

**APRIL 2000**

## **PROVINCIAL EXAMINATION**

**MINISTRY OF EDUCATION**

# **PRINCIPLES OF MATHEMATICS 12**

### **GENERAL INSTRUCTIONS**

1. Insert the stickers with your Student I.D. Number (PEN) in the allotted spaces above and on the **back** cover of this booklet. **Under no circumstance is your name or identification, other than your Student I.D. Number, to appear on this booklet.**
2. Ensure that in addition to this examination booklet, you have an **Examination Response Form**. Follow the directions on the front of the Response Form.
3. **Disqualification** from the examination will result if you bring books, paper, notes or unauthorized electronic devices into the examination room.
4. All multiple-choice answers must be entered on the Response Form using an **HB pencil**. Multiple-choice answers entered in this examination booklet will **not** be marked.
5. For each of the written-response questions, write your answer in the space provided in this booklet.
6. When instructed to open this booklet, **check the numbering of the pages** to ensure that they are numbered in sequence from page one to the last page, which is identified by

**END OF EXAMINATION**.

7. At the end of the examination, place your Response Form inside the front cover of this booklet and return the booklet and your Response Form to the supervisor.

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## PRINCIPLES OF MATHEMATICS 12 PROVINCIAL EXAMINATION

- |   | Value                  | Suggested Time     |
|---|------------------------|--------------------|
| 1. This examination consists of <b>two</b> parts: |                        |                    |
| PART A: 45 multiple-choice questions              | 45                     | 75                 |
| PART B: 8 written-response questions              | 25                     | 45                 |
|   | <b>Total: 70 marks</b> | <b>120 minutes</b> |
- Aside from an approved calculator, electronic devices including dictionaries and pagers are **not** permitted in the examination room.
  - The last **three** pages inside the back cover contain **A Summary of Basic Identities and Formulae**, **Rough Work for Graphing**, and **Rough Work for Multiple-Choice**. These pages may be detached for convenient reference prior to writing this examination.
  - You will not be provided with any additional paper since rough-work space for the written-response questions has been incorporated into the space allowed for answering each question. You may not need all of the space provided to answer each question.
  - A graphing calculator is essential for the Principles of Mathematics 12 Provincial Examination.** The calculator must be a hand-held device designed primarily for mathematical computations involving logarithmic and trigonometric functions as well as for graphing functions. Computers, calculators with a QWERTY keyboard, and electronic writing pads will not be allowed. Students must not bring any external devices to support calculators such as manuals, printed or electronic cards, printers, memory expansion chips or cards, or external keyboards. Students may have more than one calculator available during the examination of which one may be a scientific calculator. Calculators may not be shared and must not have the ability to either transmit or receive electronic signals. In addition to an approved calculator, students will be allowed to use rulers, compasses, and protractors during the examination.
  - If, in a justification, you refer to information produced by the calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem, it is important to sketch the graph, showing its general shape and indicating the appropriate window dimensions.
  - When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.
  - This examination is designed to be completed in **two hours**. *Students may, however, take up to 30 minutes of additional time to finish.*

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**PART A: MULTIPLE CHOICE**

**Value: 45 marks**

**Suggested Time: 75 minutes**

**INSTRUCTIONS:** For each question, select the **best** answer and record your choice on the Response Form provided. Using an HB pencil, completely fill in the circle that has the letter corresponding to your answer.

1. If  $x - 6$  is a factor of the polynomial  $f(x)$ , then which of the following must be true?

- A.  $f(6) = 0$
- B.  $f(-6) = 0$
- C.  $f(0) = 6$
- D.  $f(0) = -6$

2. According to the Rational Root Theorem, determine all possible rational roots of  $4x^3 - 2x^2 + 5x - 8 = 0$ .

- A.  $\pm 1, \pm 2, \pm 4$
- B.  $\pm 1, \pm 2, \pm 4, \pm 8$
- C.  $\pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{2}, \pm \frac{1}{4}$
- D.  $\pm 1, \pm 2, \pm 4, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm \frac{1}{8}$

