INSERT STUDENT I.D. NUMBER (PEN)

STICKER IN THIS SPACE

JANUARY 2000

PROVINCIAL EXAMINATION

MINISTRY OF EDUCATION

PRINCIPLES OF MATHEMATICS 12

GENERAL INSTRUCTIONS

1. Insert the stickers with your Student I.D. Number (PEN) in the allotted spaces above and on the **back** cover of this booklet. **Under no circumstance is your name or identification, other than your Student I.D. Number, to appear on this booklet.**

2. Ensure that in addition to this examination booklet, you have an **Examination Response Form**. Follow the directions on the front of the Response Form.

3. **Disqualification** from the examination will result if you bring books, paper, notes or unauthorized electronic devices into the examination room.

4. All multiple-choice answers must be entered on the Response Form using an **HB pencil**. Multiple-choice answers entered in this examination booklet will **not** be marked.

5. For each of the written-response questions, write your answer in the space provided in this booklet.

6. When instructed to open this booklet, **check the numbering of the pages** to ensure that they are numbered in sequence from page one to the last page, which is identified by

   END OF EXAMINATION.

7. At the end of the examination, place your Response Form inside the front cover of this booklet and return the booklet and your Response Form to the supervisor.

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PRINCIPLES OF MATHEMATICS 12 PROVINCIAL EXAMINATION

1. This examination consists of **two** parts:

		Value	Suggested Time
PART A:	45 multiple-choice questions	45	75
PART B:	8 written-response questions	25	45
	Total:	**70 marks**	**120 minutes**

2. Aside from an approved calculator, electronic devices including dictionaries and pagers are **not** permitted in the examination room.

3. The last **three** pages inside the back cover contain **A Summary of Basic Identities and Formulae**, **Rough Work for Graphing**, and **Rough Work for Multiple-Choice**. These pages may be detached for convenient reference prior to writing this examination.

4. You will not be provided with any additional paper since rough-work space for the written-response questions has been incorporated into the space allowed for answering each question. You may not need all of the space provided to answer each question.

5. **A graphing calculator is essential for the Principles of Mathematics 12 Provincial Examination.** The calculator must be a hand-held device designed primarily for mathematical computations involving logarithmic and trigonometric functions as well as for graphing functions. Computers, calculators with a QWERTY keyboard, and electronic writing pads will not be allowed. Students must not bring any external devices to support calculators such as manuals, printed or electronic cards, printers, memory expansion chips or cards, or external keyboards. Students may have more than one calculator available during the examination of which one may be a scientific calculator. Calculators may not be shared and must not have the ability to either transmit or receive electronic signals. In addition to an approved calculator, students will be allowed to use rulers, compasses, and protractors during the examination.

6. If, in a justification, you refer to information produced by the calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem, it is important to sketch the graph, showing its general shape and indicating the appropriate window dimensions.

7. When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.

8. This examination is designed to be completed in **two hours**. *Students may, however, take up to 30 minutes of additional time to finish.*

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PART A: MULTIPLE CHOICE

Value: 45 marks **Suggested Time: 75 minutes**

INSTRUCTIONS: For each question, select the **best** answer and record your choice on the Response Form provided. Using an HB pencil, completely fill in the circle that has the letter corresponding to your answer.

1. According to the Rational Root Theorem, determine all possible rational roots of $5x^3 - 3x^2 + x - 2 = 0$.

   A. $\pm 1,\ \pm 2$

   B. $\pm 1,\ \pm 5$

   C. $\pm 1,\ \pm 5,\ \pm\frac{1}{2},\ \pm\frac{5}{2}$

   D. $\pm 1,\ \pm 2,\ \pm\frac{1}{5},\ \pm\frac{2}{5}$

2. How many different real roots are there for the polynomial equation $x(x-3)\left(x^2+6\right) = 0$?

   A. 1
   B. 2
   C. 3
   D. 4

3. Determine the remainder: $t^2 + 2t - 4 \overline{)3t^3 - 7t^2 - 11t + 20}$

   A. $3t - 13$
   B. $-25t + 24$
   C. $-25t + 72$
   D. $27t - 32$

OVER

4. A cubic polynomial function that passes through the point $(3,\ 24)$ has zeros at $5,\ -1$ and -3. Determine an equation of this function.

A. $y = -2\,(x-5)(x+1)(x+3)$

B. $y = -\frac{1}{2}(x-5)(x+1)(x+3)$

C. $y = \frac{1}{2}(x-5)(x+1)(x+3)$

D. $y = 2\,(x-5)(x+1)(x+3)$

5. Solve the inequality: $-(x-3)(x+2)^2 < 0$

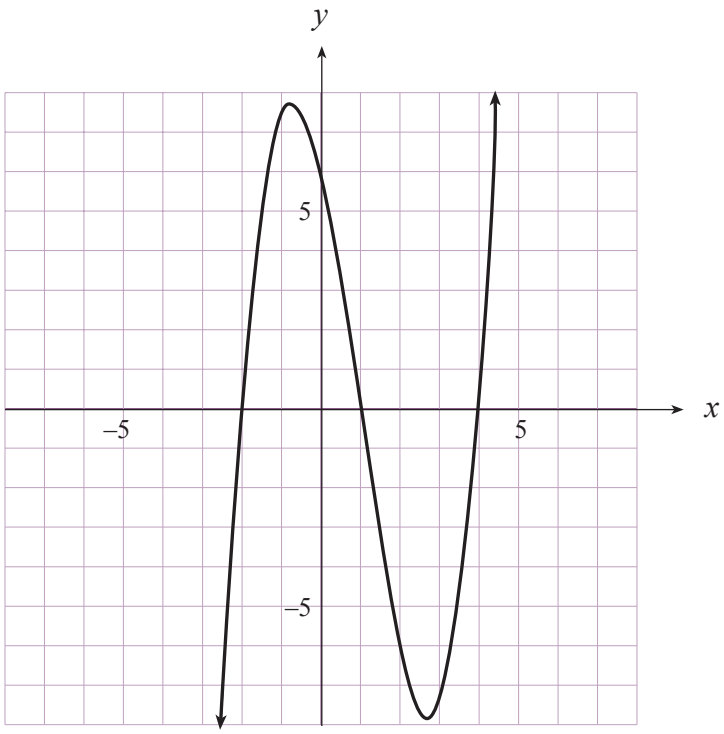
A. 

