

Insert Personal Education Number (PEN) here.

Insert **only** pre-printed PEN label here.

STUDENT INSTRUCTIONS

1. Insert the stickers with your Personal Education Number (PEN) in the allotted spaces above. **Under no circumstance is your name or identification, other than your Personal Education Number, to appear on this booklet.**
2. Ensure that in addition to this examination booklet, you have an **Examination Response Form**. Follow the directions on the front of the Response Form.
3. **Disqualification** from the examination will result if you bring books, paper, notes or unauthorized electronic devices into the examination room.
4. When instructed to open this booklet, **check the numbering of the pages** to ensure that they are numbered in sequence from page one to the last page, which is identified by

END OF EXAMINATION.

5. At the end of the examination, place your Response Form inside the front cover of this booklet and return the booklet and your Response Form to the supervisor.

PRINCIPLES OF MATHEMATICS 12

JUNE 2000

COURSE CODE = MA

Insert **only** hand-printed PEN here.

Ministry use only.

Question 1:

1. .
(3)

Question 2:

2. .
(3)

Question 3:

3. .
(3)

Question 4:

4. .
(3)

Question 5:

5. .
(3)

Question 6:

6. .
(3)

Question 7:

7. .
(3)

Question 8:

8. .
(4)



**PRINCIPLES OF
MATHEMATICS 12**

JUNE 2000

COURSE CODE = MA

GENERAL INSTRUCTIONS

1. Aside from an approved calculator, electronic devices, including dictionaries and pagers, are **not** permitted in the examination room.
2. All multiple-choice answers must be entered on the Response Form using an **HB pencil**. Multiple-choice answers entered in this examination booklet will **not** be marked.
3. For each of the written-response questions, write your answer in the space provided in this booklet.

You will not be provided with any additional paper since rough-work space for the written-response questions has been incorporated into the space allowed for answering each question. You may not need all of the space provided to answer each question.

4. Ensure that you use language and content appropriate to the purpose and audience of this examination. Failure to comply may result in your paper being awarded a zero.
5. This examination is designed to be completed in **two hours**. *Students may, however, take up to 30 minutes of additional time to finish.*

PRINCIPLES OF MATHEMATICS 12 PROVINCIAL EXAMINATION

- | | Value | Suggested Time |
|---|------------------------|--------------------|
| 1. This examination consists of two parts: | | |
| PART A: 45 multiple-choice questions | 45 | 75 |
| PART B: 8 written-response questions | 25 | 45 |
| | Total: 70 marks | 120 minutes |
2. The last **three** pages inside the back cover contain **A Summary of Basic Identities and Formulae, Rough Work for Graphing, and Rough Work for Multiple-Choice**. These pages may be detached for convenient reference prior to writing this examination.
3. **A graphing calculator is essential for the Principles of Mathematics 12 Provincial Examination.** The calculator must be a hand-held device designed primarily for mathematical computations involving logarithmic and trigonometric functions as well as for graphing functions. Computers, calculators with a QWERTY keyboard, and electronic writing pads will not be allowed. Students must not bring any external devices to support calculators such as manuals, printed or electronic cards, printers, memory expansion chips or cards, or external keyboards. Students may have more than one calculator available during the examination of which one may be a scientific calculator. Calculators may not be shared and must not have the ability to either transmit or receive electronic signals. In addition to an approved calculator, students will be allowed to use rulers, compasses, and protractors during the examination.
4. If, in a justification, you refer to information produced by the calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem, it is important to sketch the graph, showing its general shape and indicating the appropriate window dimensions.
5. When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.

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PART A: MULTIPLE CHOICE

Value: 45 marks

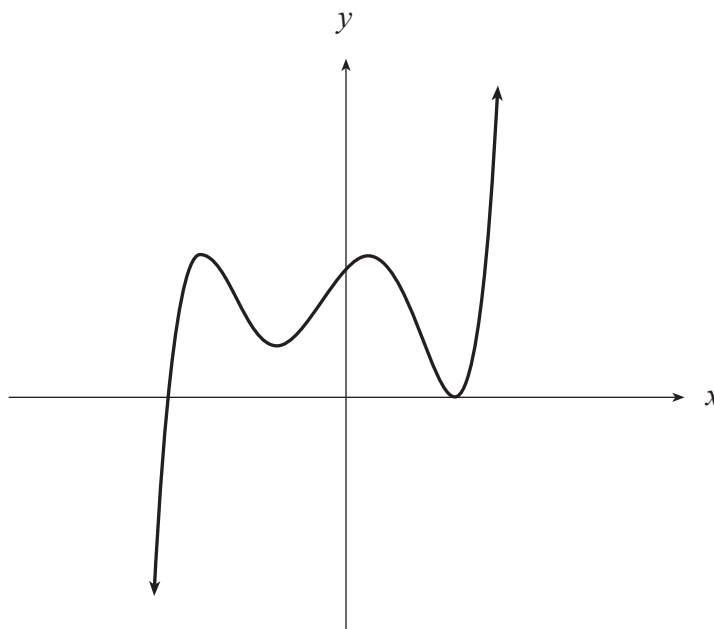
Suggested Time: 75 minutes

INSTRUCTIONS: For each question, select the **best** answer and record your choice on the Response Form provided. Using an HB pencil, completely fill in the circle that has the letter corresponding to your answer.