3. Determine the number of real roots:  $x^3 - 16x^2 - 85x - 100 = 0$

- A. 0
- B. 1
- C. 2
- D. 3

4. When  $x^4 - kx^3 + kx - 1$  is divided by  $x + 2$  the remainder is 1. Find the value of  $k$ .

A.  $-\frac{5}{2}$

B.  $-\frac{7}{3}$

C.  $-\frac{5}{3}$

D.  $\frac{7}{3}$

5. Determine the remainder when  $5x^3 + 14x^2 + 3$  is divided by  $x^2 + 3x - 1$ .

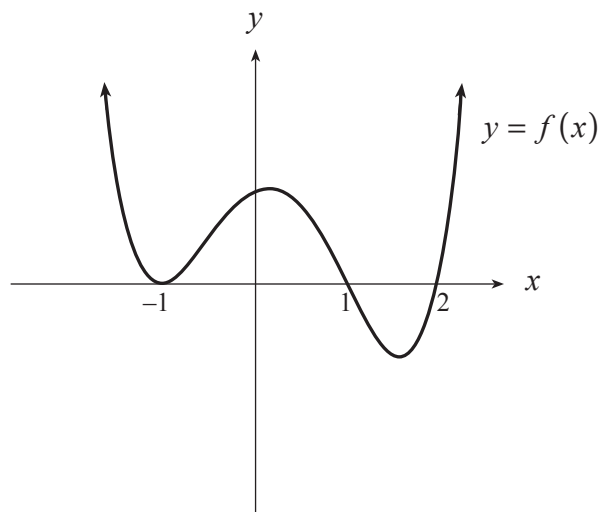
A.  $2x + 2$

B.  $5x - 1$

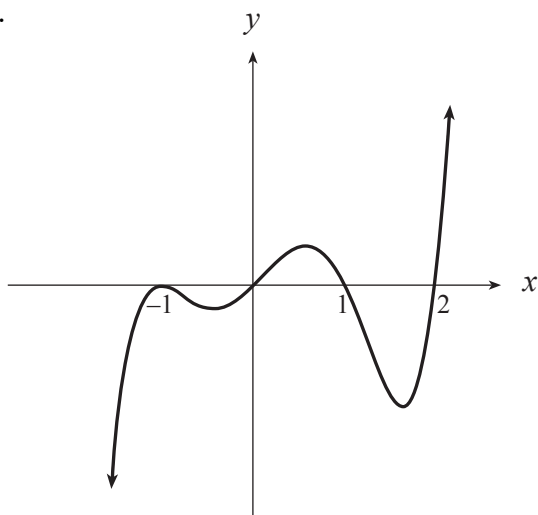
C.  $8x + 2$

D.  $11x - 1$

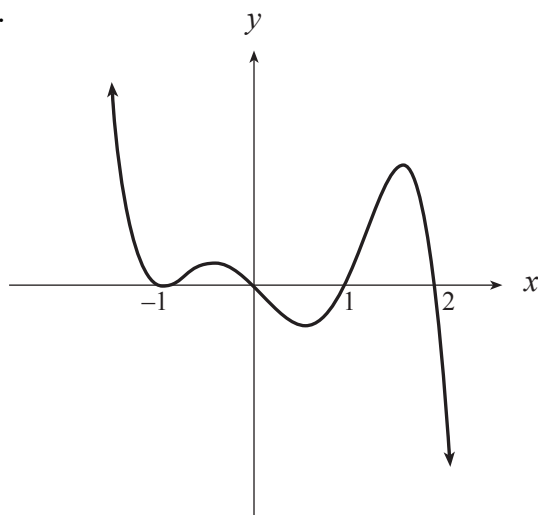
6. Given the graph of the polynomial function  $y = f(x)$  shown below, which of the following could be the graph of the polynomial function  $y = -xf(x)$ ?



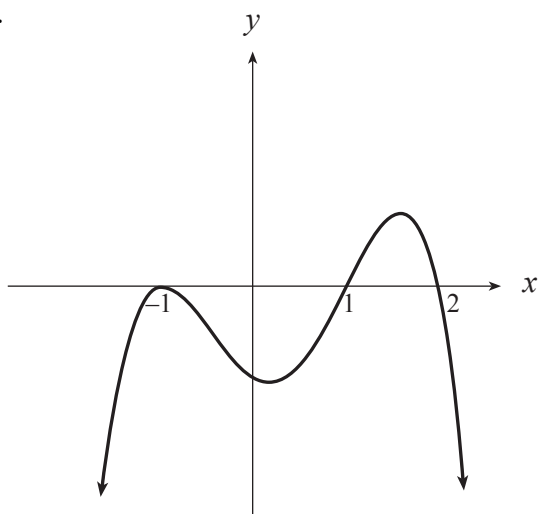
A.



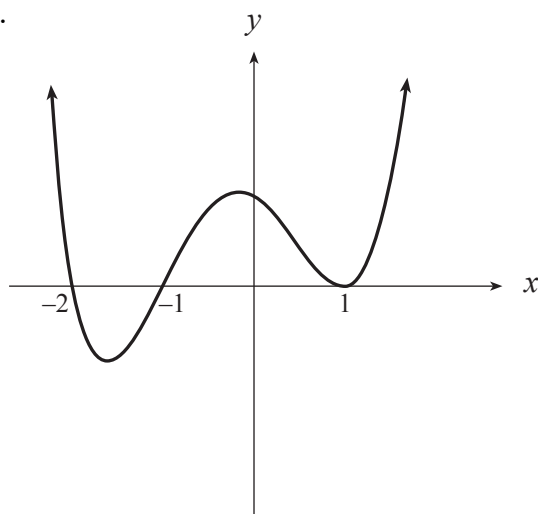
B.



C.



D.



**OVER**

7. Which conic is described by the equation  $x^2 + y = 2$  ?

- A. circle
- B. ellipse
- C. parabola
- D. hyperbola

8. Which absolute value inequality describes the solution shown?



- A.  $|x - 2| < 5$
- B.  $|x + 2| < 5$
- C.  $|x - 5| < 2$
- D.  $|x + 5| < 2$

9. Determine an equation for the set of all points  $P(x, y)$  such that the sum of the distances from  $P$  to  $A(0, 5)$  and from  $P$  to  $B(0, -5)$  is 16.

- A.  $\sqrt{x^2 + (y - 5)^2} + \sqrt{x^2 + (y + 5)^2} = 16$
- B.  $\sqrt{x^2 - (y - 5)^2} + \sqrt{x^2 - (y + 5)^2} = 16$
- C.  $x^2 + (y - 5)^2 + x^2 + (y + 5)^2 = 256$
- D.  $x^2 - (y - 5)^2 + x^2 - (y + 5)^2 = 256$

10. The midpoint of line segment  $AB$  is  $M$ . Find the coordinates of  $B$  given  $A(-4, 6)$  and  $M(2, 4)$ .

- A.  $(-1, 5)$
- B.  $(0, 2)$
- C.  $(2, 8)$
- D.  $(8, 2)$



11. Change  $5x^2 + y^2 - 10x - 20 = 0$  to standard form.

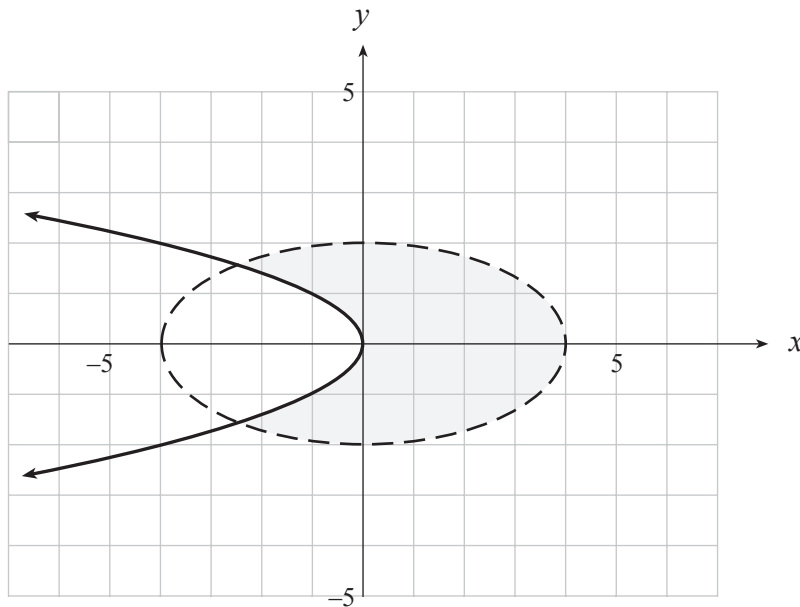
A.  $\frac{(x-1)^2}{4} + \frac{y^2}{20} = 1$