B. 

C. 

D. 

6. The graph of the polynomial function $y = f(x)$ is shown below. Find the remainder when $f(x)$ is divided by $(x - 2)$.



A. –6
B. 0
C. 1
D. 6

7. Which conic is described by the equation $5x^2 - 3x + 2y - 7 = 0$?

A. circle
B. ellipse
C. parabola
D. hyperbola

8. Determine the centre of the circle which has $(4,\ 10)$ and $(-2,\ 1)$ as endpoints of a diameter.

A. $\left(1,\ \frac{9}{2}\right)$

B. $\left(1,\ \frac{11}{2}\right)$

C. $\left(3,\ \frac{9}{2}\right)$

D. $\left(3,\ \frac{11}{2}\right)$

OVER

9. Change to standard form: $25x^2 + 4y^2 + 50x - 75 = 0$

A. $\frac{(x+1)^2}{2} + \frac{y^2}{12.5} = 1$

B. $\frac{(x+1)^2}{3} + \frac{y^2}{37.5} = 1$

C. $\frac{(x+1)^2}{4} + \frac{y^2}{25} = 1$

D. $\frac{(x+1)^2}{5} + \frac{y^2}{62.5} = 1$

10. Determine an equation of the hyperbola which has asymptotes with slopes $\pm\frac{2}{3}$, and vertices at $(0,\ 2)$ and $(0,\ -2)$.

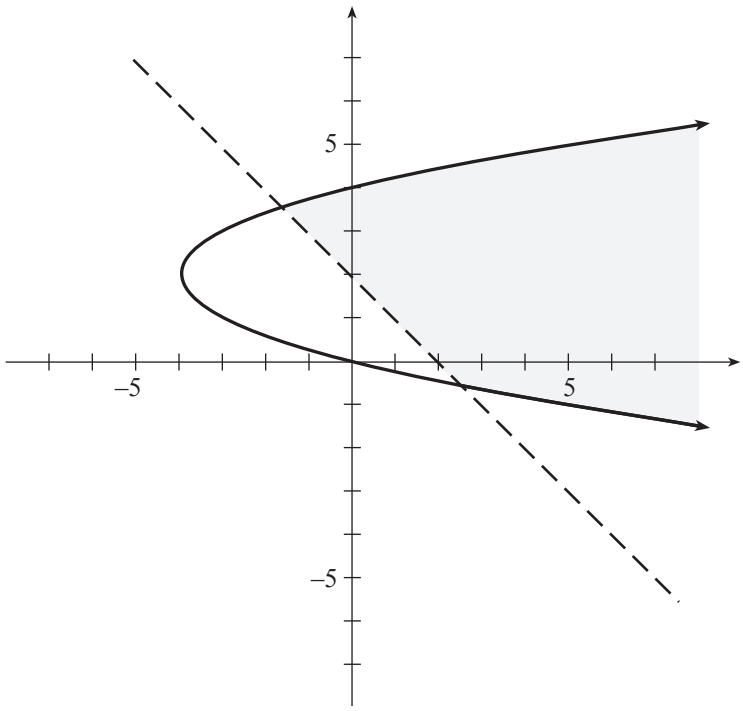
A. $\frac{x^2}{9} - \frac{y^2}{4} = -1$

B. $\frac{x^2}{9} - \frac{y^2}{4} = 1$

C. $\frac{x^2}{4} - \frac{y^2}{9} = -1$

D. $\frac{x^2}{4} - \frac{y^2}{9} = 1$

11. Which system of inequalities is represented by the shaded region shown below?



A. $y > -x + 2$
$x \geq (y-2)^2 - 4$

B. $y < -x + 2$
$x \leq (y-2)^2 - 4$

C. $y > -x + 2$
$x \leq (y-2)^2 - 4$

D. $y < -x + 2$
$x \geq (y-2)^2 - 4$

12. Solve: $|x-5| > -1$

A. $x < -6$ or $x > -4$
B. $x < 4$ or $x > 6$
C. no solution
D. all real numbers

OVER

13. Solve the system:

$$x^2 + y^2 = 14.86$$

$$y = 3x^2 - 18x + 19$$

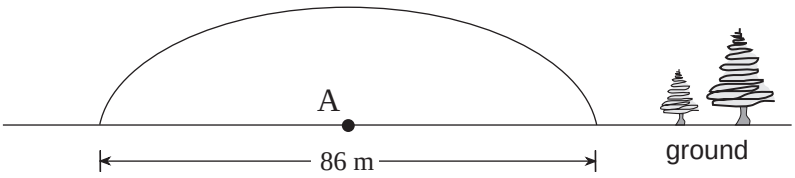
A. $(1.02,\ 3.72)$

B. $(4.92,\ 3.06)$

C. $(1.02,\ 3.72),\ (1.77,\ -3.43)$

D. $(4.92,\ 3.06),\ (4.40,\ -2.12)$

14. The cross section of the roof of an indoor tennis court has a semi-elliptical shape. If the roof spans 86 m and has a maximum height of 30 m, find the height of the roof 20 m from the centre A.