1. According to the Rational Root Theorem, determine all possible rational roots of $4x^3 - 7x^2 + 3x - 2 = 0$.

- A. $\pm 1, \pm 2$
- B. $\pm 1, \pm 2, \pm 4$
- C. $\pm 1, \pm 2, \pm 4, \pm \frac{1}{2}$
- D. $\pm 1, \pm 2, \pm \frac{1}{4}, \pm \frac{1}{2}$

2. What is the minimum degree of the polynomial function graphed below?



- A. 2
- B. 3
- C. 4
- D. 5

3. Solve: $2x^3 + 5 = 5x^2 + 5x$

- A. -1.88
- B. -0.58
- C. -1.22, 0.67, 3.05
- D. -1.00, 0.60, 3.00

4. Solve the following inequality for x , given that a , b and c are constants such that $a < b < c$.

$$(x - a)^3(x - b)^2(x - c) > 0$$

- A. $x > c$
- B. $x < a$ or $x > c$
- C. $x < c$, $x \neq a$, $x \neq b$
- D. $a < x < c$, $x \neq b$

5. Determine all values for k , such that $y = 2x^3 + 3x^2 - 12x + k$ has only one real zero.

- A. $k < -20$
- B. $k > 7$
- C. $-20 < k < 7$
- D. $k < -20$ or $k > 7$

6. Which equation represents a rectangular hyperbola?

- A. $x^2 - 3y^2 = 9$
- B. $x - y = 9$
- C. $x^2 - y = 9$
- D. $x^2 - y^2 = 9$

7. Determine the midpoint of $(2, y)$ and $(x, 3)$.

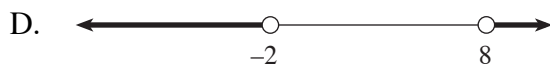
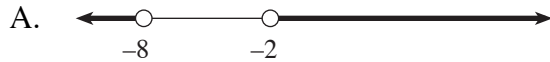
A. $\left(\frac{2+y}{2}, \frac{x+3}{2}\right)$

B. $\left(\frac{2+x}{2}, \frac{y+3}{2}\right)$

C. $\left(\frac{2-x}{2}, \frac{y-3}{2}\right)$

D. $\left(\frac{x-2}{2}, \frac{3-y}{2}\right)$

8. Solve: $|x+5| > 3$



9. Which ordered pair is a solution of the following system of inequalities?

$$x^2 + y^2 > 16$$

$$x^2 - y^2 < 4$$

A. $(1, 3)$

B. $(2, 4)$

C. $(3, 1)$

D. $(4, 2)$

10. Determine the number of different points of intersection for the following system:

$$\frac{(x+2)^2}{9} + \frac{(y-2)^2}{16} = 1$$
$$x = -(y-2)^2 - 5$$

- A. 1
- B. 2
- C. 3
- D. 4

11. Change $x^2 + 2x - 2y + 3 = 0$ to standard form.

- A. $y = \frac{1}{2}(x+1)^2 + 1$
- B. $y = \frac{1}{2}(x+1)^2 + 2$
- C. $y = \frac{1}{2}(x+1)^2 + \frac{1}{2}$
- D. $y = \frac{1}{2}(x+1)^2 - 2$

12. A point $P(x, y)$ moves such that it is always the same distance from the point $A(5, 2)$ as it is from the line $x = -1$. Determine an equation of this locus.

- A. $\sqrt{(x-5)^2 + (y-2)^2} = \sqrt{(x+1)^2 + (y-0)^2}$
- B. $\sqrt{(x-5)^2 + (y-2)^2} = \sqrt{(x+1)^2 + (y-y)^2}$
- C. $\sqrt{(x-5)^2 + (y-2)^2} = \sqrt{(x-x)^2 + (y+1)^2}$
- D. $\sqrt{(x-5)^2 + (y-2)^2} = \sqrt{(x-0)^2 + (y+1)^2}$

13. A hyperbola with vertices $(-3, 8)$ and $(-3, -4)$ has asymptotes with slopes $\pm \frac{2}{3}$. Determine the equation of the hyperbola.

A. $\frac{(x-3)^2}{9} - \frac{(y+2)^2}{4} = -1$

B. $\frac{(x+3)^2}{9} - \frac{(y-6)^2}{4} = -1$

C. $\frac{(x+3)^2}{16} - \frac{(y-2)^2}{36} = -1$

D. $\frac{(x+3)^2}{81} - \frac{(y-2)^2}{36} = -1$

14. Change $\log_a b = c$ to exponential form.

A. $b = a^c$

B. $b = c^a$

C. $a = b^c$

D. $a = c^b$

15. Evaluate: $\log 2^{2000}$

A. 0

B. 10.97

C. 602.06

D. infinitely large

16. What is the domain of $y = \log(x-a) + b$, if a and b are constants?

A. $x > a$

B. $x < a$

C. $x > b$

D. $x < b$

17. Solve: $\log_2(3-x) + \log_2 x = 1$

- A. 1
- B. 2
- C. 1, 2
- D. no solution

18. Solve for x : $3^{5x} = 8$

- A. $x = \frac{\log 8}{5 \log 3}$
- B. $x = \frac{\log 3}{5 \log 8}$
- C. $x = \frac{\log 8}{\log 3} - 5$
- D. $x = \frac{\log 3}{\log 8} - 5$