B.  $\frac{(x-1)^2}{5} + \frac{y^2}{25} = 1$

C.  $\frac{(x-1)^2}{6} + \frac{y^2}{30} = 1$

D.  $\frac{(x-1)^2}{\frac{21}{5}} + \frac{y^2}{21} = 1$

12. Which system describes the shaded region shown below?



A.  $\frac{x^2}{16} + \frac{y^2}{4} > 1$   
 $x \geq -y^2$

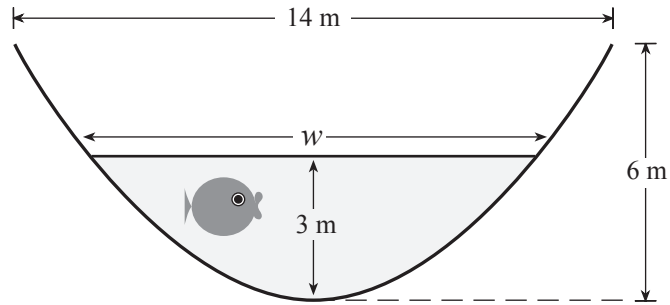
B.  $\frac{x^2}{16} + \frac{y^2}{4} < 1$   
 $x \geq -y^2$

C.  $\frac{x^2}{16} + \frac{y^2}{4} < 1$   
 $x \leq -y^2$

D.  $\frac{x^2}{16} + \frac{y^2}{4} > 1$   
 $x \leq -y^2$

**OVER**

13. A canal has a cross section that is in the shape of a parabola. The width of the canal at the top is 14 m and the maximum depth of the canal is 6 m, as shown in the diagram. The depth of the water at its deepest point is 3 m. Determine the width,  $w$ , of the water surface.



- A. 4.95 m  
 B. 7.86 m  
 C. 9.90 m  
 D. 10.41 m
14. Determine all values for  $k$  such that the following system will have exactly 2 different real solutions.

$$(x-2)^2 - \frac{(y+1)^2}{9} = 1$$

$$x = (y+1)^2 + k$$

- A.  $1 < k < 3$   
 B.  $k < 1$  or  $k > 3$   
 C.  $-1 < k < 5$   
 D.  $k < -1$  or  $k > 5$
15. Solve for  $x$ :  $\log_5 x = 3$

- A.  $\frac{3}{5}$   
 B.  $\frac{5}{3}$   
 C.  $3^5$   
 D.  $5^3$

16. Which of the following expressions is equivalent to  $\log(mn)^2$  ?

- A.  $2 \log m + \log n$
- B.  $(\log m + \log n)^2$
- C.  $\log m + 2 \log n$
- D.  $2 \log m + 2 \log n$

17. Determine an equation of the asymptote of  $y = 2 \log_3(x + 1) + 4$ .

- A.  $x = -1$
- B.  $x = 1$
- C.  $y = -4$
- D.  $y = 4$

18. Solve:  $\log_3(x + 2) - \log_3 5 = 2$

- A.  $-\frac{8}{5}$
- B.  $-\frac{1}{5}$
- C. 12
- D. 43

19. Solve:  $x^2 - 5x = \log_3 x$

- A. 5.20
- B. 5.29
- C. 0.12, 5.20
- D. 0.26, 5.29

20. Given  $f(x) = \frac{x}{x+1}$ , determine  $f^{-1}(x)$ , the inverse of  $f(x)$ .

A.  $f^{-1}(x) = \frac{x}{1-x}$

B.  $f^{-1}(x) = \frac{x}{x-1}$

C.  $f^{-1}(x) = \frac{x+1}{x}$

D.  $f^{-1}(x) = \frac{-x}{x+1}$

21. Given  $a > 1$  and  $\log_a 2 = b$ , which expression is equivalent to  $\log_a(\log_a a^8)$ ?

A.  $b^3$

B.  $b^4$

C.  $3b$

D.  $4b$

22. Determine the number of terms in the series:

$$\sum_{k=5}^{75} (3k-1)$$

A. 70

B. 71

C. 75

D. 76

23. Find the sum of the infinite geometric series:

$$60 - 20 + \frac{20}{3} - \frac{20}{9} + \dots$$

A. 15

B.  $\frac{400}{9}$

C. 45

D. 90

24. Determine the sum of the arithmetic series:

$$5 + 10 + 15 + \dots + 510$$

- A. 25 250
- B. 25 750
- C. 25 755
- D. 26 265

25. If the general term of a sequence is defined as  $t_n = \frac{n^2 + 1}{n}$ , this sequence can be described in which of the following ways?

- A. arithmetic sequence,  $d = 0.5$
- B. geometric sequence,  $r = 1.25$
- C. geometric sequence,  $r = 2.5$
- D. neither geometric nor arithmetic

26. In a geometric sequence, the 9<sup>th</sup> term is  $-5$  and the 12<sup>th</sup> term is  $40$ . Determine the common ratio of this sequence.

- A.  $-8$
- B.  $-2$
- C.  $-\frac{1}{2}$
- D.  $-\frac{1}{8}$

27. Determine the 3<sup>rd</sup> term of the sequence given by the following recursive definition:

$$t_1 = 2$$

$$t_n = nt_{n-1} + n(n-1), \quad n > 1$$

- A. 6
- B. 14
- C. 21
- D. 24

28. The first 3 terms of an arithmetic sequence are  $3m - 1$ ,  $4m + 1$ ,  $5m + 3$ . Determine the 25<sup>th</sup> term.

- A.  $27m + 23$
- B.  $27m + 47$
- C.  $28m + 24$
- D.  $28m + 49$

29. Convert  $210^\circ$  to radians.

- A. 0.37
- B. 0.52
- C. 3.67
- D. 4.19

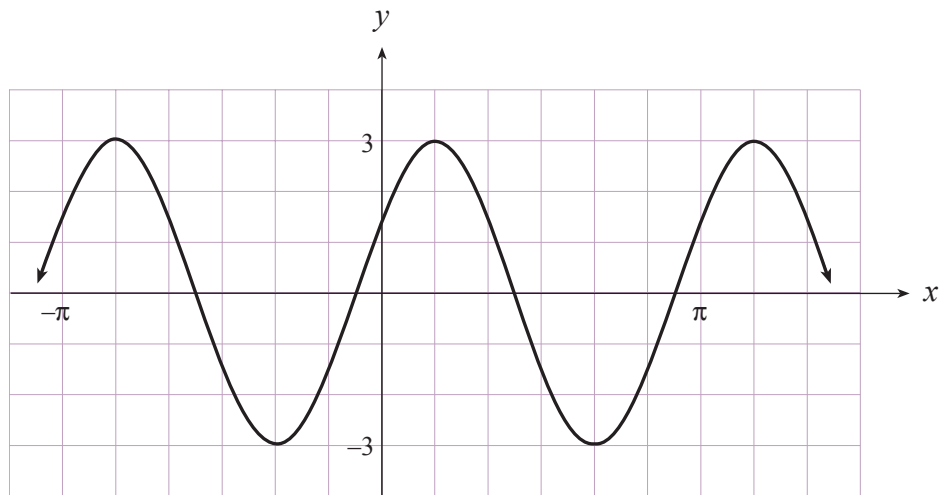
30. Determine the amplitude of the function  $y = -5 \cos 4x$ .