A. 16.05
B. 26.56
C. 27.32
D. 29.18

15. Change $a = b^c$ to logarithmic form.

A. $\log_a b = c$

B. $\log_b c = a$

C. $\log_c a = b$

D. $\log_b a = c$

16. Solve for y: $x = \log c^y$

A. $\dfrac{x}{\log c}$

B. $\log c^x$

C. $\dfrac{x}{c}$

D. $x - \log c$

17. A population of insects doubles every 5 days. If there are currently 1 200 insects, determine an equation for the population, P, of insects t days from now.

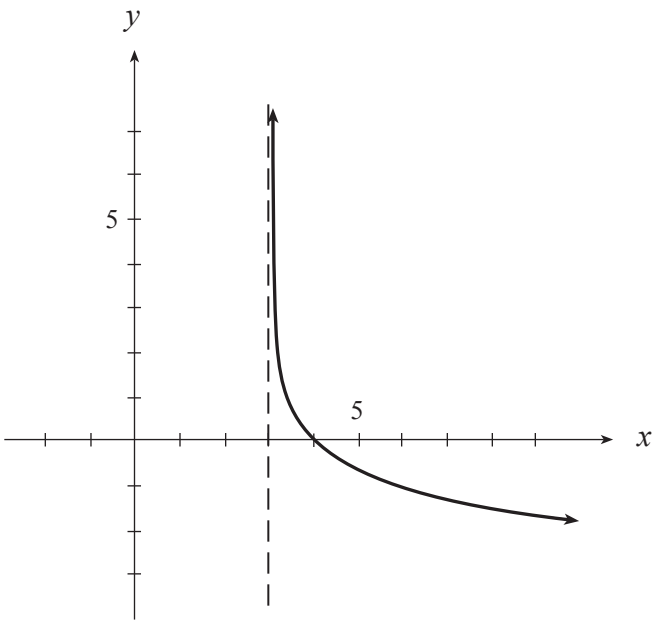
A. $P = 1\,200\left(\frac{1}{2}\right)^{\frac{t}{5}}$

B. $P = 1\,200\left(\frac{1}{2}\right)^{5t}$

C. $P = 1\,200(2)^{5t}$

D. $P = 1\,200(2)^{\frac{t}{5}}$

18. Determine the equation of the logarithmic function graphed below.



A. $y = \log_3(x - 3)$

B. $y = \log_3(x + 3)$

C. $y = -\log_3(x + 3)$

D. $y = -\log_3(x - 3)$

OVER

19. Given $f(x) = 3x + 7$, determine $f^{-1}(x)$, the inverse of $f(x)$.

A. $f^{-1}(x) = 3x - 7$

B. $f^{-1}(x) = 3x - \frac{1}{7}$

C. $f^{-1}(x) = \frac{1}{3}x + \frac{1}{7}$

D. $f^{-1}(x) = \frac{1}{3}x - \frac{7}{3}$

20. If $\log 5 = m$ and $\log 7 = n$, determine $\log \frac{35}{10}$ in terms of m and n.

A. $\frac{mn}{10}$

B. $\frac{m+n}{10}$

C. $m + n - 1$

D. $m + n - 10$

21. Solve: $(\log_2 8)^x - (\log_9 3)^{x+1} = 0$

A. –0.50
B. –0.39
C. –0.33
D. 1.71

22. Determine t_6 for $t_n = \frac{3n+1}{2n-5}$.

A. $-\frac{7}{5}$

B. $\frac{10}{7}$

C. $\frac{19}{7}$

D. $\frac{10}{3}$

23. Determine a formula for t_n in terms of n for the geometric sequence $2, 6, 18, 54, \ldots$

A. $t_n = 2(3)^n$

B. $t_n = 2(3)^{n-1}$

C. $t_n = -1 + 3n$

D. $t_n = 2 + 3n$

24. Determine the 3rd term of the sequence given by the following recursive definition:

$$t_1 = 2$$
$$t_n = 4 - 3t_{n-1}, \quad n > 1$$

A. -4
B. -2
C. 2
D. 10

25. Find the sum of the first 15 terms of the arithmetic series $125 + 100 + 75 + \ldots$

A. -937.5
B. -750
C. -225
D. 4 500

26. Determine two positive geometric means between 64 and 1.

A. 16, 4
B. 32, 16
C. 48, 16
D. 32, 8

27. Solve for x: $\sum_{k=1}^{5} (kx + 1) = 11$

A. 0.4
B. 1.5
C. 1.6
D. 2

OVER

28. In an arithmetic sequence, $t_3 = 3x + 5y$ and $t_7 = 7x + 7y$. Find the common difference, d.

A. $d = x + y$

B. $d = x + \frac{1}{2}y$

C. $d = 4x + 2y$

D. $d = \frac{5}{4}x + \frac{3}{2}y$

29. Convert 5 radians to degrees.

A. 0.09°
B. 286.48°
C. 291.39°
D. 318.31°

30. Evaluate: $\cot 4.47$

A. –0.24
B. 0.23
C. 0.25
D. 4.04

31. State the restriction(s) for $\cot x$.

A. $\sin x \neq 0$
B. $\cos x \neq 0$
C. $\cos x \neq 0,\ \sin x \neq 0$
D. no restriction(s)

32. Simplify: $\sin 5m \cos m + \cos 5m \sin m$

A. $\cos 4m$
B. $\cos 6m$
C. $\sin 4m$
D. $\sin 6m$

33. Determine the maximum value of the function $y = 3\cos 2x - 4$.

A. -2
B. -1
C. 3
D. 7

34. Solve: $2\sin x \cos x + \sin x = 0$, $0 \le x < 2\pi$

A. $0,\ 3.14$
B. $2.09,\ 4.19$
C. $0,\ 2.09,\ 3.14,\ 4.19$
D. $0,\ 2.09,\ 3.14,\ 5.24$

35. Determine $\csc\theta$ if $(-10,\ 24)$ lies on the terminal arm of angle θ in standard position.

A. $-\frac{13}{5}$
B. $-\frac{13}{12}$
C. $\frac{13}{12}$
D. $\frac{13}{5}$

36. Simplify: $\frac{\sin 2x}{1 - \cos 2x}$

A. $\cot x$
B. $\tan x$
C. $2\cot x$
D. $2\tan x$

OVER

37. Solve: $2 - x = \sin^2 x$

A. 1.06
B. 1.16
C. 2.43
D. 1.08, 1.68, 2.42

38. A sine curve has a zero at –2. The nearest zero to the right is at 3. A maximum point is located between these zeros. If the range of the function is $-1 \le y \le 1$, determine an equation of this function.