19. For which of the following functions is $f(x) = f^{-1}(x)$, where $f^{-1}(x)$ is the inverse function of $f(x)$?

- A. $f(x) = x^2$
- B. $f(x) = \frac{1}{x}$
- C. $f(x) = |x|$
- D. $f(x) = \log x$

20. If $\log_x 3 = a$ and $\log_x 49 = b$, express $\log_x \left(\frac{9}{7} \right)$ in terms of a and b .

- A. $\frac{a^2}{\sqrt{b}}$
- B. $a^2 - \sqrt{b}$
- C. $2a - \sqrt{b}$
- D. $2a - \frac{1}{2}b$

21. Which of the following is an arithmetic sequence?

A. 1, 3, 5, 7

B. 1, 2, 4, 8

C. 1, 3, 5, 8

D. 1, $\sqrt{2}$, 2, $\sqrt{8}$

22. Given $t_n = \frac{2(n+2)^2}{(n-2)^2}$, find t_4 .

A. 6

B. 12

C. 18

D. 36

23. When two geometric means are inserted between 9 and 243, which of the following is one of them?

A. 78

B. 81

C. 87

D. 165

24. Find the sum of the first 20 terms of the geometric sequence 125, 100, 80, 64, ...

A. -2 250

B. 374.89

C. 617.79

D. 42 868.09

25. What is the 3rd term of the sequence given by the following recursive definition?

$$t_1 = \frac{2}{3}$$

$$t_n = \frac{3t_{n-1}}{n-1}, \quad n > 1$$

- A. $\frac{3}{4}$
- B. 1
- C. 2
- D. 3

26. In an arithmetic sequence, $t_5 = 4$ and $t_{17} = 100$. Find the 1st term.

- A. -36
- B. -28
- C. -8
- D. $-\frac{92}{3}$

27. For what values of x , $x \neq \frac{1}{2}$, will the following infinite geometric series have a finite sum?

$$1 + (2x - 1) + (2x - 1)^2 + (2x - 1)^3 + \dots$$

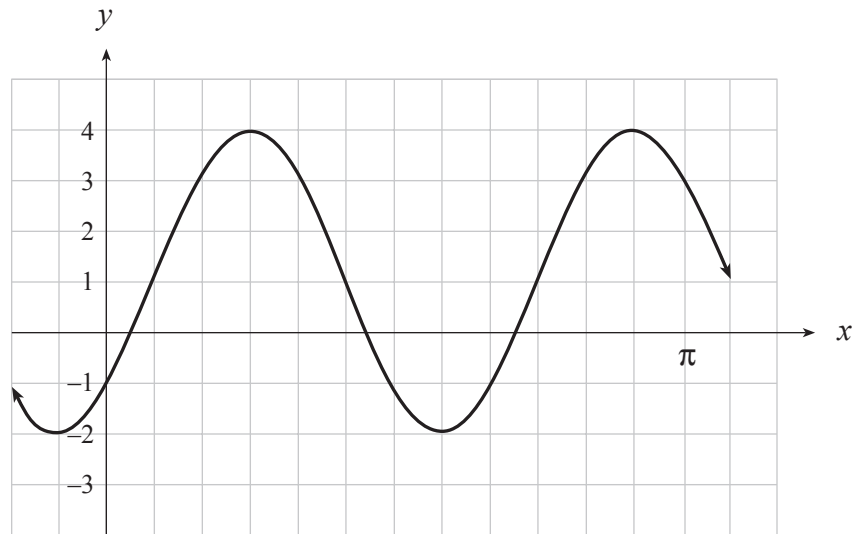
- A. $-1 < x < 0$
- B. $-1 < x < 1$
- C. $-\frac{1}{2} < x < \frac{3}{2}$
- D. $0 < x < 1$

28. Evaluate: $\sum_{k=2}^4 \log_6 k$

- A. 0.60
- B. 1.23
- C. 1.77
- D. 4.00

Use the following graph to answer questions 29 to 31.

The equation of the graph is $y = a\cos b(x - c) + d$, where a , b , c and d are all positive.



29. In the graph above, determine the value of d .

- A. 1
- B. 2
- C. 3
- D. 4

30. In the graph above, determine the value of a .

- A. 1
- B. 2
- C. 3
- D. 4

31. In the graph above, determine the value of b .

- A. $\frac{2}{3}$
- B. 3
- C. 4
- D. 8

32. Convert 3 radians to degrees. (Accurate to the nearest degree.)

- A. 150°
- B. 172°
- C. 180°
- D. 540°

33. Evaluate: $\csc \frac{2\pi}{7}$

- A. -1.00
- B. 0.90
- C. 1.28
- D. 1.60

34. Solve: $\cot \theta = -3$, $0 \leq \theta < 2\pi$

- A. 2.34, 5.48
- B. 2.80, 5.94
- C. 2.82, 5.94
- D. 2.82, 5.96

35. Solve: $\tan \theta - \cos^2 \theta = \frac{1}{2}$, $0 \leq \theta < 2\pi$

- A. 0.36, 3.50
- B. 0.79, 3.93
- C. 0.86, 2.74
- D. 0.88, 2.94, 3.26, 3.74

36. Simplify $\sin A \cos B + \cos A \sin B$ if $A = \frac{\pi}{2} - B$.

- A. -1
- B. 0
- C. 1
- D. $\frac{\pi}{2}$

37. The height of a piston in an engine can be determined by the function $h = 20 \sin \frac{\pi t}{0.025} + 20$, where height, h , is in centimetres, and time, t , is in seconds. Determine the period of this function.