- A. -5
- B. -4
- C. 4
- D. 5

31. What is the period of the function  $y = \sin \frac{1}{2}x$  ?

- A.  $\frac{\pi}{2}$
- B.  $\pi$
- C.  $2\pi$
- D.  $4\pi$

32. Determine an equation of the cosine function graphed below.



- A.  $y = 3 \cos 2\left(x - \frac{\pi}{6}\right)$
- B.  $y = 3 \cos 2\left(x + \frac{\pi}{6}\right)$
- C.  $y = 3 \cos 2(x - 1)$
- D.  $y = 3 \cos 2(x + 1)$

33. Solve:  $\sec x = -5.2$ ,  $0 \leq x < 2\pi$

- A. 1.76
- B. 2.13
- C. 1.76, 4.52
- D. 2.13, 5.28

34. Solve:  $3 \cos x - \sin x = 2$ ,  $0 \leq x < 2\pi$

- A. 0.56, 5.08
- B. 0.58, 4.93
- C. 1.36, 5.70
- D. 1.93, 3.71

35. Determine all restrictions for the expression  $\sin \theta \cot \theta$ .

- A.  $\cos \theta \neq 0$
- B.  $\sin \theta \neq 0$
- C.  $\cos \theta \neq 0, \sin \theta \neq 0$
- D. no restrictions

36. Which expression is equivalent to  $\frac{\cos x}{\sec x} - \cos x \sec x$  ?

- A.  $\cos^2 x$
- B.  $-\cos^2 x$
- C.  $\sin^2 x$
- D.  $-\sin^2 x$

37. Simplify:  $18 \sin 3x \cos 3x$

- A.  $6 \sin 3x$
- B.  $6 \sin 9x$
- C.  $9 \sin 6x$
- D.  $18 \sin 6x$

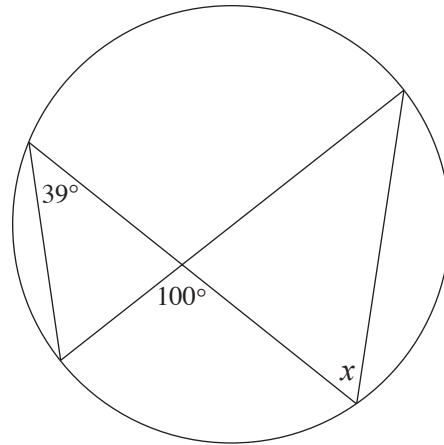
38. Determine  $\tan \theta$  if  $\cos(\pi + \theta) = \sin \theta$ .

- A.  $-1$
- B.  $0$
- C.  $1$
- D. undefined



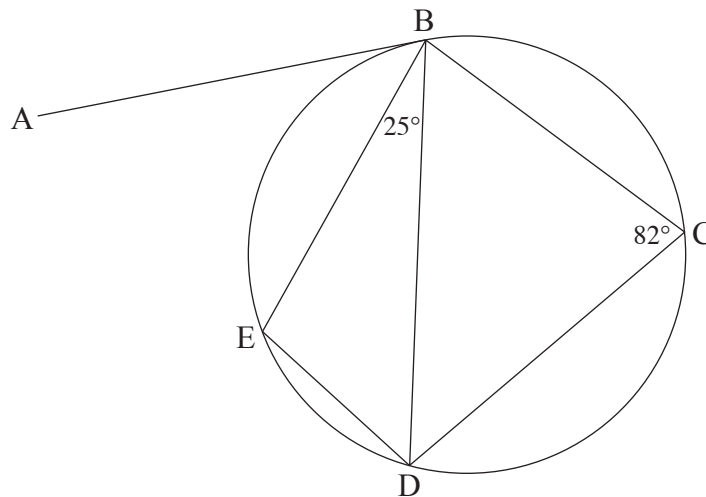
For questions 39 to 42, diagrams are not drawn to scale.

39. In the diagram below, determine the measure of  $\angle x$ .



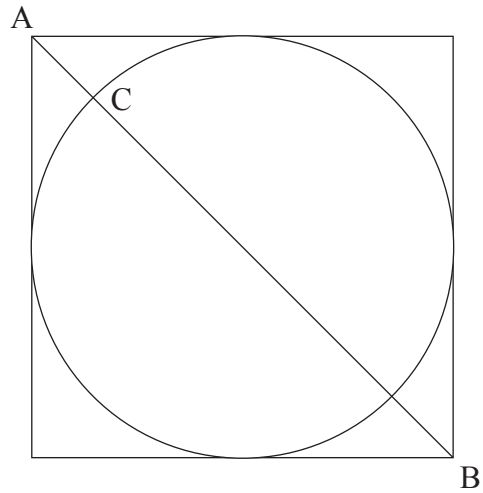
- A.  $39^\circ$
- B.  $51^\circ$
- C.  $61^\circ$
- D.  $141^\circ$

40. In the given circle,  $AB$  is a tangent. Find the measure of  $\angle ABE$ .



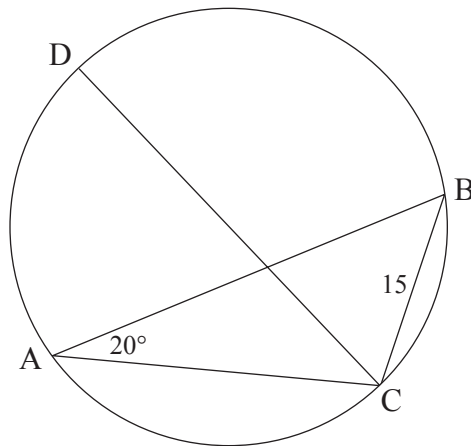
- A.  $53^\circ$
- B.  $57^\circ$
- C.  $73^\circ$
- D.  $82^\circ$

41. In the diagram below, a circle is inscribed in a square with sides of length 4 cm. If  $AB$  is a diagonal of the square and  $AB$  intersects the circle at point  $C$ , determine the length of segment  $AC$ .



