad$ $\csc\theta = \dfrac{1}{\sin\theta}$ $\quad$ $\cot\theta = \dfrac{1}{\tan\theta}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\quad$ $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Addition Identities

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\quad$ $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\quad$ $\tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

Double-Angle Identities

$\cos 2\theta = \cos^2\theta - \sin^2\theta$ $\quad$ $\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$ $\quad$ $\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

Formulae

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\quad$ $t_n = a + (n-1)d$ $\quad$ $t_n = ar^{n-1}$

$\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$ $\quad$ $S_n = \dfrac{n}{2}\left[2a + (n-1)d\right]$ $\quad$ $S_n = \dfrac{a(1 - r^n)}{1 - r}$

$c^2 = a^2 + b^2 - 2ab\cos C$ $\quad$ $S_n = \dfrac{n}{2}(a + \ell)$ $\quad$ $S_n = \dfrac{a - \ell r}{1 - r}$

$S = \dfrac{a}{1 - r}$

You may detach this page for convenient reference.
Exercise care when tearing along perforations.

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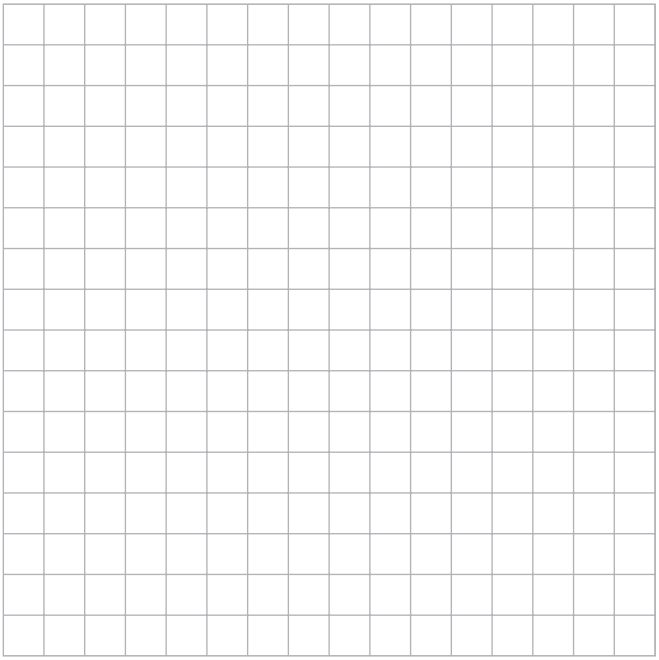
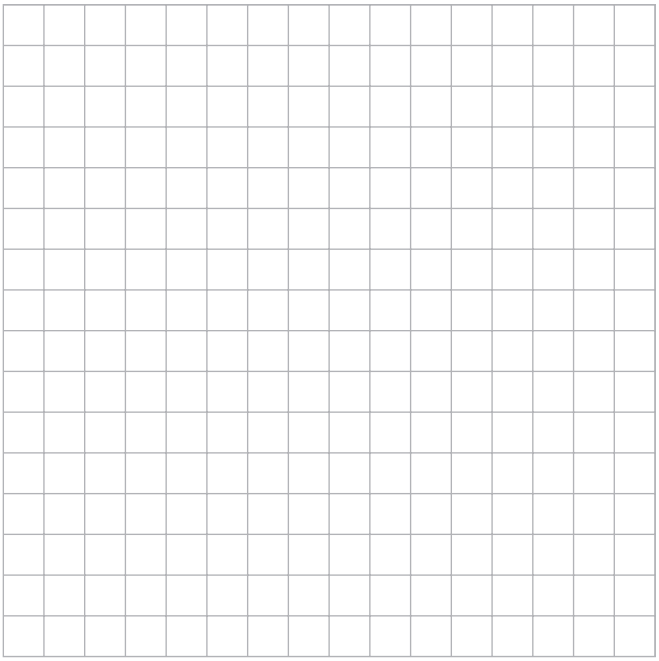
ROUGH WORK FOR GRAPHING

(No marks will be given for work done on this page.)





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ROUGH WORK FOR GRAPHING

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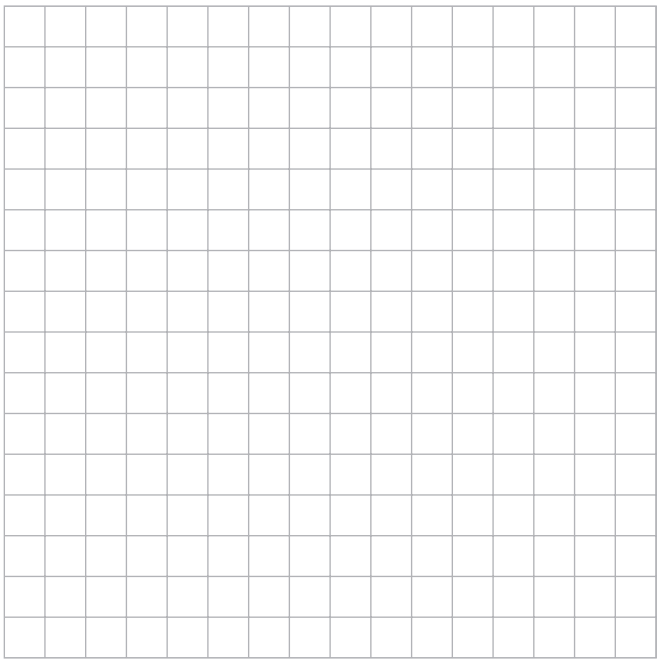

