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Principles of Mathematics 12

APRIL 2001

Course Code = MA

Student Instructions

1. Place the stickers with your Personal Education Number (PEN) in the allotted spaces above. **Under no circumstance is your name or identification, other than your Personal Education Number, to appear on this booklet.**
2. Ensure that in addition to this examination booklet, you have an **Examination Response Form**. Follow the directions on the front of the Response Form.
3. **Disqualification** from the examination will result if you bring books, paper, notes or unauthorized electronic devices into the examination room.
4. When instructed to open this booklet, **check the numbering of the pages** to ensure that they are numbered in sequence from page one to the last page, which is identified by **END OF EXAMINATION**.
5. At the end of the examination, place your Response Form inside the front cover of this booklet and return the booklet and your Response Form to the supervisor.

Question 1:

1. .

(4)

Question 2:

2. .

(4)

Question 3:

3. .

(4)

Question 4:

4. .

(5)

Question 5:

5. .

(4)

Question 6:

6. .

(4)

Question 7:

7. .

(4)

Question 8:

8. .

(5)

**PRINCIPLES OF
MATHEMATICS 12**

APRIL 2001

COURSE CODE = MA

GENERAL INSTRUCTIONS

1. Aside from an approved calculator, electronic devices, including dictionaries and pagers, are **not** permitted in the examination room.
2. All multiple-choice answers must be entered on the Response Form using an **HB pencil**. Multiple-choice answers entered in this examination booklet will **not** be marked.
3. For each of the written-response questions, write your answer in the space provided in this booklet.

You will not be provided with any additional paper since rough-work space for the written-response questions has been incorporated into the space allowed for answering each question. You may not need all of the space provided to answer each question.

4. Ensure that you use language and content appropriate to the purpose and audience of this examination. Failure to comply may result in your paper being awarded a zero.
5. This examination is designed to be completed in **two hours**. *Students may, however, take up to 30 minutes of additional time to finish.*

PRINCIPLES OF MATHEMATICS 12 PROVINCIAL EXAMINATION

- | | Value | Suggested Time |
|---|-------------------------|--------------------|
| 1. This examination consists of two parts: | | |
| PART A: 44 multiple-choice questions | 66 | 75 |
| PART B: 8 written-response questions | 34 | 45 |
| | Total: 100 marks | 120 minutes |
2. The last **three** pages inside the back cover contain **A Summary of Basic Identities and Formulae, Rough Work for Graphing, and Rough Work for Multiple-Choice**. These pages may be detached for convenient reference prior to writing this examination.
3. **A graphing calculator is essential for the Principles of Mathematics 12 Provincial Examination.** The calculator must be a hand-held device designed primarily for mathematical computations involving logarithmic and trigonometric functions as well as for graphing functions. Computers, calculators with a QWERTY keyboard or symbolic manipulation abilities, and electronic writing pads will not be allowed. Students must not bring any external devices (peripherals) to support calculators such as manuals, printed or electronic cards, printers, memory expansion chips or cards, CD-ROMs, libraries or external keyboards. Students may have more than one calculator available during the examination, of which one may be a scientific calculator. Calculators may not be shared and must not have the ability to either transmit or receive electronic signals. In addition to an approved calculator, students will be allowed to use rulers, compasses, and protractors during the examination.
- Calculators must not have any information programmed into the memory which would not be acceptable in paper form. Specifically, calculators must not have any built-in notes, definitions, or libraries. There is no requirement to clear memories at the beginning of the examination but the use of calculators with built-in notes is equivalent to the use of notes in paper form. Any student deemed to have cheated on a provincial examination will receive a “0” on that examination and will be permanently disqualified from the Provincial Examination Scholarship Program.
4. If, in a justification, you refer to information produced by the calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem, it is important to sketch the graph, showing its general shape and indicating the appropriate window dimensions.
5. When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.

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PART A: MULTIPLE CHOICE

Value: 66 marks

Suggested Time: 75 minutes

INSTRUCTIONS: For each question, select the **best** answer and record your choice on the Response Form provided. Using an HB pencil, completely fill in the circle that has the letter corresponding to your answer.

1. According to the Rational Root Theorem, list the possible rational roots of the equation $5x^3 + mx^2 + nx + 2 = 0$, where m and n are integers.