- A. 0.83 cm
- B. 0.94 cm
- C. 1.41 cm
- D. 1.66 cm

42. Given a circle with diameter  $DC$ , if  $BC = 15$  and  $\angle BAC = 20^\circ$ , determine the length of  $DC$ .



- A. 31.93
- B. 41.21
- C. 43.86
- D. 46.67

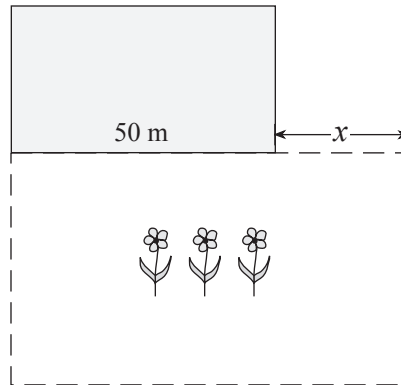
43. Male bees, called drones, hatch from unfertilized eggs; therefore they have a mother but no father. Fertilized eggs hatch into female bees. The female bee, therefore, has both a mother and a father. How many ancestors does the female bee have in the 4<sup>th</sup> generation back?

- A. 5
- B. 8
- C. 13
- D. 16

44. Determine an algebraic expression for  $\tan(\sin^{-1}(x))$ ,  $0 < x < 1$ .

- A.  $\sqrt{1-x^2}$
- B.  $\frac{1}{\sqrt{1-x^2}}$
- C.  $\frac{\sqrt{1-x^2}}{x}$
- D.  $\frac{x}{\sqrt{1-x^2}}$

45. A garden is to be planted beside a 50 m long building, as shown in the diagram. If 200 m of fencing is used to enclose the garden, determine an equation for the area,  $A$ , of the garden in terms of  $x$ .



- A.  $A = (x + 50)(50 - x)$
- B.  $A = (x + 50)(100 - x)$
- C.  $A = (x + 50)\left(\frac{150 - x}{2}\right)$
- D.  $A = (x + 50)(75 - x)$

**This is the end of the multiple-choice section.  
Answer the remaining questions directly in this examination booklet.**

## PART B: WRITTEN RESPONSE

Value: 25 marks

Suggested Time: 45 minutes

**INSTRUCTIONS:** Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

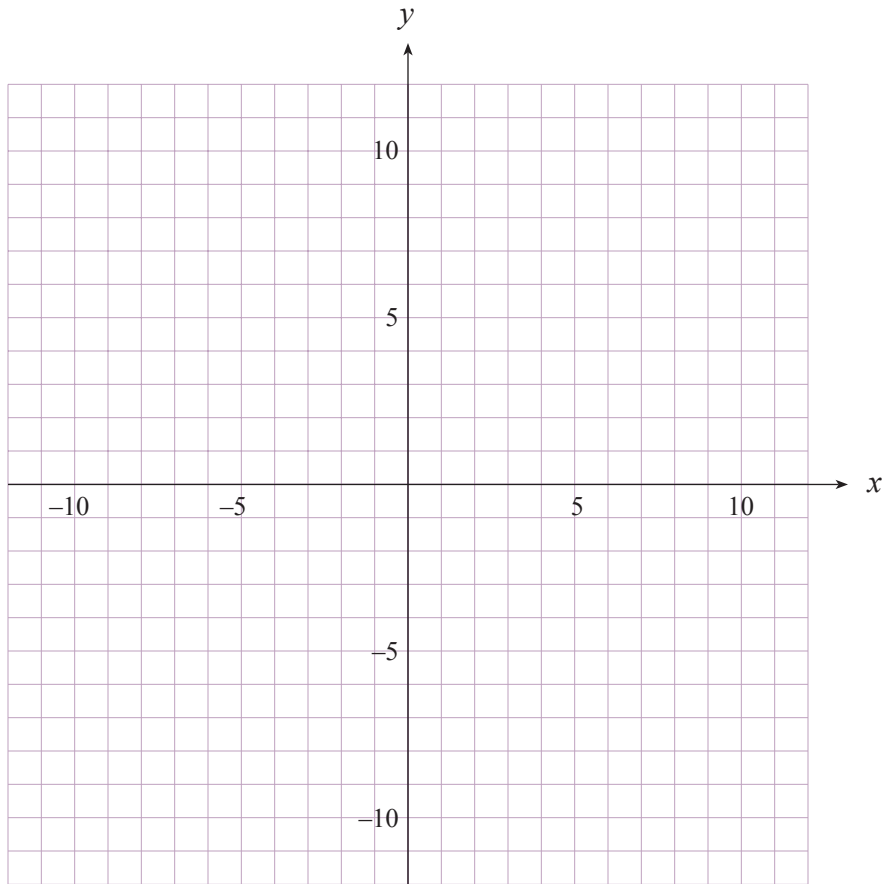
If, in a justification, you refer to information produced by the calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem, it is important to sketch the graph, showing its general shape and indicating the appropriate window dimensions.

When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.

**Full marks will NOT be given for the final answer only.**

1. A hyperbola has vertices at  $(1, -4)$  and  $(1, 8)$ . If the asymptotes have slopes  $\pm 2$ , determine the equation of the hyperbola in standard form. **(3 marks)**

**A grid is provided for rough work only.**



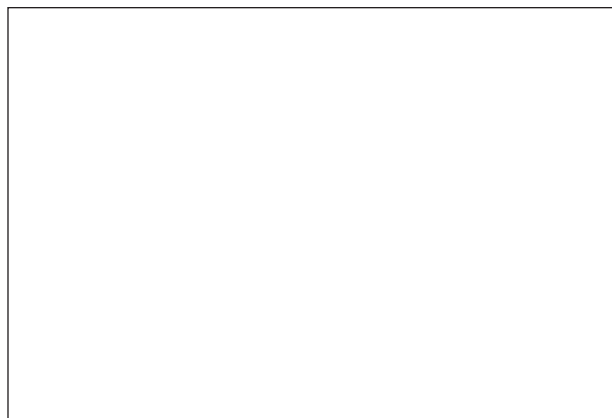
ANSWER:

2. Solve the following inequality using a graphing calculator.

(3 marks)

$$x^3 - 3x^2 - x > 2x - 4$$

Sketch the graph in the viewing window below and indicate appropriate window dimensions. State the function(s) used in your graph. Ensure that the relative maximum and relative minimum points of the function(s) are visible within the viewing window. The solution may be given in algebraic form or shown on a number line.



Y<sub>1</sub> =

Y<sub>2</sub> =

Y<sub>3</sub> =

Y<sub>4</sub> =

[            ,            ]            [            ,            ]

$x$   
min             $x$   
max

$y$   
min             $y$   
max



ANSWER:

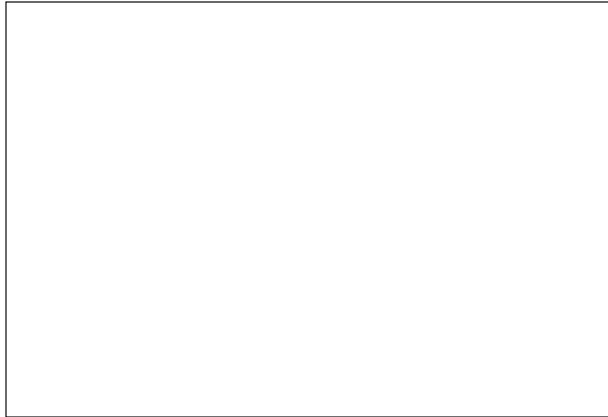
3. Solve the following system using a graphing calculator, when  $0 \leq x < 2\pi$ .