- A. 0.025
- B. 0.05
- C. 0.25
- D. 0.5

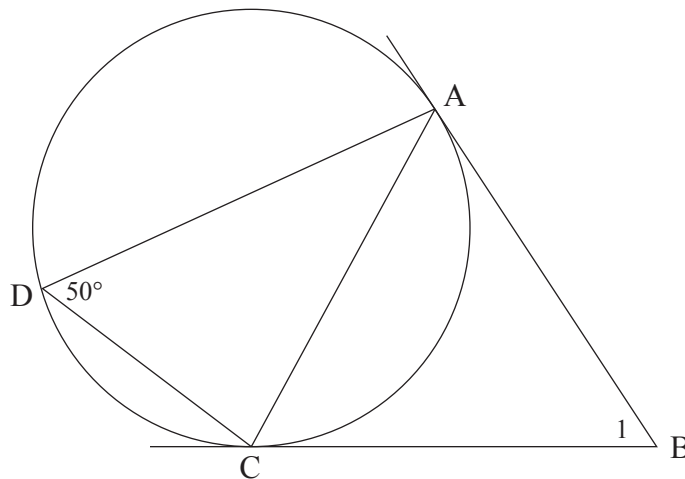
38. Determine the number of solutions for:

$$\csc \theta(2 \sec \theta + 1) = 0, \quad 0 \leq \theta < 2\pi$$

- A. 0
- B. 2
- C. 3
- D. 4

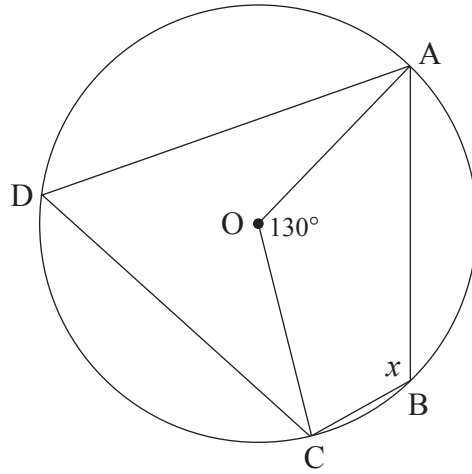
For questions 39 to 42, diagrams are not drawn to scale.

39. In the diagram below, BA and BC are tangents. Determine the measure of $\angle 1$.



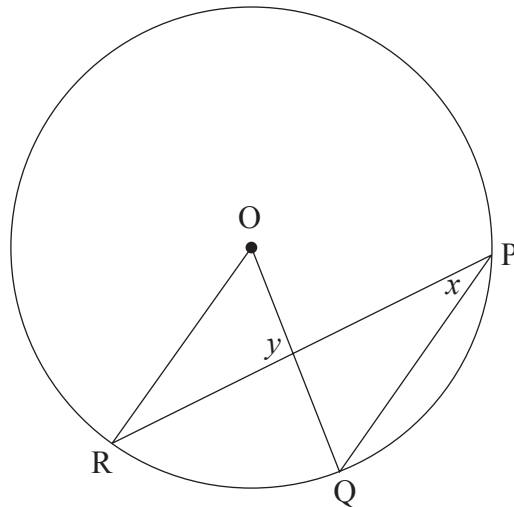
- A. 50°
- B. 60°
- C. 65°
- D. 80°

40. Given a circle with centre O and $\angle AOC = 130^\circ$, determine the measure of $\angle x$.



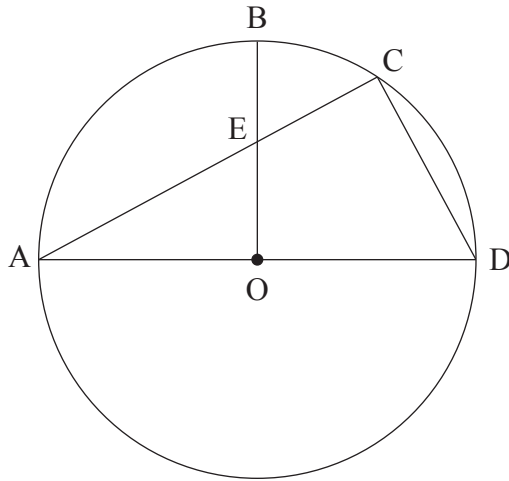
- A. 50°
- B. 65°
- C. 115°
- D. 130°

41. In the diagram below, O is the centre of the circle and $OR \parallel PQ$. Determine an expression for y in terms of x .



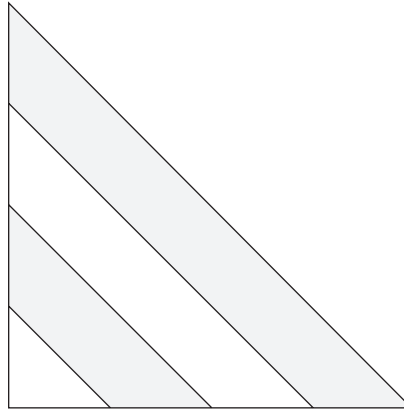
- A. $2x$
- B. $3x$
- C. $180^\circ - 2x$
- D. $180^\circ - 3x$

42. In a circle with centre O , AD is a diameter, $OB \perp AD$, $AD = 13$ and $CD = 5$. Determine the length OE .



- A. 2.33
B. 2.50
C. 2.60
D. 2.71
43. Determine the product of all of the positive divisors of 72.
- A. 72^5
B. 72^6
C. 72^{10}
D. 72^{12}