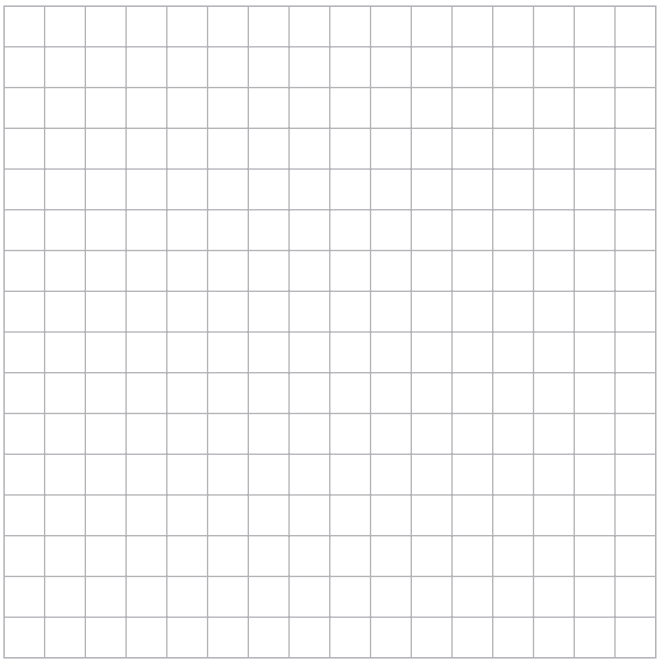

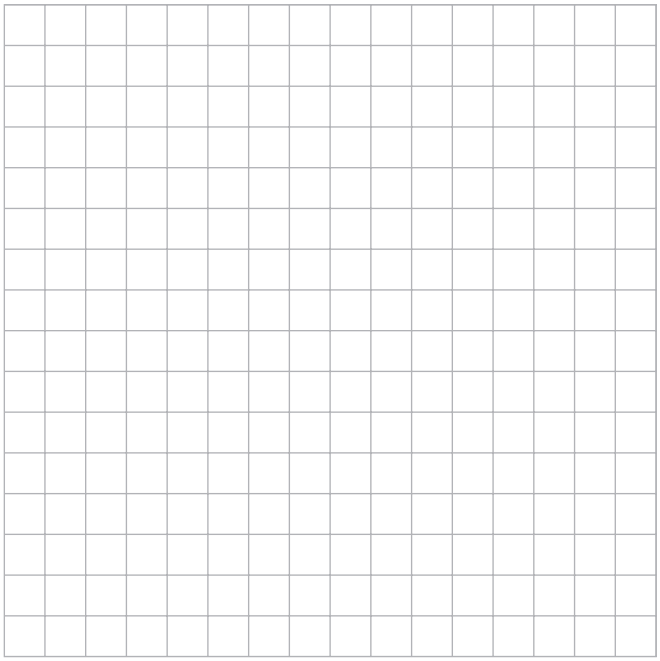

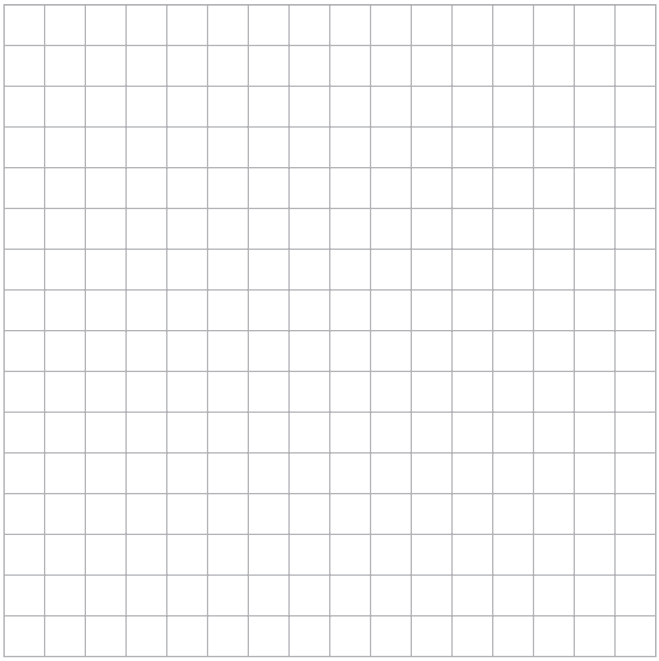

ROUGH WORK FOR MULTIPLE-CHOICE

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