A. $\pm 1, \pm 2$

B. $\pm 1, \pm 5$

C. $\pm 1, \pm 5, \pm \frac{1}{2}, \pm \frac{5}{2}$

D. $\pm 1, \pm 2, \pm \frac{1}{5}, \pm \frac{2}{5}$

2. If $x + 7$ is a factor of $P(x)$, then which of the following must be true?

A. $P(-7) = 0$

B. $P(7) = 0$

C. $P(0) = -7$

D. $P(0) = 7$

3. Determine the number of real roots of: $x^3 - 12x^2 - x - 3.2 = 0$

A. 0

B. 1

C. 2

D. 3

4. Determine the remainder:

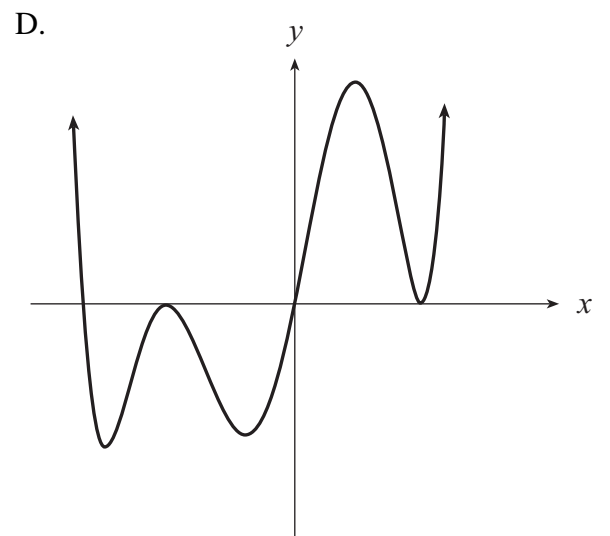
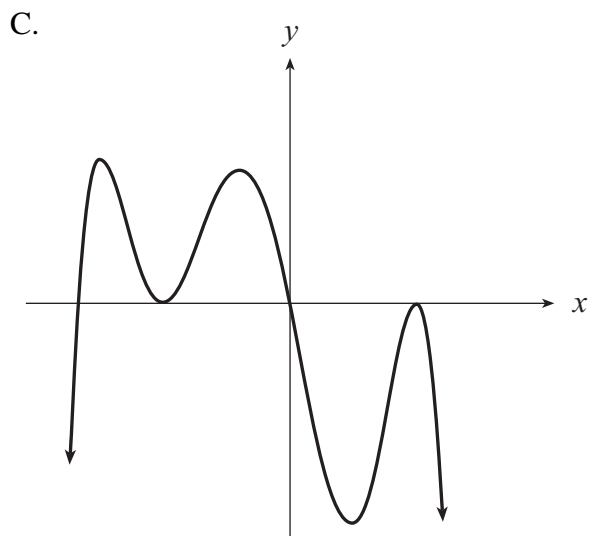
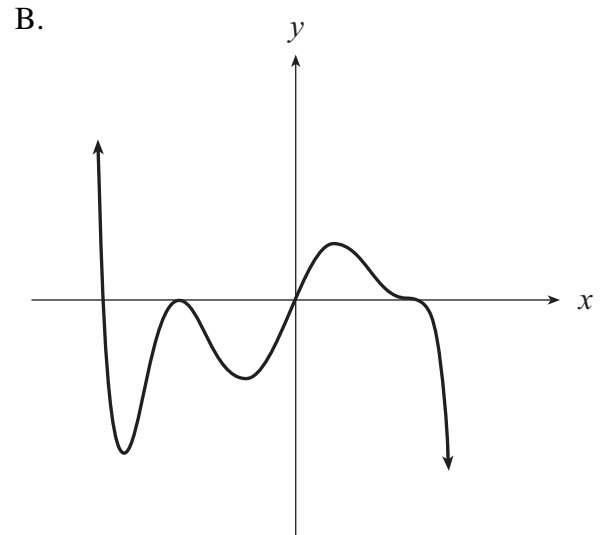
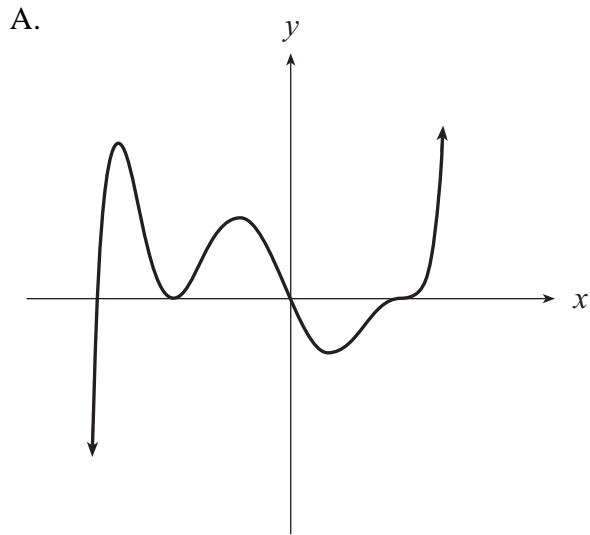
$$x^2 + x + 1 \overline{)2x^4 + 2x^3 - x^2 + 3x - 1}$$

- A. 2
- B. $2x$
- C. $2x - 2$
- D. $6x + 2$

5. The function $H(x)$ is the product of a 3rd degree polynomial function and a 2nd degree polynomial function. What is the maximum number of zeros of $H(x)$?

- A. 2
- B. 3
- C. 5
- D. 6

6. Which graph could represent $f(x) = x(a-x)(x-b)^2(x-c)^3$, where a , b and c are constants?



7. Determine the midpoint of the line segment joining the points $(-3, -5)$ and $(1, -3)$.

- A. $(-2, -1)$
- B. $(-2, 1)$
- C. $(-1, -1)$
- D. $(-1, -4)$

8. Which conic is represented by $3x^2 - 3y^2 + 12x - 6y - 5 = 0$?

- A. circle
- B. ellipse
- C. parabola
- D. hyperbola

9. Change to standard form: $x^2 + 2y^2 + 4y - 2 = 0$

- A. $\frac{x^2}{2} + \frac{(y+1)^2}{1} = 1$
- B. $\frac{x^2}{4} + \frac{(y+1)^2}{2} = 1$
- C. $\frac{x^2}{6} + \frac{(y+1)^2}{3} = 1$
- D. $\frac{x^2}{10} + \frac{(y+2)^2}{5} = 1$

