

Applications of Mathematics 12  
**August 2006 — Form A**  
 Provincial Examination — Answer Key / Scoring Guide

**Cognitive Processes**

**K** = Knowledge  
**U** = Understanding  
**H** = Higher Mental Processes

**Question Types**

**44** = Multiple Choice (MC)  
**8** = Written Response (WR)

**Topics**

1. Problem Solving
2. Number (Number Operations I)
3. Number (Number Operations II)
4. Patterns and Relations (Patterns)
5. Shape and Space (Measurement)
6. Shape and Space (3-D Objects and 2-D Shapes)
7. Statistics and Probability (Chance and Uncertainty)

**Prescribed Learning Outcomes (PLOs)**

- A  
 B  
 C  
 D  
 E  
 F  
 G

**Weightings**

integrated throughout  
 10% – 15%  
 10% – 15%  
 25%  
 10% – 15%  
 10% – 15%  
 25%

Question Number	Keyed Response	Cognitive Process	Mark	Topic	PLO	Question Type
1.	B	K	1.5	2	B1	MC
2.	D	U–	1.5	2	B1	MC
3.	C	U+	1.5	2	B1	MC
4.	C	U+	1.5	2	B2	MC
5.	D	H	1.5	2	B2	MC
6.	D	K	1.5	3	C2	MC
7.	A	U–	1.5	3	C4	MC
8.	A	U–	1.5	3	C3	MC
9.	B	U–	1.5	3	C2	MC
10.	A	U+	1.5	3	C4	MC
11.	C	H	1.5	3	C3	MC
12.	A	K	1.5	4	D1	MC
13.	A	U–	1.5	4	D1	MC
14.	B	U–	1.5	4	D3	MC
15.	D	U–	1.5	4	D1	MC
16.	C	H	1.5	4	D1	MC
17.	D	U–	1.5	4	D4	MC
18.	C	U–	1.5	4	D4	MC
19.	B	U+	1.5	4	D4	MC
20.	D	U+	1.5	4	D4	MC
21.	C	U+	1.5	4	D6	MC
22.	B	H	1.5	4	D4	MC

Question Number	Keyed Response	Cognitive Process	Mark	Topic	PLO	Question Type
23.	B	K	1.5	5	E1	MC
24.	C	U-	1.5	5	E1	MC
25.	A	U-	1.5	5	E2	MC
26.	D	U+	1.5	5	E1	MC
27.	B	U+	1.5	5	E1, E2	MC
28.	B	H	1.5	5	E1, E4	MC
29.	C	K	1.5	6	F1	MC
30.	A	U-	1.5	6	F2	MC
31.	C	U+	1.5	6	F4, F9	MC
32.	B	U+	1.5	6	F4	MC
33.	D	H	1.5	6	F3	MC
34.	B	U-	1.5	7	G6	MC
35.	D	U-	1.5	7	G5	MC
36.	A	U+	1.5	7	G3	MC
37.	C	U+	1.5	7	G8	MC
38.	D	H	1.5	7	G4	MC
39.	C	U+	1.5	7	G5	MC
40.	D	K	1.5	7	G1	MC
41.	C	U-	1.5	7	G1	MC
42.	A	U-	1.5	7	G1	MC
43.	A	U+	1.5	7	G2	MC
44.	B	H	1.5	7	G2	MC

Question Number	Keyed Response	Cognitive Process	Mark	Topic	PLO	Question Type
1.	-	U+	3	2	B2	WR
2.	-	U-/H	3	3	C4	WR
3.	-	U-/U+	3	4	D2, D3	WR
4.	-	U/H	3	4	D4	WR
5.	-	U+	3	5	E1, E2	WR
6.	-	U/H	3	6	F3	WR
7.	-	U+/H	3	7	G8	WR
8.	-	U+	3	7	G2	WR

1. In a survey of diet cola drinkers, 60% drank Brand A while 40% drank Brand B. Market research shows that each month:
- 10% of Brand A drinkers switch to Brand B
  - 35% of Brand B drinkers switch to Brand A

Determine what percentage of diet cola drinkers will be drinking Brand B after 3 months; **and then**, if the pattern continues, determine what the final market share will be for Brand B in the longer term. **(3 marks)**