(3 marks)

$$y = \sec x$$

$$y = \sin^2 x + 1.5$$

Sketch the graph in the viewing window below. State the function(s) that you entered to obtain your graph and your solution. Indicate the dimensions of the viewing window that will show enough of the graph so that all intersection points are visible.



$Y_1 =$

$Y_2 =$

$Y_3 =$

$Y_4 =$

[            ,            ]            [            ,            ]

$x$   
min         $x$   
max

$y$   
min         $y$   
max

ANSWER:



4. Determine the dimensions of a rectangle that has an area of  $10 \text{ cm}^2$  and a diagonal of length  $2\sqrt{13} \text{ cm}$ . (Solve algebraically.) **(3 marks)**

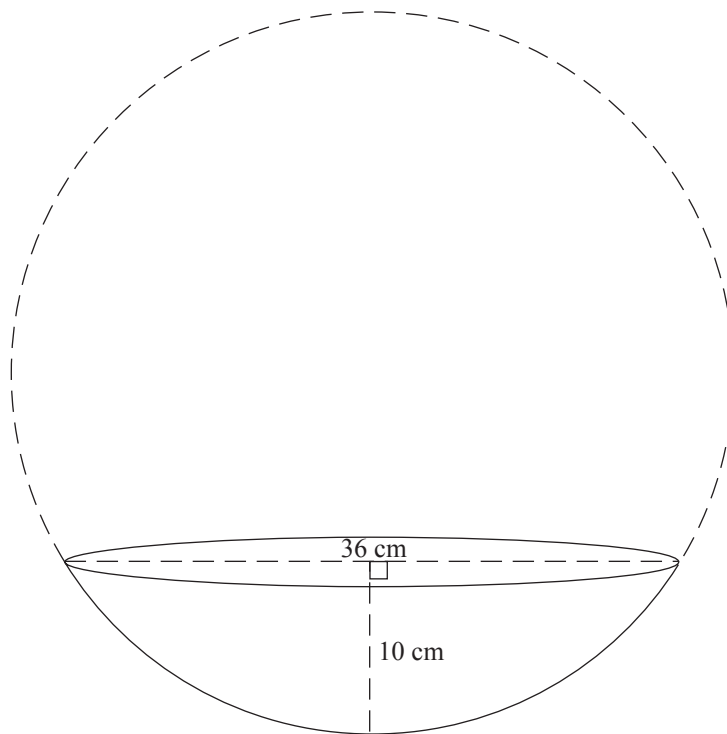
ANSWER:



5. The population of a culture of bacteria doubles every 4 hours. If the present population is 5 000 bacteria, how long will it take for the population to reach 70 000 bacteria? (Answer in hours, accurate to two decimal places.) **(3 marks)**

ANSWER:

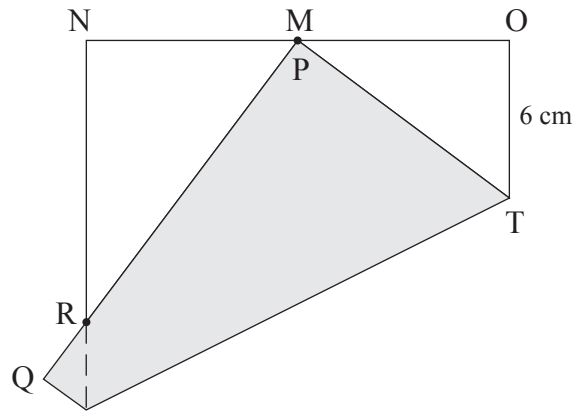
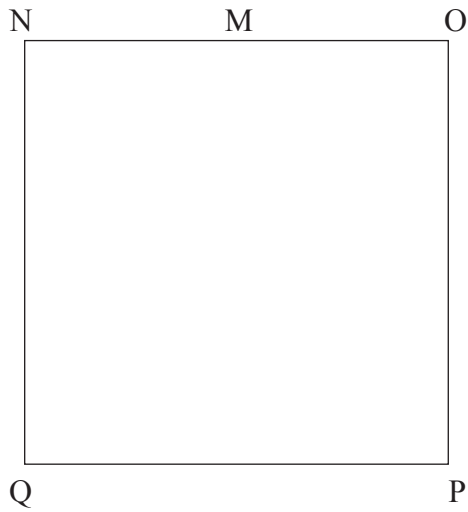
6. A hollow spherical ball has been sliced to form a bowl that measures 36 cm across the centre of the opening as shown in the diagram. If the bowl is 10 cm deep, determine the original radius of the spherical ball. **(3 marks)**





ANSWER:

7. A square piece of paper  $16\text{ cm} \times 16\text{ cm}$  is folded in such a way that the lower right hand corner at  $P$  just touches the midpoint  $M$  of the top side, as shown in the diagram. If  $OT = 6\text{ cm}$ , determine the length of side  $QR$ . **(3 marks)**



ANSWER:



**Students should choose one or the other method of proof.**

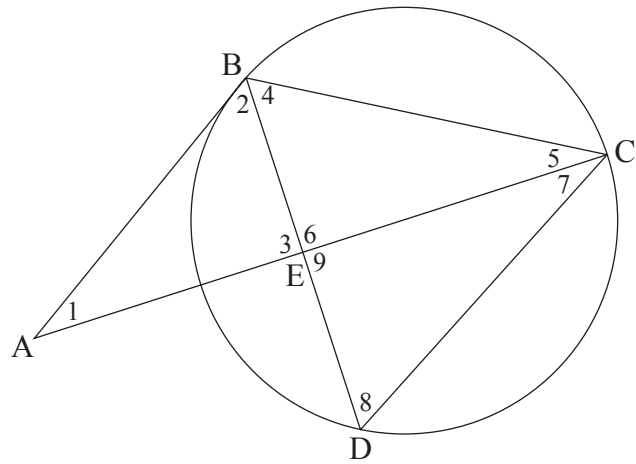
8. Complete the proof.