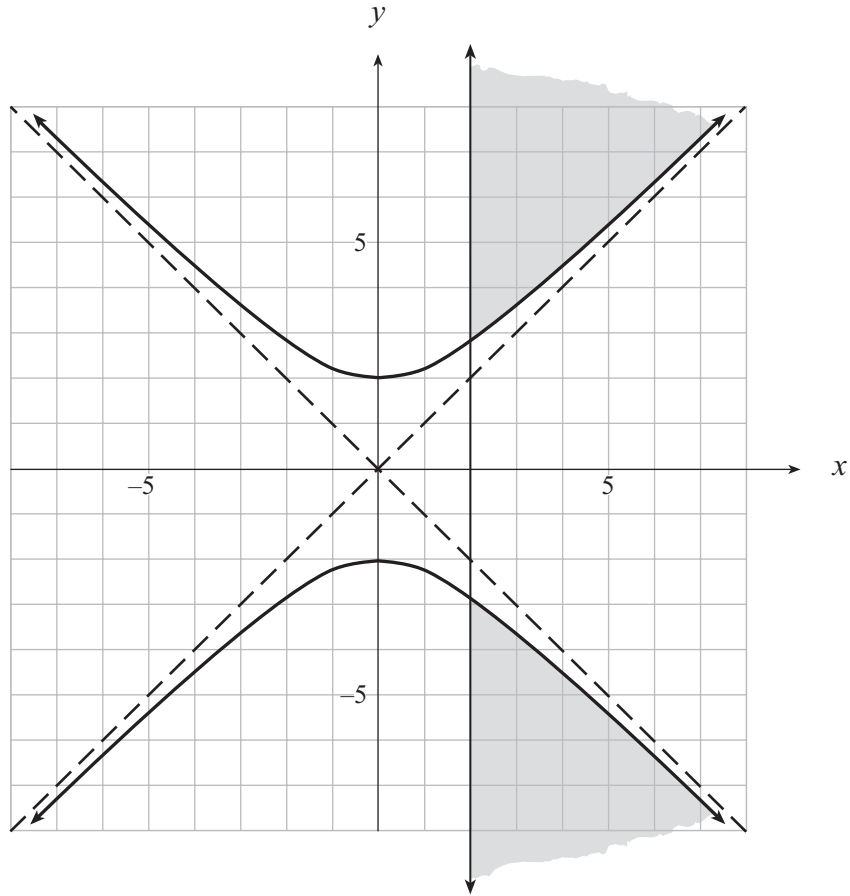
10. Determine an equation of the ellipse with centre $(-1, 2)$, a horizontal major axis of length 8 and a minor axis of length 4.

- A. $\frac{(x+1)^2}{16} + \frac{(y-2)^2}{64} = 1$
- B. $\frac{(x+1)^2}{8} + \frac{(y-2)^2}{4} = 1$
- C. $\frac{(x+1)^2}{64} + \frac{(y-2)^2}{16} = 1$
- D. $\frac{(x+1)^2}{16} + \frac{(y-2)^2}{4} = 1$

11. Determine the absolute value inequality whose solution is $x < -3$ or $x > 5$.

- A. $|x - 2| > 3$
- B. $|x - 1| < 4$
- C. $|x - 1| > 4$
- D. $|x + 1| > 4$

12. The shaded region below is the solution of which of the following systems?



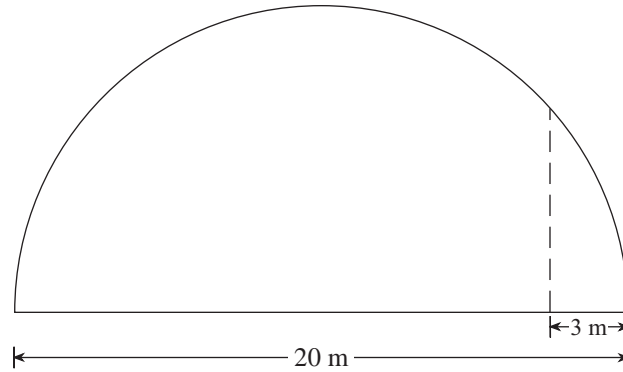
A. $x^2 - y^2 \leq -4$
 $y \geq 2$

B. $x^2 - y^2 \leq -4$
 $x \geq 2$

C. $x^2 - y^2 \geq -4$
 $y \geq 2$

D. $x^2 - y^2 \geq -4$
 $x \geq 2$

13. An arch is in the shape of a semi-circle. If the base of the arch measures 20 m, how high is the arch 3 m from the outside edge of the base?



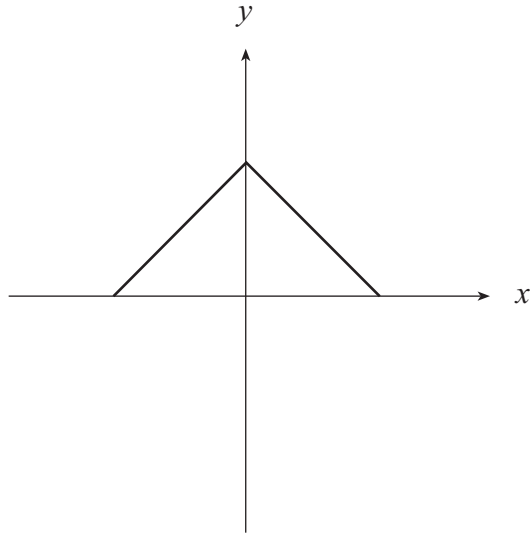
- A. 7.14 m
B. 8.00 m
C. 9.27 m
D. 9.54 m
14. Which condition on b , $b > 0$, will result in the following system always having exactly 4 different solutions if $0 < a < 9$?

$$x^2 + y^2 = 9$$

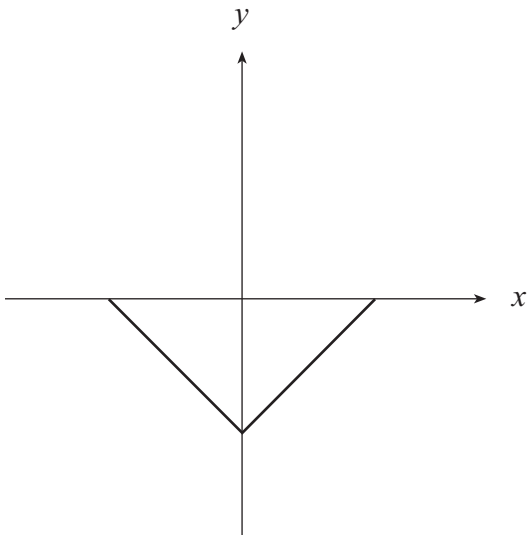
$$\frac{x^2}{a} + \frac{y^2}{b} = 1$$

- A. $b > 9$
B. $b \geq 9$
C. $b < 9$
D. $b \leq 9$
15. Which expression is equivalent to $\log_2 7$?
- A. $\frac{\log 7}{\log 2}$
B. $\log 7 - \log 2$
C. $\log 3.5$
D. $7 \log 2$