### **Solution**

$$\begin{array}{c}
 \frac{1}{2} \text{ mark} \\
 \downarrow \\
 \text{To} \\
 \begin{array}{cc}
 & \text{A} & \text{B} \\
 \text{From} & \begin{bmatrix} 0.90 & 0.10 \\ 0.35 & 0.65 \end{bmatrix}^3 & = [0.7482 \quad 0.2518] \\
 \begin{bmatrix} 0.60 & 0.40 \end{bmatrix} & & \\
 \uparrow & \uparrow & \\
 \frac{1}{2} \text{ mark} & \frac{1}{2} \text{ mark} & 
 \end{array}
 \end{array}$$

25% of diet cola drinkers will be drinking Brand B.  $\leftarrow \frac{1}{2} \text{ mark}$

Multiply the initial state matrix by successively higher powers of the transition matrix until the steady state is reached.

$$\begin{array}{c}
 \frac{1}{2} \text{ mark for a power of 38 or higher} \\
 \downarrow \\
 [0.60 \quad 0.40] \begin{bmatrix} 0.90 & 0.10 \\ 0.35 & 0.65 \end{bmatrix}^{38} = [0.777\dots \quad 0.222\dots]
 \end{array}$$

22.22% or  $\frac{2}{9}$  would be the final market share for Brand B.  $\leftarrow \frac{1}{2} \text{ mark}$

2. Hilary deposits \$200 at the beginning of every month into an RRSP account paying  $8\frac{1}{2}\%$  per annum compounded monthly. She deposits the money regularly for 25 years.

Determine what the balance in Hilary's RRSP account will be after 25 years; **and then**, determine by how much her account grew from the end of the 24<sup>th</sup> year to the end of the 25<sup>th</sup> year. **(3 marks)**

### **Solution**



Using TVM Solver:

$$N = 300$$

$$I = 8.5$$

$$PV = 0$$

$$PMT = -200$$

$$P/Y = 12$$

$$C/Y = 12$$

PMT : BEGIN

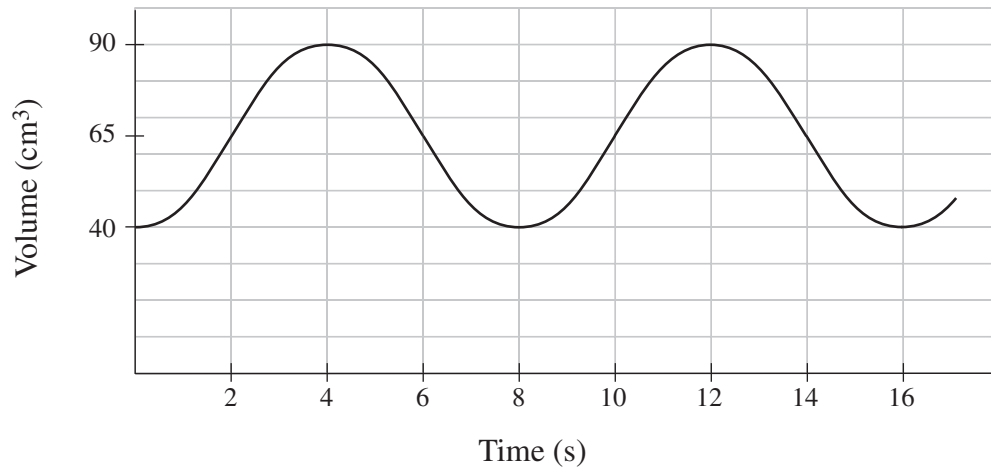
$$\text{Solve for FV} = \$207\,873.7446\dots \quad \leftarrow \mathbf{1 \text{ mark}}$$

$$\text{Amount after 24}^{\text{th}} \text{ year (N = 288)} \quad \leftarrow \mathbf{1 \text{ mark}}$$

$$FV = 188\,682.5056\dots$$

$$207\,873.7446\dots - 188\,682.5056\dots = \$19\,191.24 \quad \leftarrow \mathbf{1 \text{ mark}}$$

3. The sinusoidal graph below represents the volume,  $V$ , of air remaining in a cylinder as a piston moves up and down over time,  $t$ .



Using the sine regression equation, determine the amount of time during each period the volume is greater than  $60 \text{ cm}^3$ .