A. $y = \sin \frac{\pi}{5}(x - 2)$

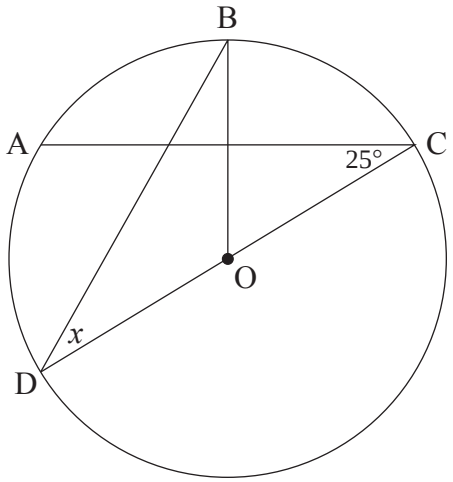
B. $y = \sin \frac{2\pi}{5}(x - 2)$

C. $y = \sin \frac{2\pi}{5}(x + 2)$

D. $y = \sin \frac{\pi}{5}(x + 2)$

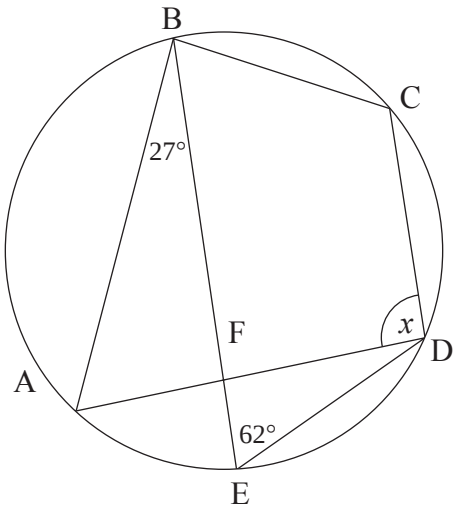
For questions 39 to 42, diagrams are not drawn to scale.

39. In the given circle, O is the centre and $BO \perp AC$. Find the measure of $\angle x$.



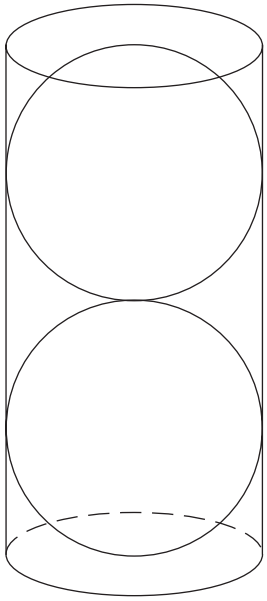
A. 25°
B. 30.5°
C. 32.5°
D. 65°

40. In the diagram below, $BE \,||\, CD$. Find the measure of $\angle x$.



A. 63°
B. 85°
C. 91°
D. 118°

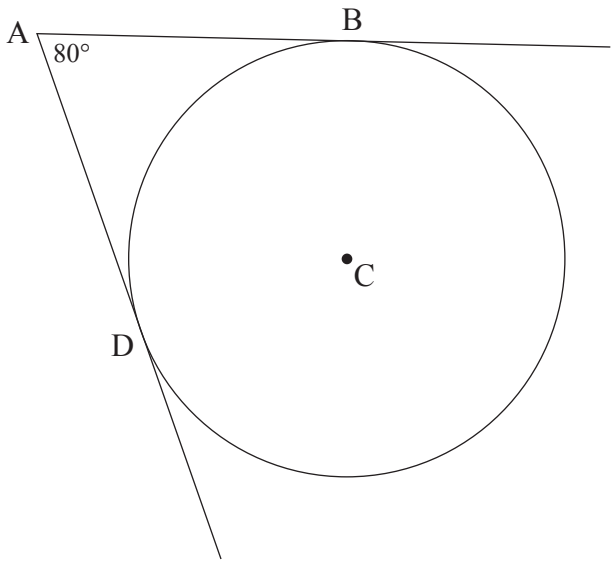
41. Two spherical balls are tightly packed into a cylinder, as shown in the diagram. If the radius of each ball is 3 cm, determine the volume of the cylinder. $V_{cylinder} = \pi r^2 h$



A. 36π cm^3
B. 54π cm^3
C. 108π cm^3
D. 216π cm^3

OVER

42. In the diagram below, BA and DA are tangents to a circle with centre C. Determine the radius of the circle if $AB = 10$.



A. 5.67
B. 6.43
C. 7.66
D. 8.39

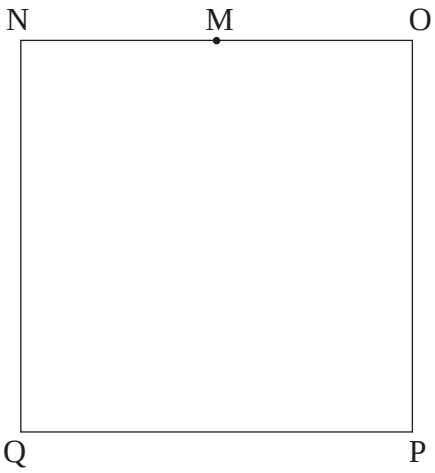
43. Male bees, called drones, hatch from unfertilized eggs; therefore they have a mother but no father. Fertilized eggs hatch into female bees. A female bee, therefore, has both a mother and a father. Determine the total number of ancestors that a male bee has in the first 5 generations preceding him.