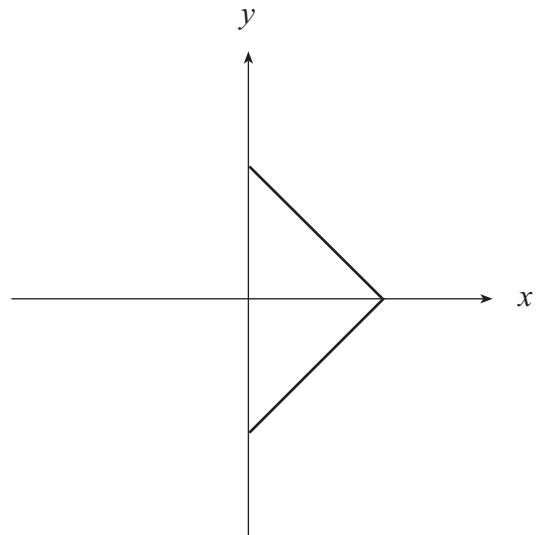
16. Which graph best represents the inverse relation of the graph shown?



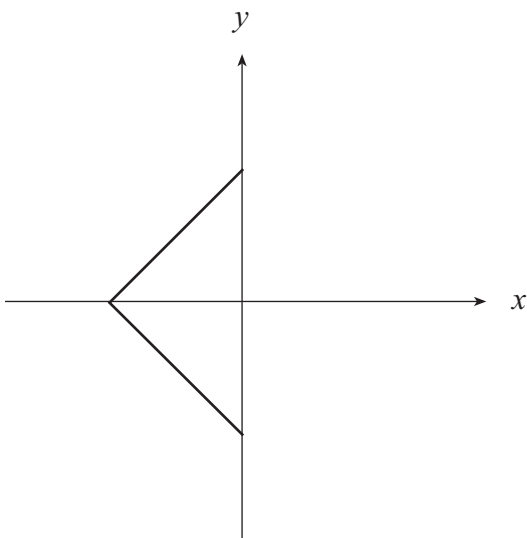
A.



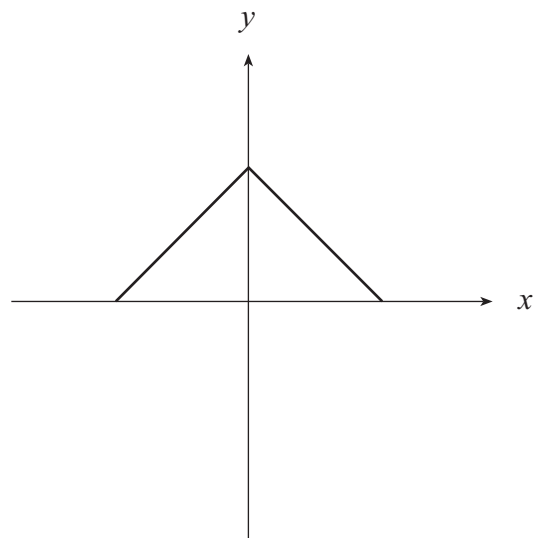
B.



C.



D.



17. Solve: $\log x - \log 2 = 1$

- A. 2
- B. 3
- C. 12
- D. 20

18. Solve: $2^x = 3 - \log(x + 2)$

- A. 1.31
- B. 1.84
- C. 2.07
- D. 2.48

19. If $\log_a 3 = x$ and $\log_a 5 = y$, determine an expression for $\log_a\left(\frac{15}{a}\right)$ in terms of x and y .

- A. xy
- B. $x + y$
- C. $xy - 1$
- D. $x + y - 1$

20. Simplify: $16^{\log_2 x}$

- A. 2^{x^4}
- B. 2^{4x}
- C. $4x$
- D. x^4

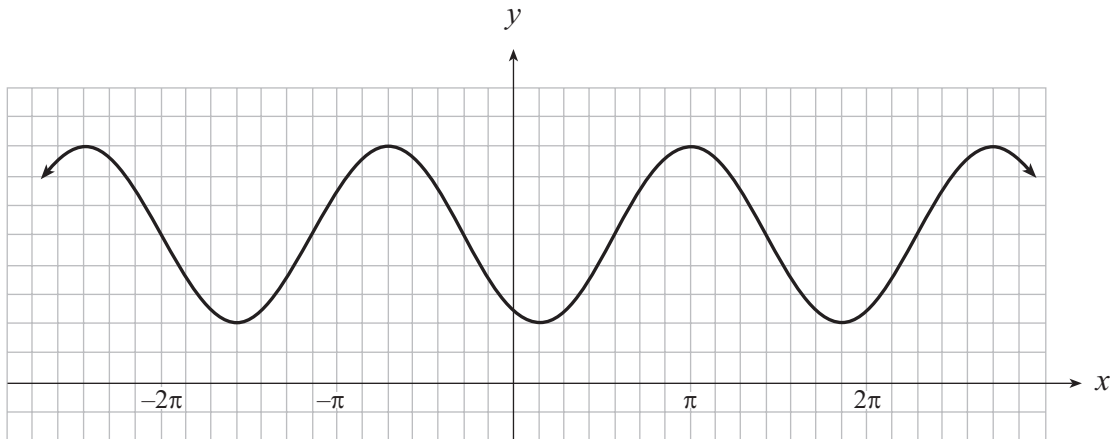
21. If the graphs of $y = 2^x$ and $y = a^{x-4}$ intersect at a point where $x = 2$, determine the value of a .

