

Applications of Mathematics 12
Sample Examination 2007/08
 Provincial Examination — Answer Key

Cognitive Processes

K = Knowledge
U = Understanding
H = Higher Mental Processes

Weightings

10%
 70%
 20%

Question Types

44 = Multiple Choice (MC)
7 = Written Response (WR)

Topics

1. Number (Matrices and Finance)
2. Patterns and Relations (Sinusoidal Functions)
3. Shape and Space (Measurement, 3-D Objects and 2-D Shapes)
4. Statistics and Probability (Chance and Uncertainty)

Prescribed Learning Outcomes (PLOs)

A
 B
 C
 D

Weightings

26
 21
 38
 15

Question Number	Keyed Response	Cognitive Process	Mark	Topic	PLO	Question Type
1.	B	K	1.5	1	A1	MC
2.	D	U	1.5	1	A1	MC
3.	A	U	1.5	1	A2	MC
4.	A	U	1.5	1	A2	MC
5.	B	U	1.5	1	A2	MC
6.	A	H	1.5	1	A1	MC
7.	C	U	1.5	1	A4	MC
8.	B	U	1.5	1	A4	MC
9.	C	K	1.5	1	A3	MC
10.	B	U	1.5	1	A4, A6	MC
11.	C	H	1.5	1	A3, A4	MC
12.	B	H	1.5	1	A6	MC
13.	B	K	1.5	2	B1	MC
14.	D	U	1.5	2	B1	MC
15.	A	U	1.5	2	B3	MC
16.	B	U	1.5	2	B1	MC
17.	D	U	1.5	2	B1	MC
18.	C	U	1.5	2	B2	MC
19.	A	U	1.5	2	B3	MC
20.	A	H	1.5	2	B1	MC
21.	C	H	1.5	2	B1	MC
22.	C	U	1.5	3	C1, C2	MC

Question Number	Keyed Response	Cognitive Process	Mark	Topic	PLO	Question Type
23.	A	U	1.5	3	C1, C2	MC
24.	C	U	1.5	3	C2	MC
25.	B	U	1.5	3	C1, C2	MC
26.	B	U	1.5	3	C1, C2	MC
27.	D	H	1.5	3	C1, C2	MC
28.	C	K	1.5	3	C5	MC
29.	B	U	1.5	3	C7, C8	MC
30.	D	U	1.5	3	C7	MC
31.	D	U	1.5	3	C6	MC
32.	B	U	1.5	3	C7, C8	MC
33.	A	U	1.5	3	C7	MC
34.	A	U	1.5	3	C7	MC
35.	B	U	1.5	3	C8	MC
36.	A	H	1.5	3	C8	MC
37.	D	H	1.5	3	C7	MC
38.	C	K	1.5	4	D1	MC
39.	C	U	1.5	4	D2	MC
40.	A	U	1.5	4	D1, D4	MC
41.	C	U	1.5	4	D2	MC
42.	A	U	1.5	4	D4	MC
43.	C	U	1.5	4	D2, D4	MC
44.	D	U	1.5	4	D2	MC

Question Number	Keyed Response	Cognitive Process	Mark	Topic	PLO	Question Type
1.	–	U	3	1	A2	WR
2.	–	U/H	3	1	A4	WR
3.	–	U	5	2	B2, B3	WR
4.	–	U	3	4	D3, D4	WR
5.	–	U	2	3	C7	WR
6.	–	U	3	3	C8	WR
7.	–	U	5	3	C1, C2, C3	WR

Applications of Mathematics 12
Sample Examination 2007/08
 Provincial Examination — Scoring Guide

1. In a certain community, 70% of the homes currently get their television service from a cable connection while 30% use a satellite connection. A survey predicts that each year 8% of those who use cable will switch to satellite, while 95% of those who use satellite will continue to use satellite.