**(4 marks)**

Diagram clarification: A, E, C are collinear

Given: AB is a tangent  
BD = CD  
AB = BC

Prove:  $\angle 3 = \angle 6$



**Paragraph proof method:**

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**Two-column proof method:**

STATEMENT	REASON

**END OF EXAMINATION**

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## A SUMMARY OF BASIC IDENTITIES AND FORMULAE

### Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

### Reciprocal and Quotient Identities

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

### Addition Identities

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

### Double-Angle Identities

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - 2 \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

### Formulae

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t_n = a + (n-1)d$$

$$t_n = ar^{n-1}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$S_n = \frac{n}{2}(a + \ell)$$

$$S_n = \frac{a - \ell r}{1-r}$$

$$S = \frac{a}{1-r}$$

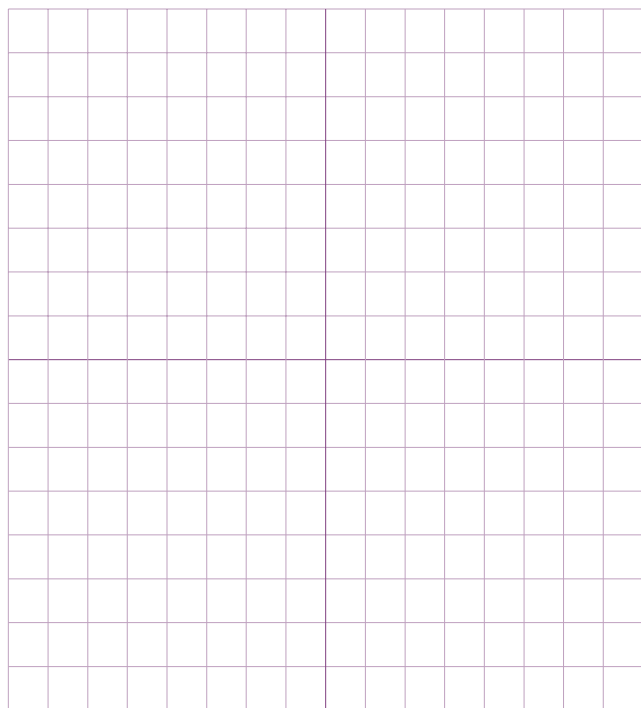
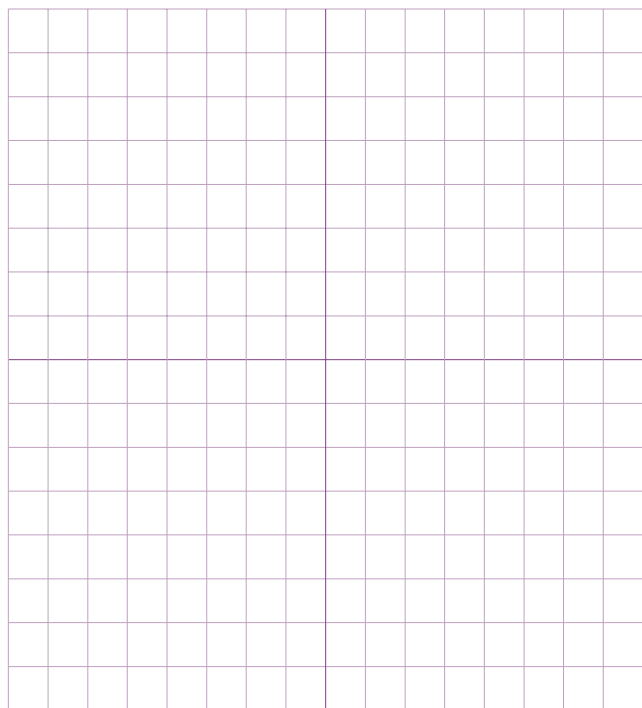
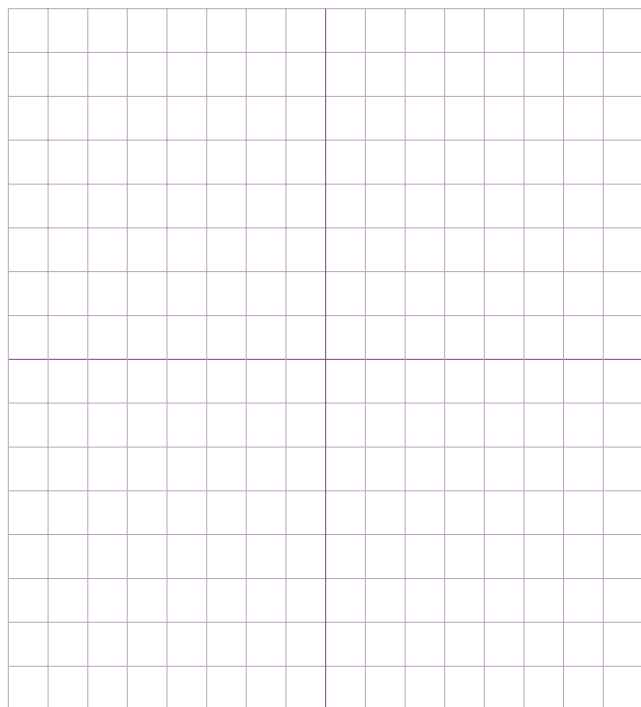
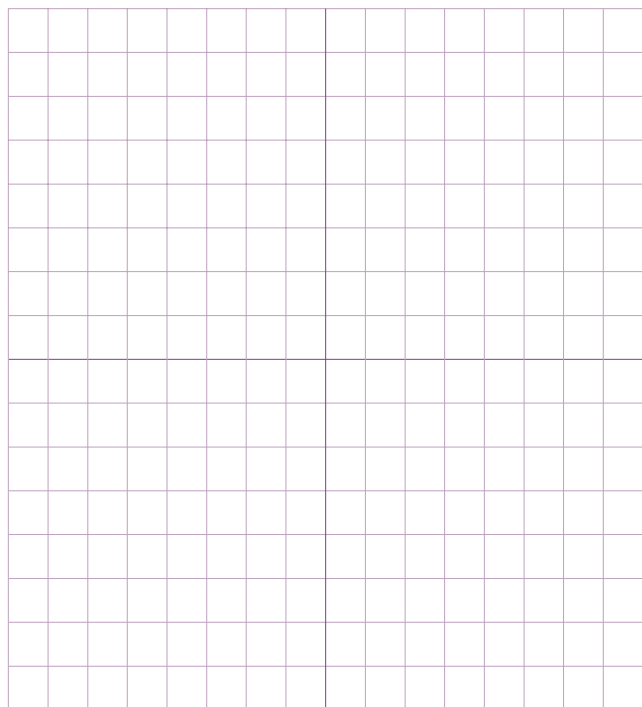
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**ROUGH WORK FOR GRAPHING**