- A. -2
- B. $\frac{1}{16}$
- C. $\frac{1}{2}$
- D. 2

22. Which of the following is a geometric sequence?
- A. 5, 3, 2, 1
 - B. 6, 4, 2, 0
 - C. 7, 4, 2, 1
 - D. 8, 4, 2, 1
23. Given that $t_n = 4n^3 - n$, determine the 2nd term.
- A. 18
 - B. 22
 - C. 30
 - D. 34
24. If 3 arithmetic means are inserted between 0 and 48, which of the following would be one of them?
- A. 16
 - B. 30
 - C. 32
 - D. 36
25. In a geometric sequence, $t_1 = 16$ and $t_5 = 81$. Find t_7 .
- A. 107
 - B. 121.5
 - C. 182.25
 - D. 237.39
26. Evaluate: $\sum_{k=4}^{30} (3k + 2)$
- A. 1 378
 - B. 1 431
 - C. 1 455
 - D. 1 590

27. The n^{th} term of an infinite series is given by $t_n = 5\left(\frac{2}{3}\right)^{n-1}$. Find the sum of the series.
- A. 3
 - B. 10
 - C. 15
 - D. no finite sum
28. The sum of n terms in a series is given by $S_n = 2n^2 - 3n$. Find t_3 , the 3rd term of this series.
- A. 3
 - B. 4
 - C. 7
 - D. 9
29. Express $\frac{14}{3}$ radians in degrees.
- A. 0.08°
 - B. 267.38°
 - C. 534.76°
 - D. 840°
30. Evaluate: $2 \sec 3$
- A. -2.02
 - B. -0.51
 - C. 3.54
 - D. 14.17
31. What is the amplitude of $f(x) = -3 \cos(4x + \pi) + 2$?
- A. -3
 - B. 2
 - C. 3
 - D. 4

32. Determine the period of the sine function shown below.



- A. π
- B. 2π
- C. $\frac{6\pi}{7}$
- D. $\frac{12\pi}{7}$

33. What is the phase shift of $f(x) = 2 \sin(3x + \pi) + 6$?

- A. $\frac{\pi}{3}$ to the left
- B. $\frac{\pi}{2}$ to the left
- C. $\frac{2\pi}{3}$ to the left
- D. π to the left

34. Solve: $\tan^3 x - \sin x = 1$, $0 \leq x < 2\pi$

- A. 0.88, 3.78
- B. 2.50, 5.40
- C. 0.88, 1.54, 3.78, 4.68
- D. 1.54, 2.50, 4.68, 5.40

35. Determine the number of solutions for: $(\tan \theta + 1)(2 \sin \theta + 1)(\cos \theta + 2) = 0$, where $0 \leq \theta < 2\pi$.

- A. 2
- B. 3
- C. 4
- D. 6

36. Which expression is equivalent to $4 \sin^2 2x - 2$?

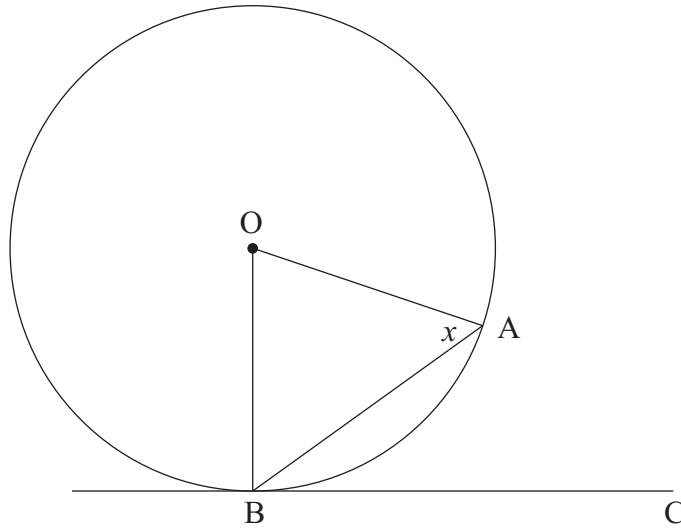
- A. $-2 \cos 2x$
- B. $-2 \cos 4x$
- C. $-\cos 2x$
- D. $-\cos 4x$

37. If $\csc \theta = a$ and θ terminates in quadrant II, determine an expression for $\cos \theta$.

- A. $\frac{\sqrt{a^2 - 1}}{a}$
- B. $\frac{-\sqrt{a^2 - 1}}{a}$
- C. $\frac{a}{\sqrt{a^2 - 1}}$
- D. $\frac{-a}{\sqrt{a^2 - 1}}$

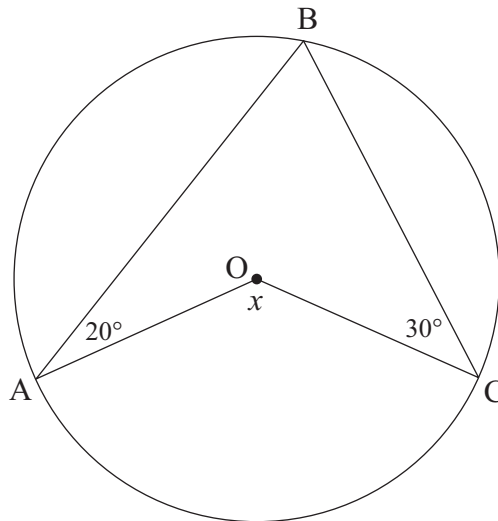
For questions 38 to 41, diagrams are not drawn to scale.