44. In the diagram below, what fraction of the entire figure is shaded? All bands are of equal width.



- A. $\frac{1}{2}$
B. $\frac{3}{5}$
C. $\frac{5}{8}$
D. $\frac{2}{3}$
45. A farmer has 40 m of fencing to enclose a rectangular pasture. One side of the pasture must include part of one side of his building. If the side of the building is 60 m, determine the maximum area the farmer can enclose.
- A. 100 m^2
B. 200 m^2
C. 400 m^2
D. 625 m^2

**This is the end of the multiple-choice section.
Answer the remaining questions directly in this examination booklet.**

PART B: WRITTEN RESPONSE

Value: 25 marks

Suggested Time: 45 minutes

INSTRUCTIONS: Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

If, in a justification, you refer to information produced by the calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem, it is important to sketch the graph, showing its general shape and indicating the appropriate window dimensions.

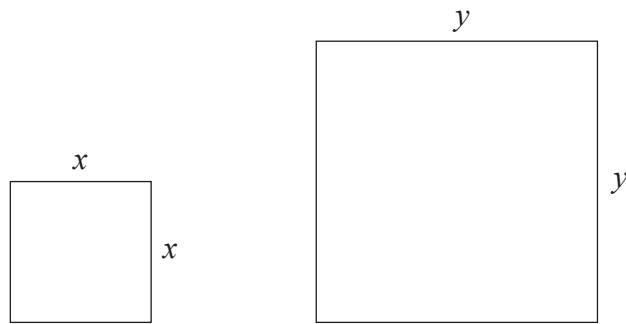
When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.

Full marks will NOT be given for the final answer only.

1. Solve the following problem algebraically.

(3 marks)

The sum of the areas of two separate squares is 234 cm^2 . The sum of their perimeters is 72 cm. Determine the measure of the sides of each square.



ANSWER:

2. When $2x^3 - 8x^2 + kx + 18$ is divided by $x + 2$, the remainder is -14 . Find k , then use a graphing calculator to find all real roots of $2x^3 - 8x^2 + kx + 18 = 0$. (**Note:** It is not necessary to show the viewing window.) **(3 marks)**

ANSWER:

$k =$

roots:

OVER

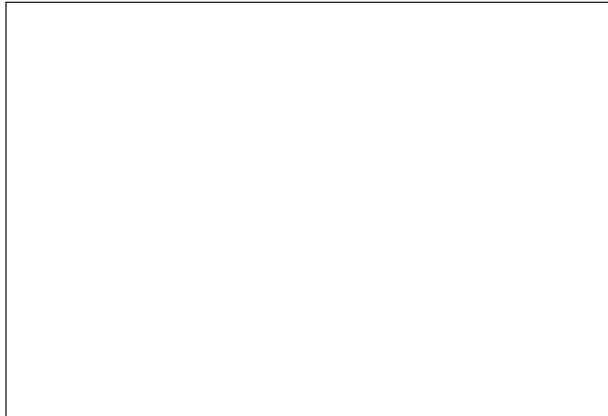
3. Solve the following system using a graphing calculator.

(3 marks)

$$y = \log_2(x + 4)$$

$$y = 2^{x+1} - 3$$

Sketch the graph in the viewing window below. State the function(s) that you entered to obtain your graph and your solution. Indicate the dimensions of the viewing window that will show enough of the graph so that recognizable characteristics of the function(s) and all intersection points are visible.



$Y_1 =$

$Y_2 =$

$Y_3 =$

$Y_4 =$

[,] [,]

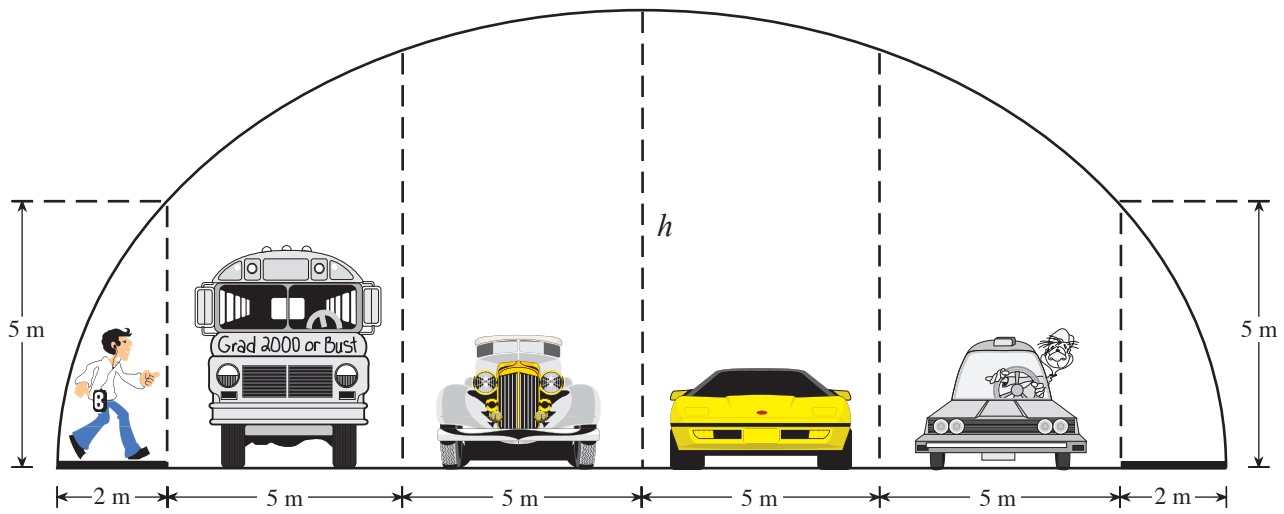
x
min max

y
min max

ANSWER:



4. A tunnel is semi-elliptical in shape. It has four traffic lanes, each 5 m wide, as well as two service walkways, each 2 m wide, as shown in the diagram. The tunnel has a height of 5 m at the edge of the roadway. Determine the height, h , of the tunnel at its highest point. **(3 marks)**



ANSWER:



5. Prove the identity:

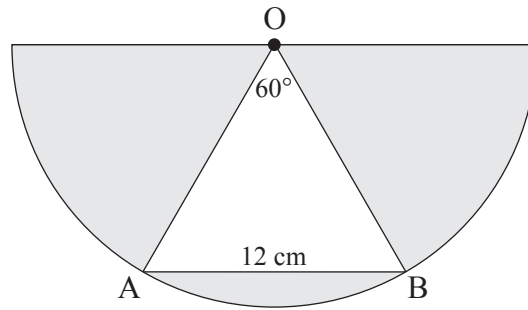
(3 marks)

$$\frac{1}{\sec \theta + \tan \theta} = \frac{1 - \sin \theta}{\cos \theta}$$

LEFT SIDE

RIGHT SIDE

6. In a semi-circle with centre O, $\angle AOB = 60^\circ$ and $AB = 12$ cm. Determine the area of the shaded region. **(3 marks)**



ANSWER:



7. Solve for y and state all restrictions on x and y .

(3 marks)

$$\frac{1}{\log_y 3} = \log_{\frac{1}{3}} 27 + 2 \log_3 x$$

ANSWER:



Students should choose one or the other method of proof.

8. Complete the proof.

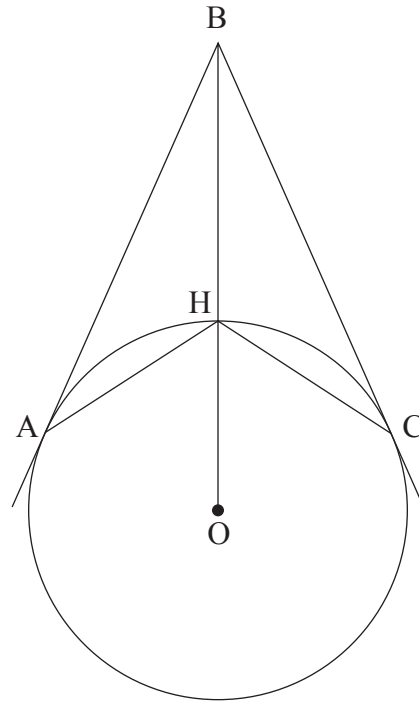
(4 marks)

Diagram clarification: O is the centre
B, H, O are collinear

Given: AB and CB are tangents

Prove: $AH = CH$

Note: Students are encouraged
to number angles.



Paragraph proof method:

Two-column proof method:

STATEMENT	REASON

END OF EXAMINATION

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A SUMMARY OF BASIC IDENTITIES AND FORMULAE

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Reciprocal and Quotient Identities

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Addition Identities

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

Double-Angle Identities

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - 2 \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Formulae

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t_n = a + (n-1)d$$

$$t_n = ar^{n-1}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$S_n = \frac{n}{2}(a + \ell)$$