**(3 marks)**

### **Solution**

Using points from the graph and sine regression:

$t$	$V$
0	40
2	65
4	90
6	65
8	40

$\frac{1}{2}$  mark       $\frac{1}{2}$  mark  
 ↓                    ↓

$$y = 25 \sin(0.79x - 1.57) + 65$$

↑                    ↑  
 $\frac{1}{2}$  mark       $\frac{1}{2}$  mark

In the first period, if  $y_1 = 25 \sin(0.79x - 1.57) + 65$  and  $y_2 = 60$ ,  
 the intersection of these two graphs yields  $x = 1.7436\dots$  and  $x = 6.2563\dots$

$\therefore$  the amount of time is  $6.26 - 1.74 = 4.52 \text{ s}$

**← 1 mark**

4. A certain prescription drug calls for a first dosage of 400 mg and then a dosage of 50 mg on each successive day. The body eliminates 65% of the medication from the bloodstream every day.

Determine the number of milligrams of medication in the bloodstream immediately before the 5<sup>th</sup> dosage **and** the maintenance level of this drug. **(3 marks)**

 **Solution**

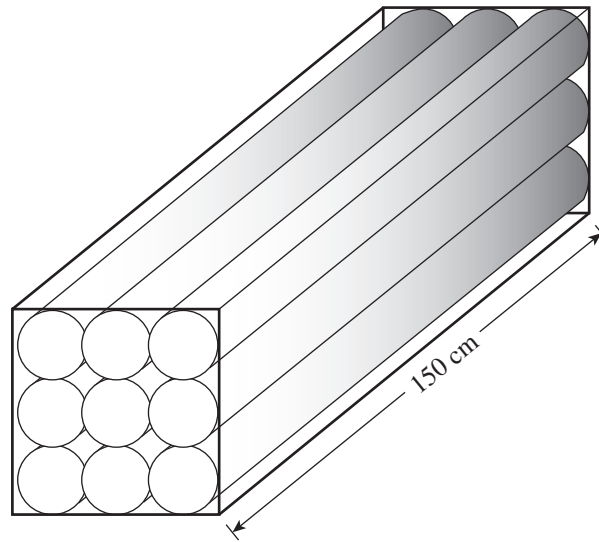
Dosage	Amount
1	400
2	190
3	116.5
4	90.775
5	$81.77125 - 50$
⋮	$= 31.77125 \text{ mg}$

} ← 1 mark

} ← 1 mark

The maintenance level is 76.9230... mg. ← 1 mark

5. A company orders 30 boxes of fluorescent light bulbs that are pre-packaged in cardboard rectangular boxes with closed ends containing 9 light bulbs each, as shown in the diagram below.



- Each light bulb has a diameter of 3 cm and a length of 150 cm.
- Each light bulb costs \$2.50.
- Cardboard costs \$0.0002/cm<sup>2</sup>.

What is the total cost of the 30 boxes of light bulbs and the cardboard packaging?

**(3 marks)**

**Solution**

$$A = 2(9)(9) + 4(9)(150) \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$= 162 + 5400 = 5562 \text{ cm}^2 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$9 \times 30 \text{ boxes} = 270 \text{ light bulbs} \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$5562 \times 30 \times 0.0002 = 33.372 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$270 \times 2.50 = 675.00 \quad \leftarrow \frac{1}{2} \text{ mark}$$