38. Given a circle with centre O and tangent BC, determine the measure of $\angle x$ if $\angle ABC = 40^\circ$.



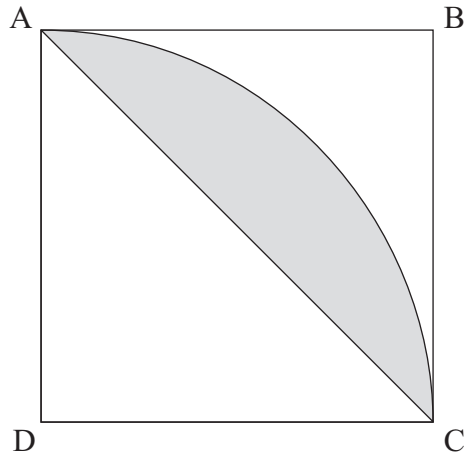
- A. 40°
- B. 50°
- C. 60°
- D. 70°

39. Given that O is the centre of the circle, determine the measure of $\angle x$.



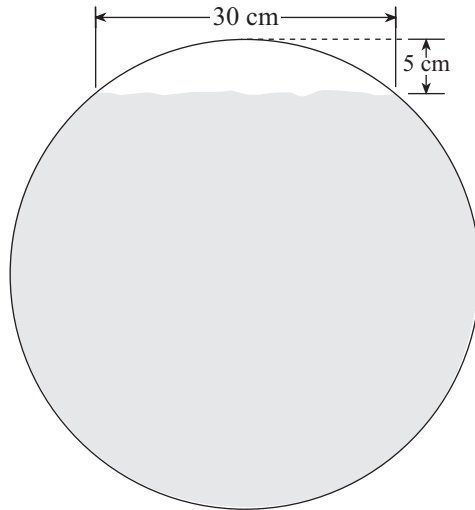
- A. 100°
- B. 120°
- C. 130°
- D. 140°

40. Square ABCD has sides of length 10 cm. A quarter circle with centre D is drawn in the square, as shown in the diagram. Determine the area of the shaded region.



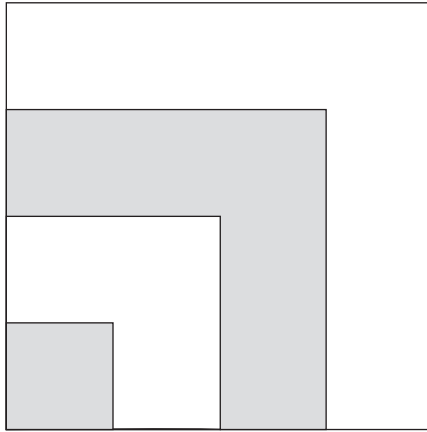
- A. 12.83
- B. 25.66
- C. 28.54
- D. 66.04

41. A circular pipe is filled with water to the level shown in the diagram. The water surface has a width of 30 cm and the space from the water surface to the top of the pipe is 5 cm. Determine the depth of the water.



- A. 25 cm
- B. 40 cm
- C. 45 cm
- D. 50 cm

42. In the diagram below, what fraction of the entire figure is shaded? All bands are of equal width.



- A. $\frac{3}{8}$
B. $\frac{7}{12}$
C. $\frac{3}{5}$
D. $\frac{2}{3}$
43. Determine the product of all the positive divisors of 216.
- A. $(2^{18})(3^{18})$
B. $(2^{21})(3^{21})$
C. $(2^{24})(3^{24})$
D. $(2^{27})(3^{27})$
44. Determine the domain of: $y = \log_{x-2} 5 + \log \sqrt{x-1}$
- A. $x > 2$
B. $x > 0$
C. $x > 0, x \neq 1$
D. $x > 2, x \neq 3$

**This is the end of the multiple-choice section.
Answer the remaining questions directly in this examination booklet.**

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PART B: WRITTEN RESPONSE

Value: 34 marks

Suggested Time: 45 minutes

INSTRUCTIONS: Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

If, in a justification, you refer to information produced by the calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem, it is important to sketch the graph, showing its general shape and indicating the appropriate window dimensions.

When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.

Full marks will NOT be given for the final answer only.

1. Solve the following using a graphing calculator.

(4 marks)

$$x^4 - x^3 \geq 8x^2 + 2$$

Sketch the graph in the viewing window below and indicate appropriate window dimensions. State the function(s) used in your graph. Ensure that the relative maximum and relative minimum points of the function(s) are visible within the viewing window.



$Y_1 =$

$Y_2 =$

$Y_3 =$

$Y_4 =$

[,] [,]

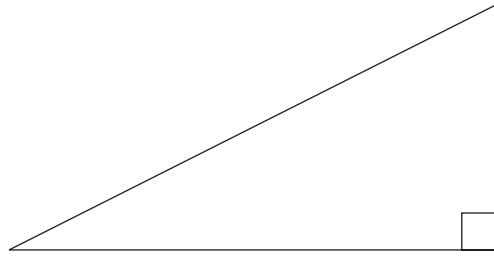
x x
min max

y y
min max

ANSWER:



2. A right triangle has a perimeter of 56 cm. If the hypotenuse measures 25 cm, determine the lengths of the other two sides of the triangle. (Solve algebraically.) **(4 marks)**



ANSWER:



3. The population of a certain type of bacteria doubles every 20 hours. At this rate, in how many hours will the original population of 100 bacteria grow to 450 bacteria?
(Answer in hours, accurate to at least 2 decimal places.) **(4 marks)**

ANSWER:



4. A point $P(x, y)$ moves such that it is the same distance from point $A(3, 6)$ as it is from the line $y = 2$. Determine the equation of this locus. Express your answer in standard form.

(5 marks)

ANSWER:



5. Prove the identity:

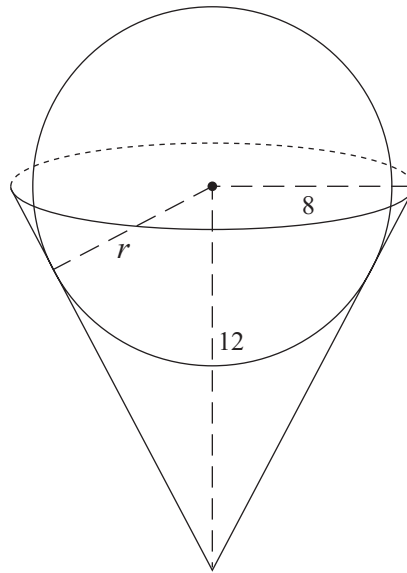
(4 marks)

$$(1 - \sin \theta)(\sec \theta + \tan \theta) = \frac{1}{\sec \theta}$$

LEFT SIDE

RIGHT SIDE

6. A cone has radius 8 cm and height 12 cm. Determine the radius, r , of a sphere that will just fit into the cone so that its centre is level with the top of the cone, as shown in the diagram. **(4 marks)**



ANSWER:

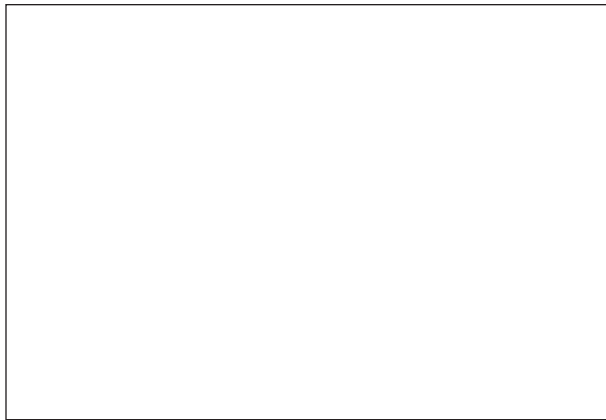


7. Solve the following equation using a graphing calculator.

(4 marks)

$$|x - 3| = 1.15|x + 3|$$

Sketch the graph in the viewing window below and indicate appropriate window dimensions. State the function(s) used in your graph. Ensure that the relative maximum and relative minimum points of the function(s) are visible within the viewing window.



$Y_1 =$

$Y_2 =$

$Y_3 =$

$Y_4 =$

[,]

[,]

x
min x
max

y
min y
max

ANSWER:

Students must choose one or the other method of proof.

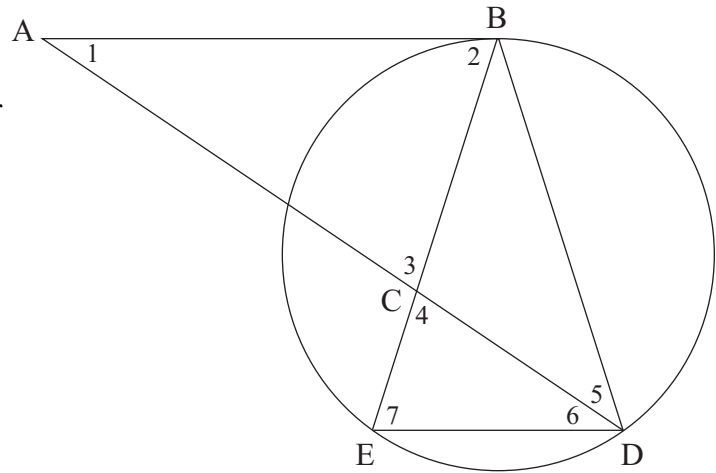
8. Complete the proof.

(5 marks)

Diagram clarification: A, C, D are collinear
B, C, E are collinear

Given: $BE = BD$
AB is tangent to the circle

Prove: $\angle 1 = \angle 6$



Paragraph proof method:

Two-column proof method:

STATEMENT	REASON

END OF EXAMINATION

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A SUMMARY OF BASIC IDENTITIES AND FORMULAE

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Reciprocal and Quotient Identities

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Addition Identities

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

Double-Angle Identities

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - 2 \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Formulae

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t_n = a + (n-1)d$$

$$t_n = ar^{n-1}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$S_n = \frac{n}{2}(a + \ell)$$

$$S_n = \frac{a - \ell r}{1-r}$$

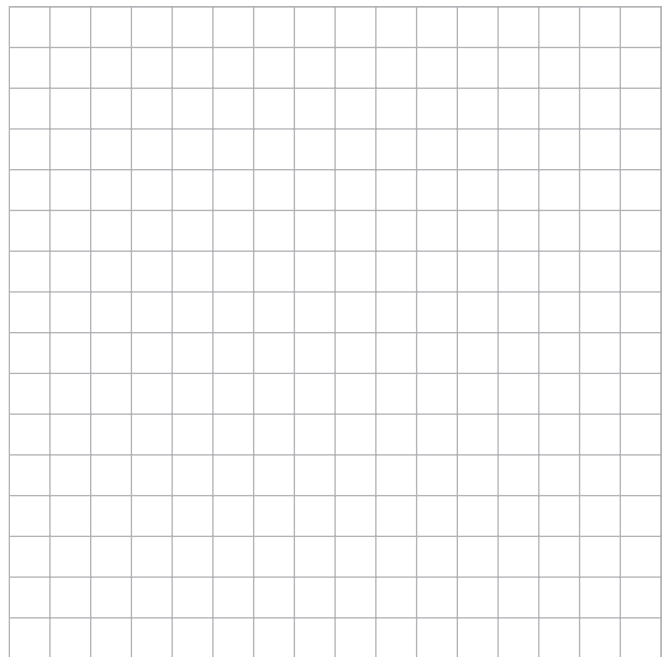
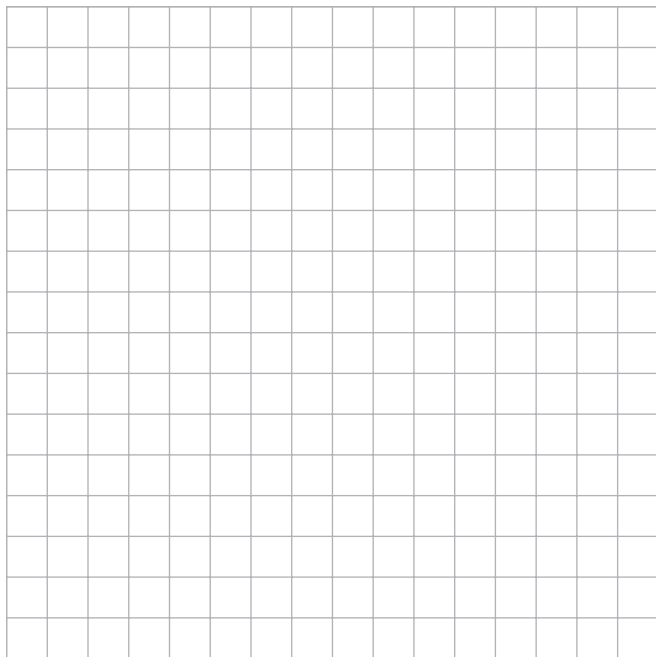
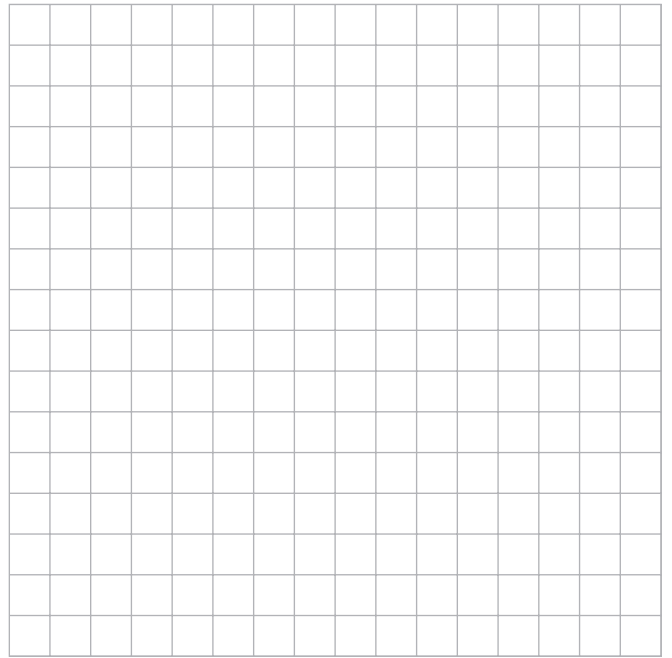
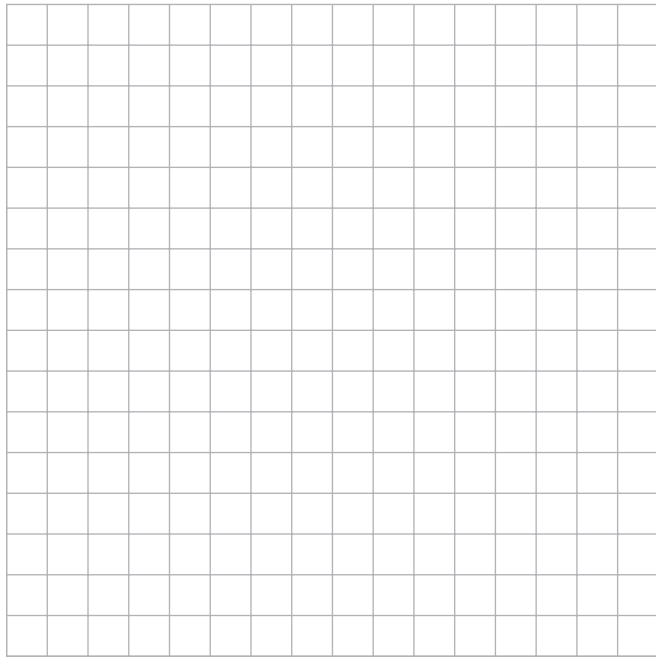
$$S = \frac{a}{1-r}$$