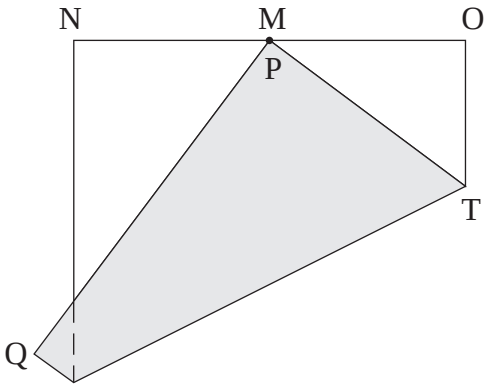
A. 12
B. 19
C. 32
D. 62

44. How many solutions does $|x+3| = 1.5\,|x-2|$ have?

A. 0
B. 1
C. 2
D. 3

45. A square piece of paper, 10×10, is folded in such a way that the lower right hand corner at P just touches the midpoint M of the top side, as shown in the diagram below. A new triangle, ΔMOT is formed. Determine the length of MT.





A. 3.75
B. 6.13
C. 6.25
D. 7.50

This is the end of the multiple-choice section.
Answer the remaining questions directly in this examination booklet.

OVER

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PART B: WRITTEN RESPONSE

Value: 25 marks **Suggested Time: 45 minutes**

INSTRUCTIONS: Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

If, in a justification, you refer to information produced by the calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem, it is important to sketch the graph, showing its general shape and indicating the appropriate window dimensions.

When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.

Full marks will NOT be given for the final answer only.

OVER

1. Solve the following equation using a graphing calculator. **(3 marks)**

$$x^3 + 10x^2 = 22 - 10x$$

Sketch the graph in the viewing window below and indicate appropriate window dimensions. State the function(s) used in your graph. Ensure that the relative maximum and relative minimum points of the function(s) are visible within the viewing window.

$Y_1 =$

$Y_2 =$

$Y_3 =$

$Y_4 =$

[,] [,]

x min x max y min y max

ANSWER:

OVER

2. A population of wolves decreases by 2% each year. At the present time, there are 8 000 wolves. How long will it take for the population to become 500 wolves? (Answer to the nearest year.)

(3 marks)

ANSWER:

OVER

3. A point $P(x, y)$ moves such that it is always the same distance from $A(12, 0)$ as it is from $B(3, 1)$. Determine an equation, in standard form, of this locus. **(3 marks)**

ANSWER:

OVER

4. Prove the identity: **(3 marks)**

$$\csc^2 x + \sec^2 x \quad = \quad \csc^2 x \sec^2 x$$

LEFT SIDE	RIGHT SIDE

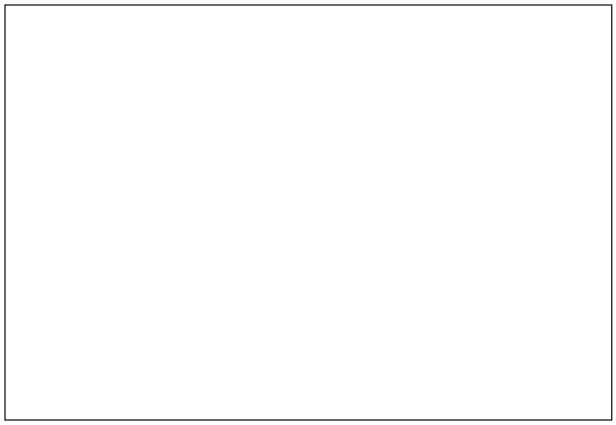
OVER

5. Solve the following system using a graphing calculator. **(3 marks)**

$$xy = 12$$

$$y = \frac{1}{4}x^2 - 8$$

Sketch the graph in the viewing window below. State the function(s) that you entered to obtain your graph and your solution. Indicate the dimensions of the viewing window that will show enough of the graph so that recognizable characteristics of the function(s) and all intersection points are visible.



$Y_1 =$

$Y_2 =$

$Y_3 =$

$Y_4 =$

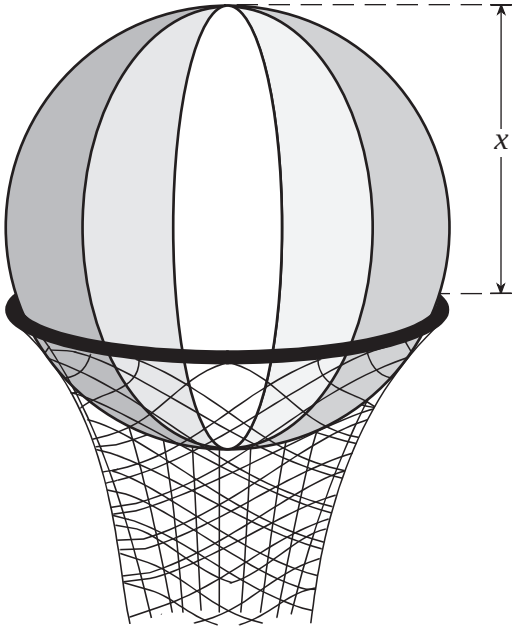
[,] [,]

x min x max y min y max

ANSWER:

OVER

6. A child throws her beach ball with radius 28 cm into a basketball hoop with an inside diameter of 46 cm. The ball is too big and gets stuck, as shown in the diagram. What is the vertical distance x from the top of the ball to the level of the hoop? **(3 marks)**