$$S_n = \frac{a - \ell r}{1-r}$$

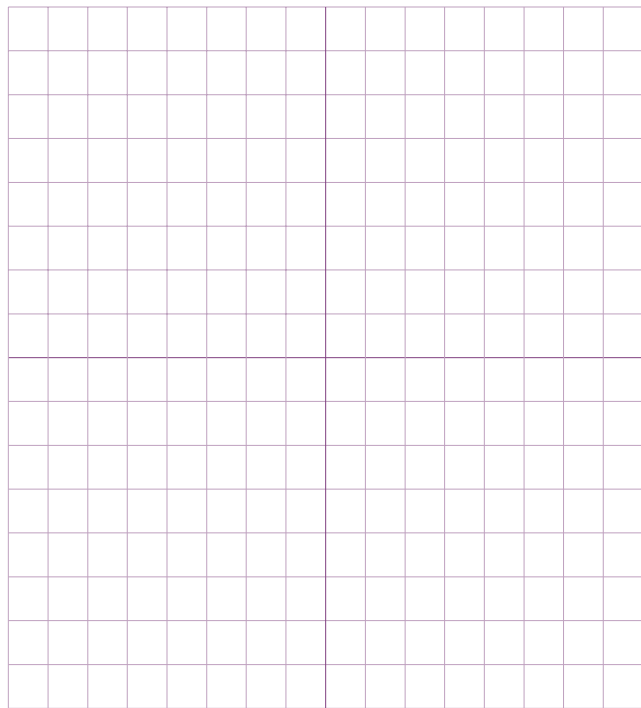
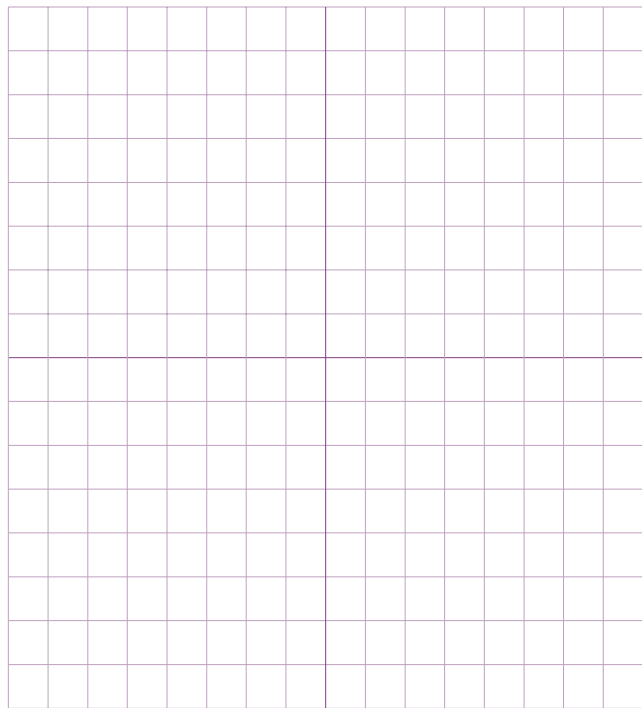
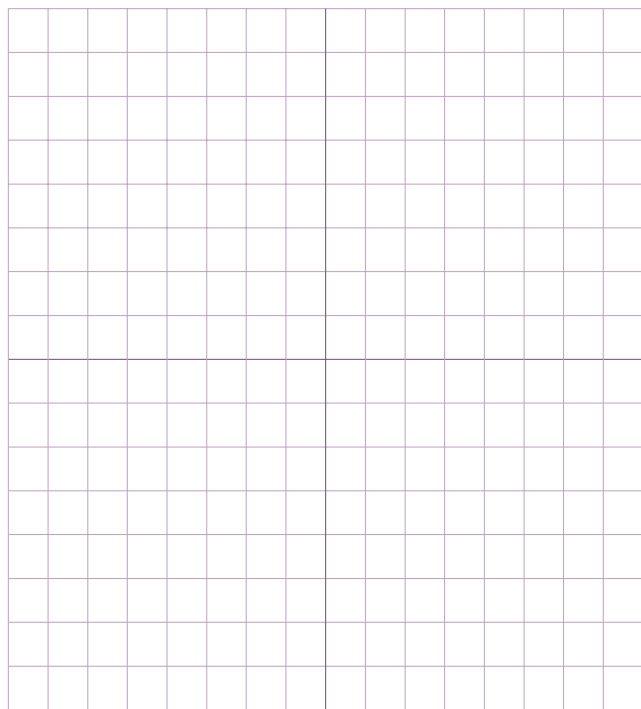
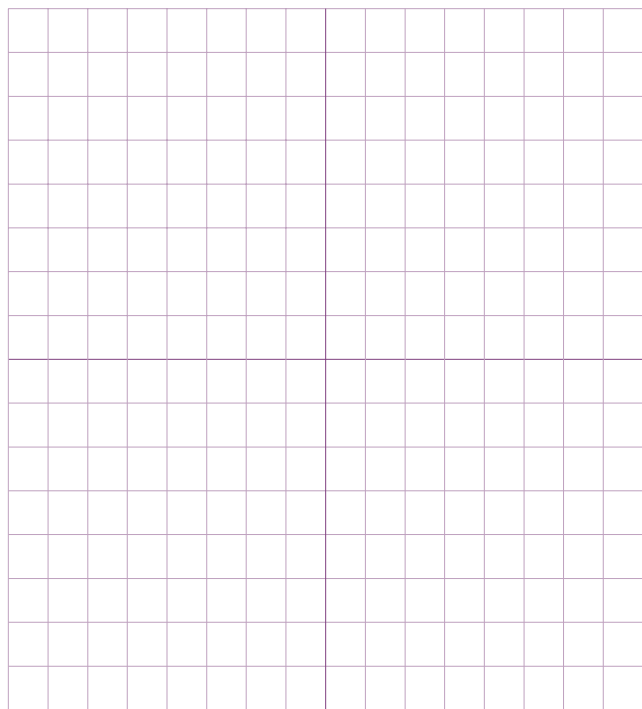
$$S = \frac{a}{1-r}$$