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ROUGH WORK FOR GRAPHING

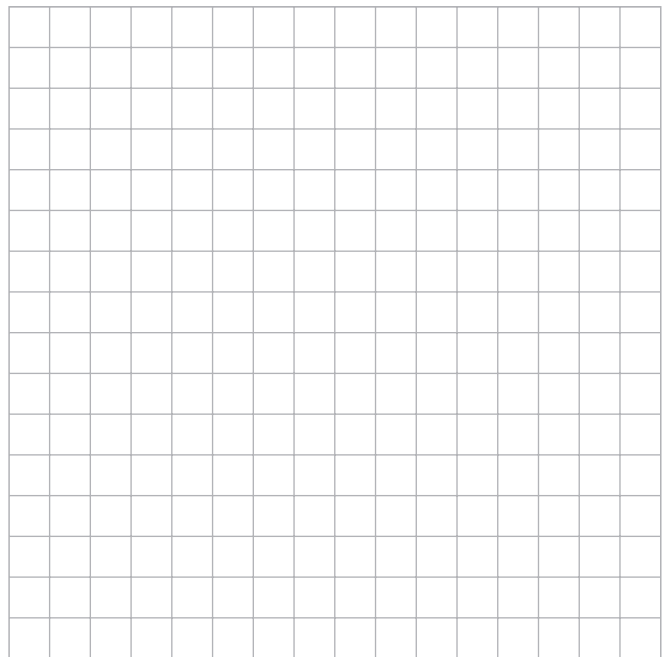
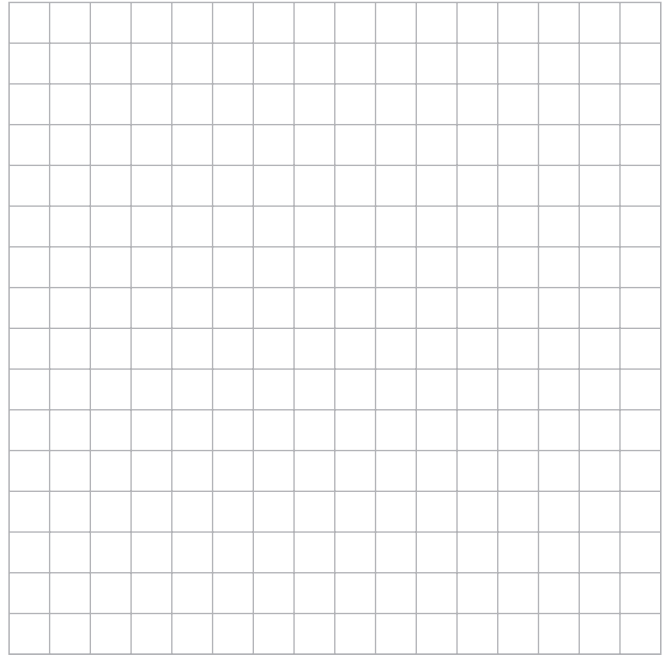
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ROUGH WORK FOR GRAPHING

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ROUGH WORK FOR MULTIPLE-CHOICE

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ROUGH WORK FOR MULTIPLE-CHOICE