ANSWER:

OVER

7. Given $\frac{1}{\log_y 4} = \log_{\frac{1}{4}} \frac{1}{8x}$, express y as a polynomial function of x. State the restrictions on x and y. **(3 marks)**

ANSWER:

OVER

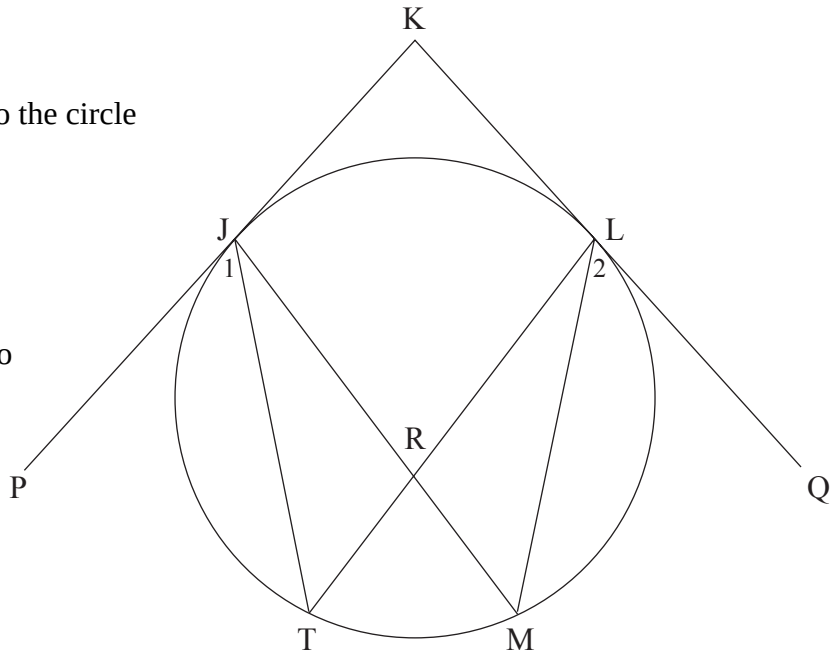
Students should choose one or the other method of proof.

8. Complete the proof. **(4 marks)**

Given: PK and QK are tangents to the circle
$\angle 1 = \angle 2$

Prove: JR = LR

Note: Students are encouraged to number angles.



Paragraph proof method:

Two-column proof method:

STATEMENT	REASON

END OF EXAMINATION

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A SUMMARY OF BASIC IDENTITIES AND FORMULAE

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$ $\qquad 1 + \tan^2\theta = \sec^2\theta$ $\qquad 1 + \cot^2\theta = \csc^2\theta$

Reciprocal and Quotient Identities

$\sec\theta = \dfrac{1}{\cos\theta}$ $\qquad \csc\theta = \dfrac{1}{\sin\theta}$ $\qquad \cot\theta = \dfrac{1}{\tan\theta}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\qquad \cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Addition Identities

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\qquad \tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\qquad \tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

Double-Angle Identities

$\cos 2\theta = \cos^2\theta - \sin^2\theta$ $\qquad \sin 2\theta = 2\sin\theta\cos\theta$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$ $\qquad \tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

Formulae

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\qquad t_n = a + (n-1)d$ $\qquad t_n = ar^{n-1}$

$\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$ $\qquad S_n = \dfrac{n}{2}\left[2a + (n-1)d\right]$ $\qquad S_n = \dfrac{a\left(1 - r^n\right)}{1 - r}$

$c^2 = a^2 + b^2 - 2ab\cos C$ $\qquad S_n = \dfrac{n}{2}(a + \ell)$ $\qquad S_n = \dfrac{a - \ell r}{1 - r}$

$S = \dfrac{a}{1 - r}$

You may detach this page for convenient reference.
Exercise care when tearing along perforations.