**You may detach this page for convenient reference.
Exercise care when tearing along perforations.**

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ROUGH WORK FOR GRAPHING

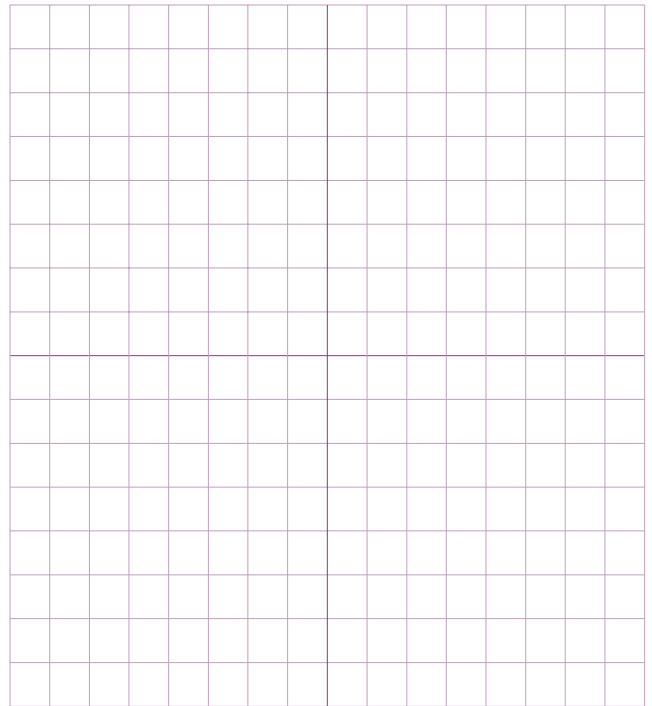
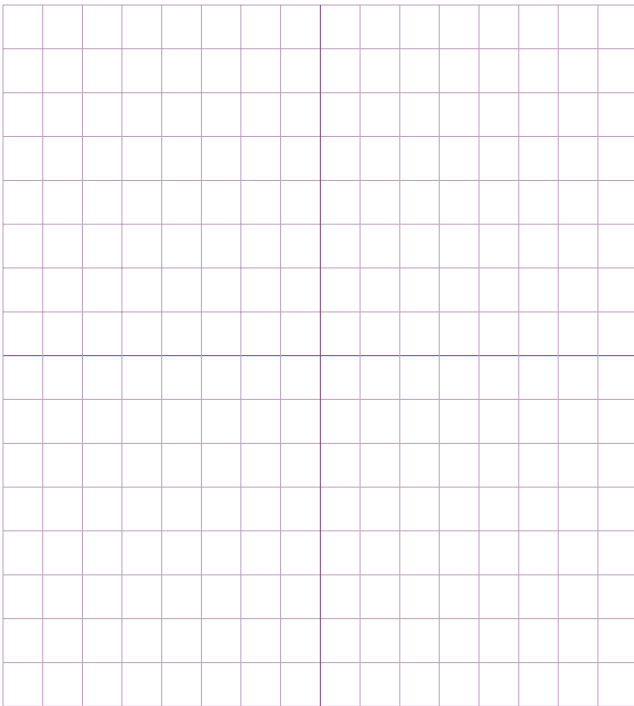
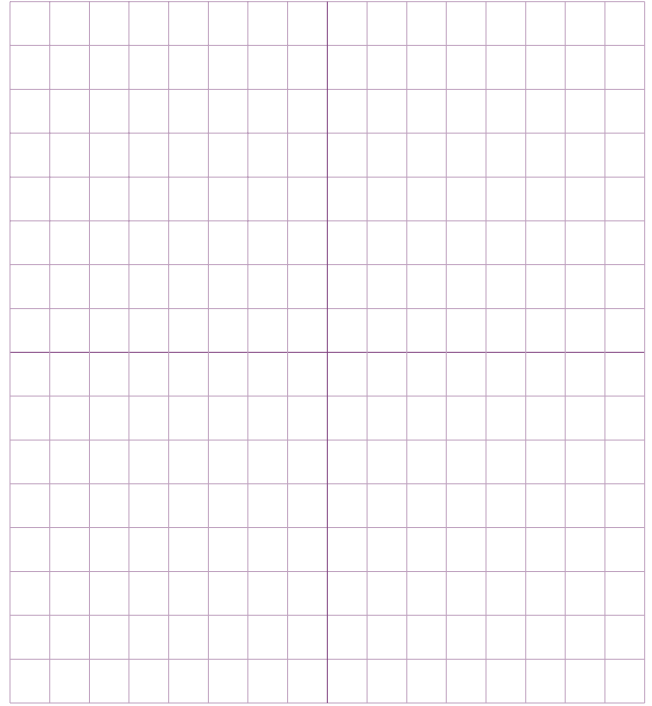
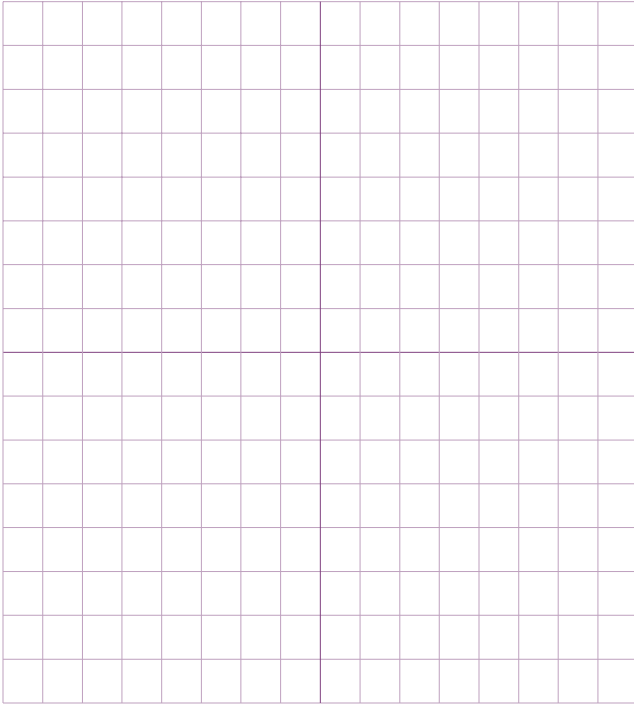
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ROUGH WORK FOR GRAPHING

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ROUGH WORK FOR MULTIPLE-CHOICE

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ROUGH WORK FOR MULTIPLE-CHOICE