Determine:

- an initial matrix and transition matrix
- the percentage of satellite users after 3 years
- the percentage of satellite users over the long term if this trend continues

(3 marks)

 Solution

<p>Initial Matrix</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td></td> <td style="text-align: center;">Cable</td> <td style="text-align: center;">Satellite</td> </tr> <tr> <td style="text-align: right;">$I =$</td> <td style="text-align: center;">[0.7</td> <td style="text-align: center;">0.3]</td> </tr> </table> <p style="text-align: center;">OR</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td></td> <td style="text-align: center;">Cable</td> <td style="text-align: center;">Satellite</td> </tr> <tr> <td style="text-align: right;">$I =$</td> <td style="text-align: center;">[70</td> <td style="text-align: center;">30]</td> </tr> </table> <p style="text-align: center;">↑ $\frac{1}{2}$ mark</p>		Cable	Satellite	$I =$	[0.7	0.3]		Cable	Satellite	$I =$	[70	30]	<p>Transition Matrix</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td></td> <td colspan="2" style="text-align: center;">To</td> </tr> <tr> <td></td> <td style="text-align: center;">C</td> <td style="text-align: center;">S</td> </tr> <tr> <td style="text-align: right;">$T =$</td> <td style="text-align: center;">From</td> <td style="text-align: center;">[0.92 0.08]</td> </tr> <tr> <td></td> <td style="text-align: center;">C</td> <td style="text-align: center;">S</td> </tr> <tr> <td></td> <td style="text-align: center;">S</td> <td style="text-align: center;">[0.05 0.95]</td> </tr> </table> <p style="text-align: center;">↑ $\frac{1}{2}$ mark</p>		To			C	S	$T =$	From	[0.92 0.08]		C	S		S	[0.05 0.95]
	Cable	Satellite																										
$I =$	[0.7	0.3]																										
	Cable	Satellite																										
$I =$	[70	30]																										
	To																											
	C	S																										
$T =$	From	[0.92 0.08]																										
	C	S																										
	S	[0.05 0.95]																										

$$IT^3 = [0.5922... \quad 0.4077...]$$

After three years, 41% will be satellite users.

← 1 mark

$$IT^{100} = [0.3846... \quad 0.6153...]$$

Over the long term, 62% will be satellite users.

← 1 mark

2. Gary bought a condominium for \$220 000. He obtained a mortgage for the total amount at 4.75% per annum, compounded semi-annually, for 25 years with monthly payments.

Determine:

- his monthly payment
- the total amount paid over 25 years
- how much is still owing after 5 years

(3 marks)

Solution



Using TVM Solver:

Calculate Payment:

$$N = 25 \times 12 = 300$$

$$I\% = 4.75$$

$$PV = -220\,000$$

$$PMT = 1248.4022\dots$$

$$FV = 0 \quad \leftarrow 1 \text{ mark}$$

$$P/Y = 12$$

$$C/Y = 2$$

PMT : END

$$\text{Payment} = \$1248.40$$

Calculate Future Value:

$$N = 5 \times 12 = 60$$

$$I\% = 4.75$$

$$PV = -220\,000$$

$$FV = 193\,942.8812\dots$$

$$PMT = 1\,248.4022\dots \quad \leftarrow 1 \text{ mark}$$

$$P/Y = 12$$

$$C/Y = 2$$

PMT : END

$$\text{Amount still owing} = \$193\,942.88$$

Total Amount Paid:

$$= 300 \times \$1248.4022\dots = \$374\,520.66 \quad \leftarrow 1 \text{ mark}$$

$\frac{1}{2}$ mark off for each incorrect entry.

3. A boat is moored in a bay and its height above the ocean floor is measured using a depth sounder at various times during the day, as shown in the table below. The first measurement is at midnight.

Time (hours)	Depth (metres)
0:00	4.30
4:00	5.06
8:00	6.05
12:00	6.29
16:00	5.06

For this data:

- determine the sinusoidal regression for this equation
- graph the sinusoidal function over a 24-hour period and label the axes (i.e., provide title and scale on the axes).
- determine the length of time during this 24-hour period that the depth of the water is greater than 6.0 m

(5 marks)

Solution

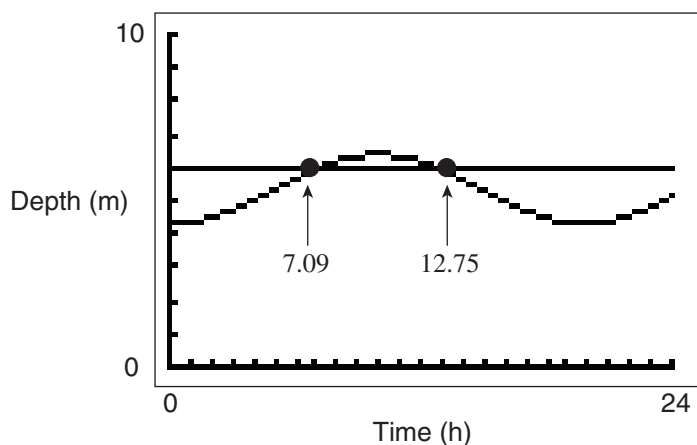


Using the SinReg function of the graphing calculator:

$$Y_1 = 1.048 \dots \sin(0.319 \dots X - 1.599 \dots) + 5.351 \dots$$

← 1 mark

$$Y_2 = 6$$



← $\frac{1}{2}$ mark for each intersection point

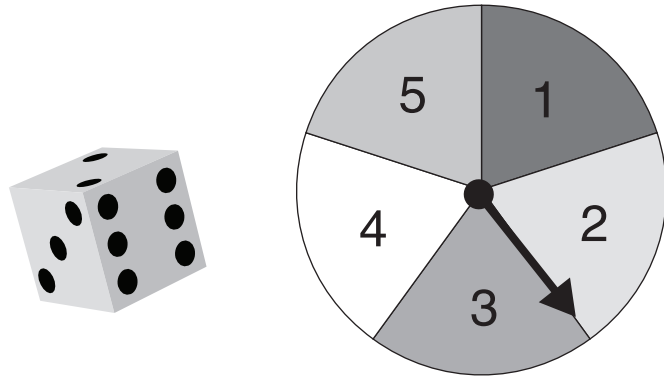
← 2 marks for labelled graph

$$\text{The length of time over 6.0 m} = 12.75 - 7.09 = 5.66 \text{ h}$$

← 1 mark

Note: Answers provided by Sharp calculators may vary slightly.

4. A game consists of rolling a fair, six-sided die and spinning a spinner as shown below.



For the game above:

- create a sample space
- determine the probability of both numbers being a 4
- determine the probability the sum of the two numbers is greater than 3

(3 marks)

Solution

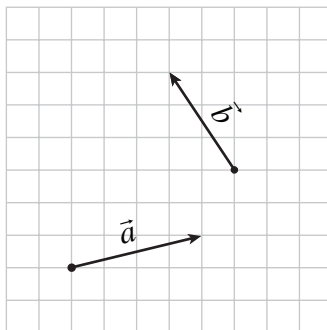
		Spinner				
		1	2	3	4	5
Die	1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)
	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)
	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)
	5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)
	6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)

← 1 mark

$$P(\text{both are 4}) = \frac{1}{30} \quad \leftarrow 1 \text{ mark}$$

$$P(\text{sum} > 3) = \frac{27}{30} \quad \text{OR} \quad = 1 - P(\text{complement}) = 1 - \frac{3}{30} = \frac{27}{30} \quad \leftarrow 1 \text{ mark}$$

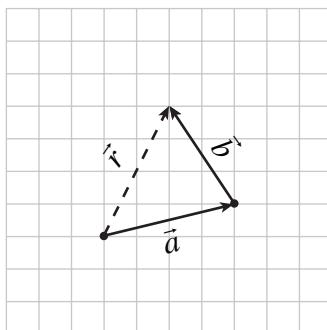
5. Given the vectors in the diagram below:



Draw the vector diagram that shows the resultant of $\vec{a} + \vec{b}$.

(1 mark)

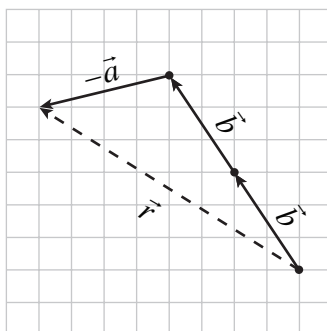
Solution



Draw the vector diagram that shows the resultant of $2\vec{b} - \vec{a}$.

(1 mark)