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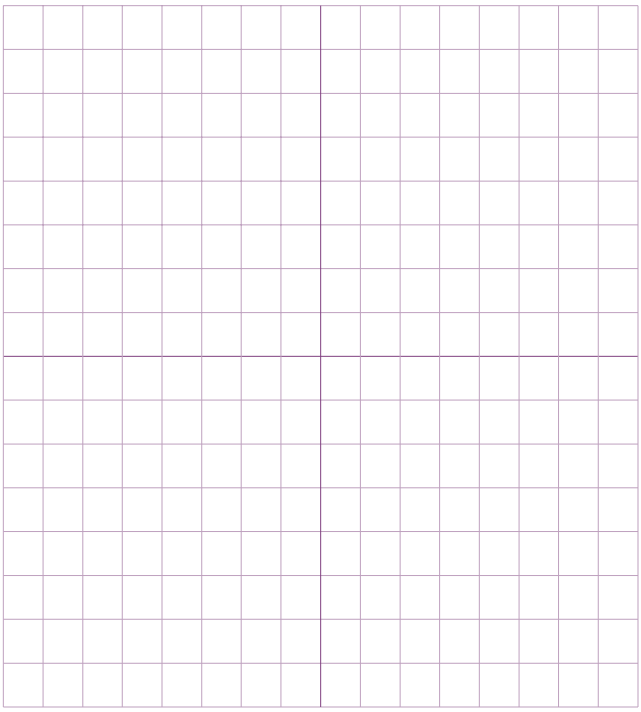
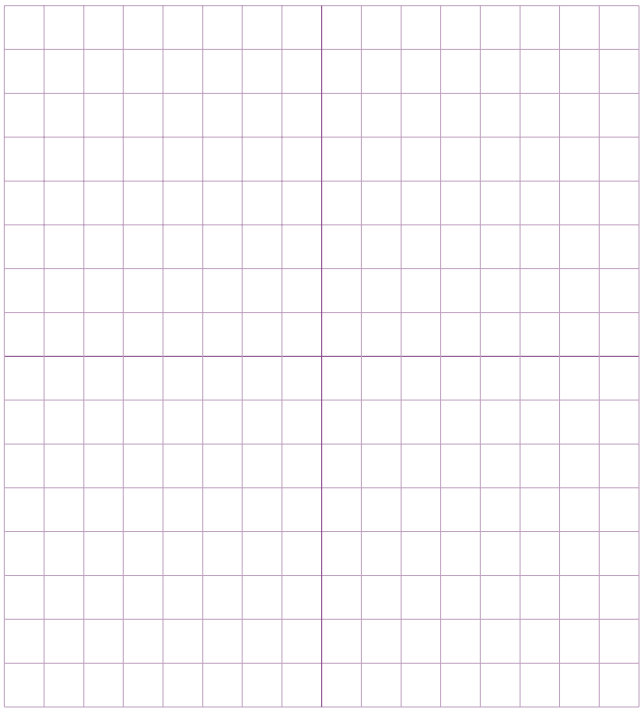
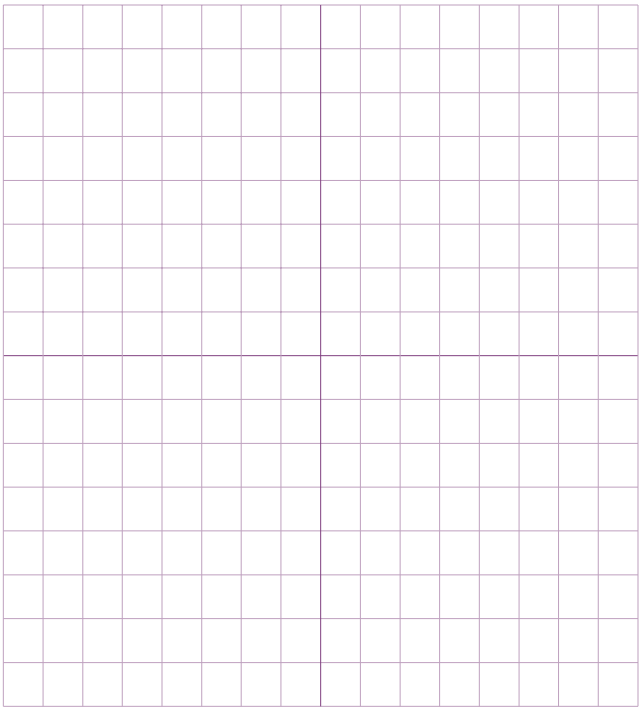
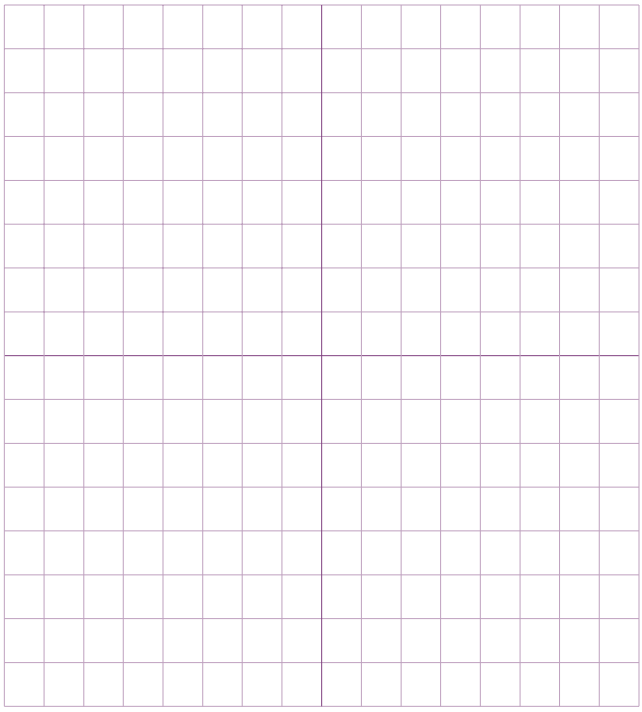
ROUGH WORK FOR GRAPHING

(No marks will be given for work done on this page.)