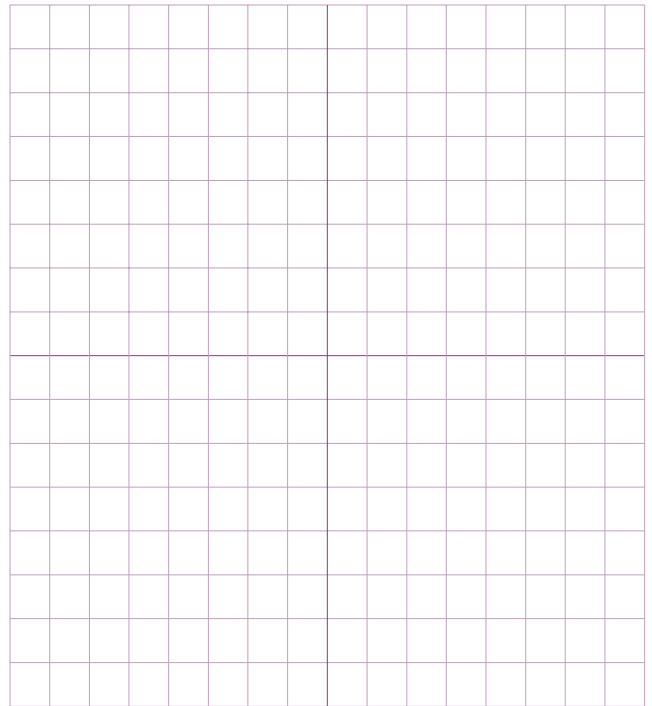
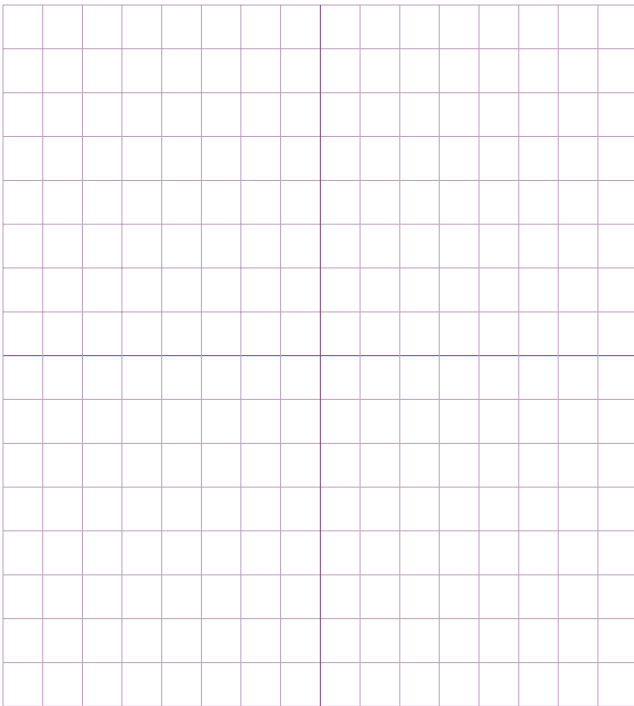
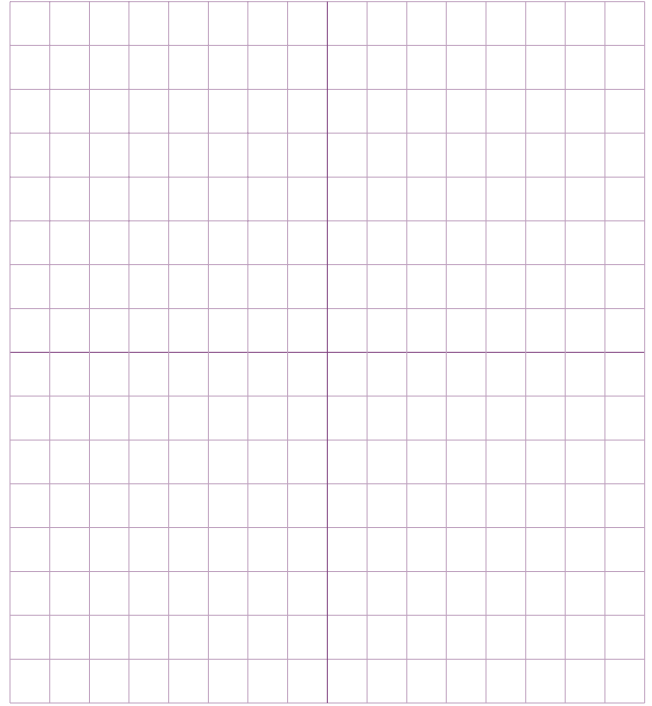
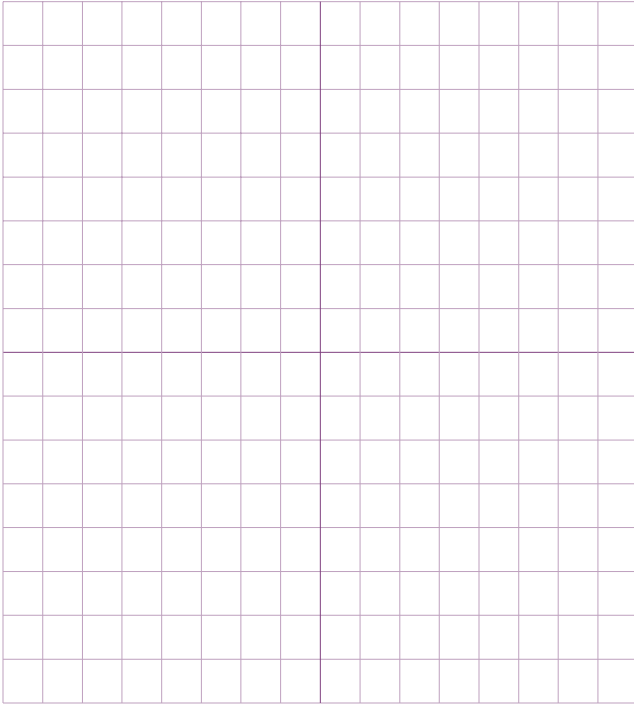
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# ROUGH WORK FOR GRAPHING

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**ROUGH WORK FOR MULTIPLE-CHOICE**

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## ROUGH WORK FOR MULTIPLE-CHOICE

Question 1:

1.

(3)

Question 2:

2.

(3)

Question 3:

3.

(3)

Question 4:

4.

(3)

Question 5:

5.

(3)

Question 6:

6.

(3)

Question 7:

7.

(3)

Question 8:

8.

(4)

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# PRINCIPLES OF MATHEMATICS 12

**April 2000**

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