Solution

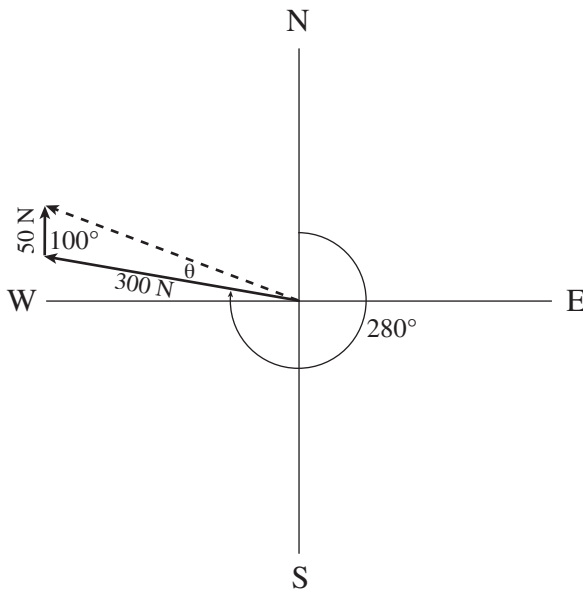


6. A plane flies at 300 km/h at a heading of $[280^\circ]$. A wind blows from the south at 50 km/h.

- Create a vector diagram.
- Determine the magnitude of the resultant.
- Determine the direction of the resultant.

(3 marks)

Solution



← 1 mark for diagram

$$r^2 = 300^2 + 50^2 - 2(300)(50)(\cos 100^\circ)$$

$$r^2 = 97\,709.44533\dots$$

$$r = 312.5851\dots$$

← 1 mark

$$\frac{\sin \theta}{50} = \frac{\sin 100}{312.5851\dots}$$

$$\theta = 9.063\dots^\circ$$

← $\frac{1}{2}$ mark

∴ the plane is flying at 312.59 km/h at a heading of $[289^\circ]$. ← $\frac{1}{2}$ mark

Note: Full marks for any other correctly expressed direction.

Alternate Solution

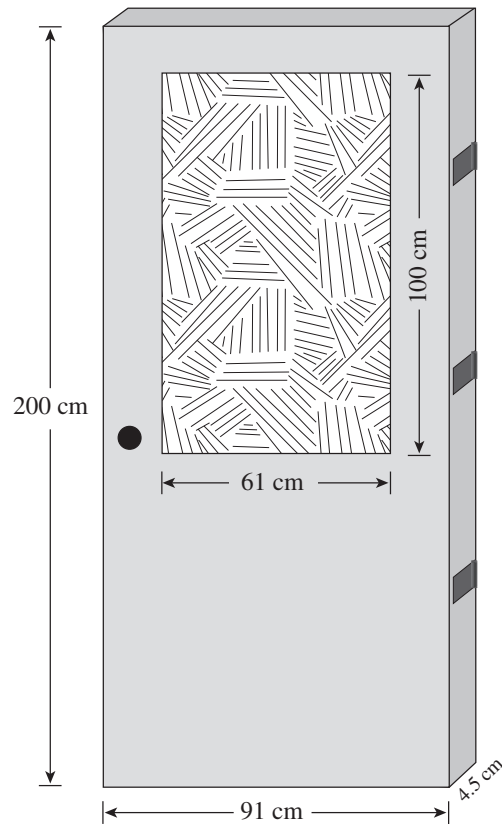
For θ :

$$\cos \theta = \frac{r^2 + 300^2 - 50^2}{2(r)(300)}$$

$$\theta = 9.06^\circ$$

7. Starnight Doors manufactures solid wood doors for new homes. One particular door is 200 cm tall, 91 cm wide and 4.5 cm thick. The door has three hinges, one door knob-and-lock set, and a stained-glass insert. The glass insert measures 61 cm \times 100 cm. Costs for the door are as follows:

- oak costs $\$0.002/\text{cm}^3$
- stained glass costs $\$0.005/\text{cm}^2$
- hinges cost $\$0.45$ each
- one door knob-and-lock set costs $\$44.50$



Using the given information, determine the total cost of materials for the door.

(5 marks)

Solution

$$\text{Volume of door} = 200 \times 91 \times 4.5 = 81\,900 - (61 \times 100 \times 4.5) = 54\,450 \text{ cm}^3 \quad \leftarrow \text{1 mark}$$

$$\text{Cost of oak} = 54\,450 \times 0.002 = \$108.90 \quad \leftarrow \text{1 mark}$$

$$\text{Cost of glass} = 61 \times 100 = 6100 \times 0.005 = \$30.50 \quad \leftarrow \text{1 mark}$$

$$\text{Cost of hinges} = 0.45 \times 3 = \$1.35 \quad \leftarrow \text{1 mark}$$

$$\begin{aligned} \text{Total cost} &= 108.90 + 30.50 + 1.35 + 44.50 \\ &= \$185.25 \quad \leftarrow \text{1 mark} \end{aligned}$$