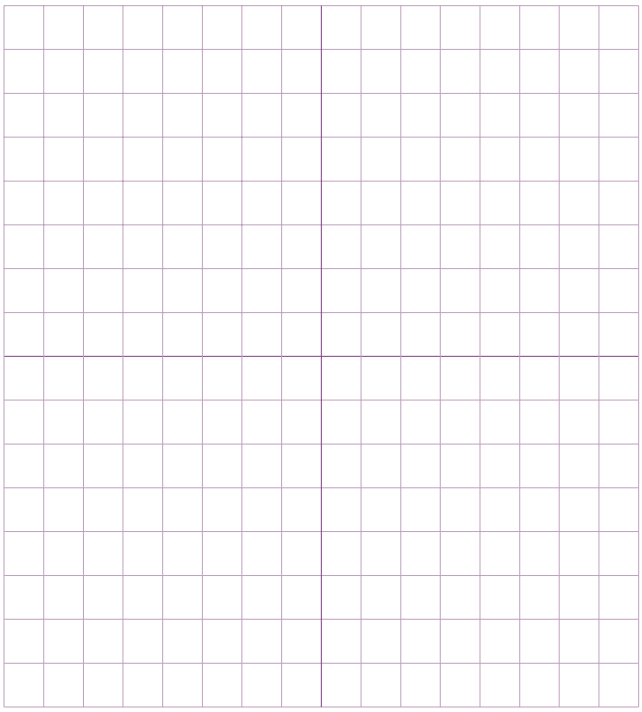


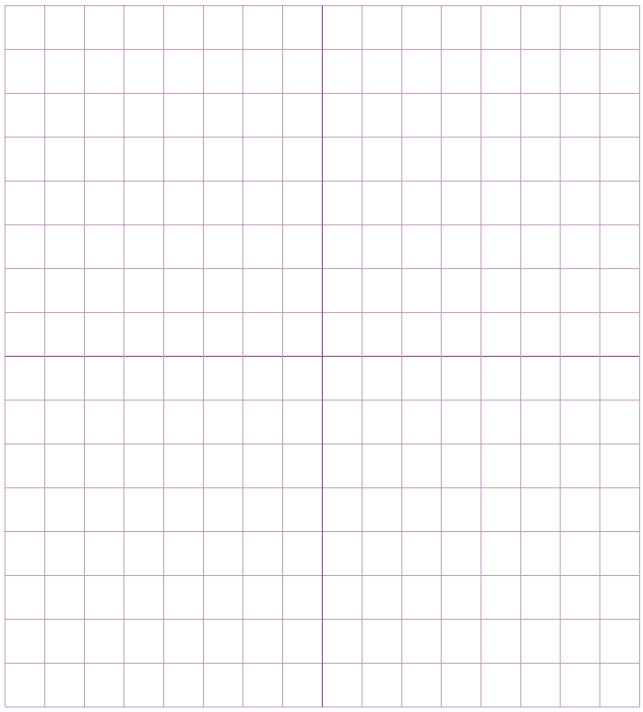


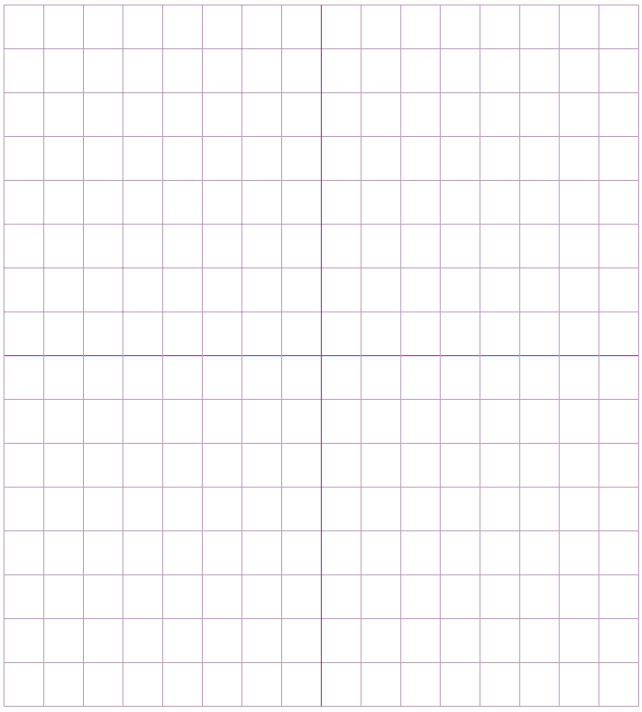
You may detach this page for convenient reference.
Exercise care when tearing along perforations.

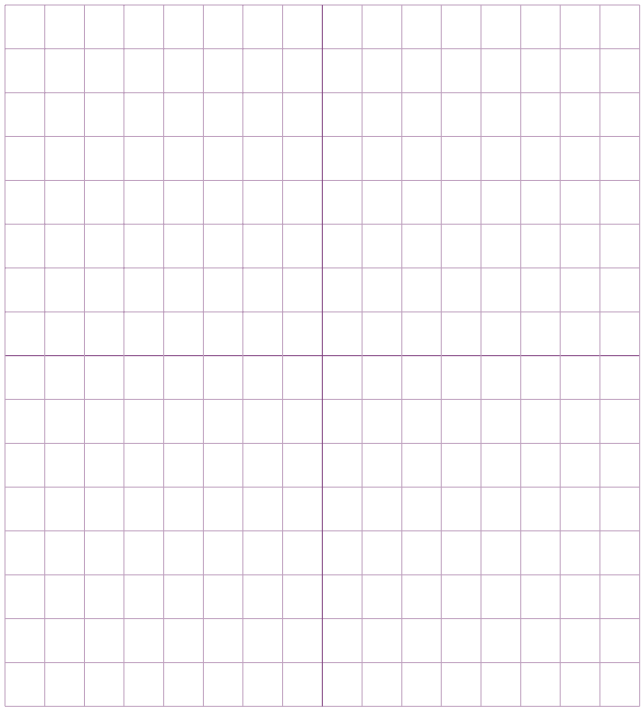
ROUGH WORK FOR GRAPHING

(No marks will be given for work done on this page.)









ROUGH WORK FOR MULTIPLE-CHOICE

You may detach this page for convenient reference.
Exercise care when tearing along perforations.

Question 1:

1. ____ . ____

(3)

Question 2:

2. ____ . ____

(3)

Question 3:

3. ____ . ____

(3)

Question 4:

4. ____ . ____

(3)

Question 5:

5. ____ . ____

(3)

Question 6:

6. ____ . ____

(3)

Question 7:

7. ____ . ____

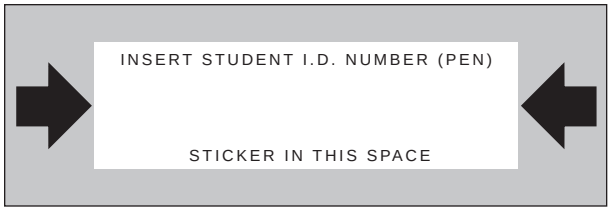
(3)

Question 8:

8. ____ . ____

(4)

❒ ❒ ❒ ❒



batch and sequence number

PRINCIPLES OF MATHEMATICS 12

January 2000

Course Code = MA

Use this space if I.D. sticker is **not** available.