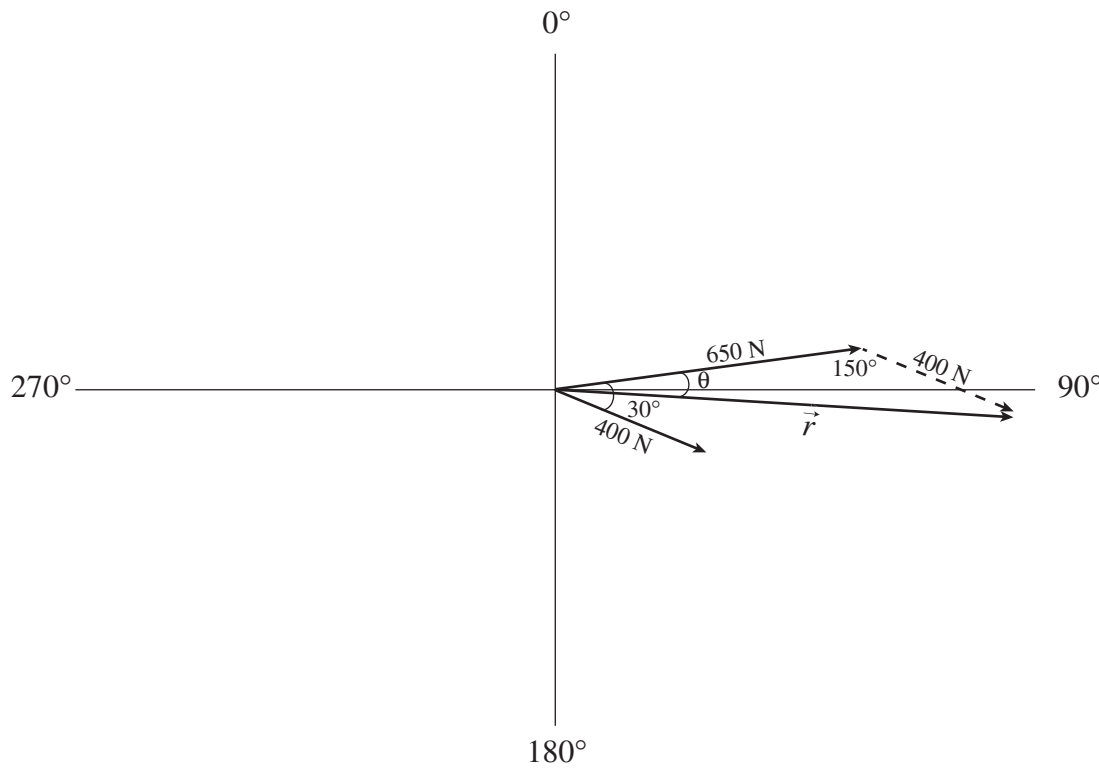
$$= 708.37 \quad \leftarrow \frac{1}{2} \text{ mark}$$

6. Two ropes are attached to a tree. A boy pulls on one of the ropes exerting a force of 400 N toward  $110^\circ$ . A man pulls on the second rope with a force of 650 N toward  $080^\circ$ .

Determine the magnitude and the direction of the resultant force vector. Show your vector diagram below.

**(3 marks)**

**Solution**



**1 mark  
for diagram**

$$r^2 = 650^2 + 400^2 - 2(650)(400)\cos 150^\circ \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$r = 1016.28402\dots$$

$$r \approx 1016.28 \text{ N} \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$\left. \begin{aligned} \frac{\sin \theta}{400} &= \frac{\sin 150^\circ}{1016.284\dots} \\ \theta &= 11.3496\dots \\ \theta &\approx 11.35^\circ \end{aligned} \right\} \leftarrow \frac{1}{2} \text{ mark}$$

$$\therefore \text{ the direction is } 80^\circ + \theta = 91.35^\circ \quad \leftarrow \frac{1}{2} \text{ mark}$$



7. In a coastal region of British Columbia, the probability that it will rain on any given day in January is 0.3.

Determine the probability it will rain at least 12 days out of the 31 days in January.

**(3 marks)**

 **Solution**



$$P(\text{at least } 12) = 1 - \text{binomialcdf}(31, 0.3, 11) = 0.1924141\dots \approx 0.19$$

$\uparrow$                      $\uparrow$                      $\uparrow$                      $\uparrow$   
**1 mark**     $\frac{1}{2}$  **mark**                    **1 mark**                     $\frac{1}{2}$  **mark**

8. Acme Co. manufactures balls. The diameters of these balls are normally distributed with a mean of 5 cm and a standard deviation of 0.2. Balls with diameters less than 4.77 cm and greater than 5.35 cm are rejected.

In a production run of 1000 balls, how many balls will be rejected?

**(3 marks)**

 **Solution**



**1 mk for concept**

**1 mark**

↓

$$1 - \text{normalcdf}(4.77, 5.35, 5, 0.2) = 1 - 0.83486\dots \approx 0.1651\dots$$

$$\text{number of balls rejected} = (0.1651)(1000) = 165 \text{ balls} \quad \leftarrow \text{1 mark}$$

**END OF KEY**