

Applications of Mathematics 12

August 2005 Provincial Examination

ANSWER KEY / SCORING GUIDE

CURRICULUM:

Organizers	Sub-Organizers
A. Problem Solving	A Integrated Throughout
B. Number	B Number Operations I
C. Number	C Number Operations II
D. Patterns and Relations	D Patterns
E. Shape and Space	E Measurement
F. Shape and Space	F 3-D Objects and 2-D Shapes
G. Statistics and Probability	G Chance and Uncertainty

Part A: Multiple Choice

Q	K	C	S	CO	PLO	Q	K	C	S	CO	PLO
1.	A	K	1.5	B	B1	21.	C	K	1.5	E	E4
2.	D	U	1.5	B	B1	22.	C	U	1.5	E	E1
3.	D	U	1.5	B	B1	23.	D	U	1.5	E	E1
4.	B	U	1.5	B	B2	24.	D	U	1.5	E	E1
5.	D	U	1.5	B	B1	25.	D	U	1.5	E	E2
6.	D	H	1.5	B	B2	26.	B	H	1.5	E	E1
7.	D	U	1.5	C	C1	27.	A	K	1.5	F	F1
8.	C	U	1.5	C	C3	28.	C	U	1.5	F	F2
9.	C	U	1.5	C	C4	29.	B	U	1.5	F	F3
10.	A	U	1.5	C	C3	30.	A	H	1.5	F	F3
11.	A	U	1.5	C	C4	31.	B	H	1.5	F	F4
12.	B	H	1.5	C	C2	32.	A	K	1.5	G	G8
13.	C	K	1.5	D	D1	33.	C	U	1.5	G	G1
14.	B	U	1.5	D	D3	34.	D	U	1.5	G	G8
15.	B	U	1.5	D	D1	35.	C	U	1.5	G	G4
16.	C	U	1.5	D	D4	36.	B	U	1.5	G	G5
17.	D	U	1.5	D	D4	37.	D	U	1.5	G	G6, G8
18.	C	U	1.5	D	D6	38.	A	U	1.5	G	G2
19.	B	H	1.5	D	D5	39.	B	H	1.5	G	G8
20.	C	H	1.5	D	D1	40.	A	H	1.5	G	G8

Multiple Choice = 60 marks

Part B: Written Response

Q	C	S	CO	PLO
1.	U	2	C	C3
2.	U	1	C	C3
3.	U	2	C	C3
4.	U	2	D	D2, D3
5.	U	1	D	D2, D3
6.	U	2	D	D2, D3
7.	U	2	B	B2
8.	U	2	B	B2
9.	U	1	B	B2
10.	U	1	F	F4
11.	U	2	F	F4
12.	U	2	F	F4
13.	U	2	G	G2
14.	U	2	G	G2
15.	U	1	G	G2
16.	U	2	E	E1
17.	U	2	E	E1
18.	U	1	E	E1

Written Response = 30 marks

Multiple Choice = 60 (40 questions)

Written Response = 30 (18 questions)

EXAMINATION TOTAL = 90 marks

LEGEND:

Q = Question Number

K = Keyed Response

C = Cognitive Level

S = Score

CO = Curriculum Organizer

PLO = Prescribed Learning Outcome

PART B: WRITTEN RESPONSE

Value: 30 marks

Suggested Time: 45 minutes

Use the following information to answer questions 1 to 3.

Emily wishes to obtain a new car with a total price of \$18 752.

Option 1: Purchase the car now with a down payment of \$4000 and a loan at 2% per annum compounded monthly and paid monthly for 3 years.

Option 2: Lease the car with no down payment and pay \$325 per month for 3 years and then purchase the car outright at its lease-end value of \$8000.

1. What is the monthly payment for Option 1? (2 marks)

Solution



Using TVM Solver:

$$N = 36$$

$$I = 2$$

$$PV = 14\,752 \quad \leftarrow \text{1 mark}$$

$$PMT = -422.54$$

$$FV = 0$$

$$P/Y = 12$$

$$C/Y = 12$$

PMT : END

$$\text{Solve for } PMT = 422.54 \quad \leftarrow \text{1 mark}$$

2. What is the total paid in Option 2? (1 mark)

Solution

$$325 * 36 + 8000 = \$19\,700 \quad \leftarrow \text{1 mark}$$

3. Which option will cost Emily the least amount and by how much?

(2 marks)

 Solution

$$422.54 * 36 + 4000 = \$19\,211.44$$

← **1 mark**

Emily should choose Option 1 because $19\,700 - 19\,211.44 = \$488.56$

← **1 mark**

Use the following information to answer questions 4 to 6.

The following table describes the height of a seat on a Ferris wheel as the wheel rotates.

Time (s)	0	5	10	15	20	25	30
Height (m)	1.5	9.79	23.21	23.21	9.79	1.5	9.79

4. Determine a sinusoidal regression equation for this data. (2 marks)

Solution

$$y = 12 \sin(0.2513 \dots x - 1.57 \dots) + 13.5$$

$\frac{1}{2}$ mark $\frac{1}{2}$ mark
↓ ↓
 $\frac{1}{2}$ mark $\frac{1}{2}$ mark

5. Determine the radius of the Ferris wheel. (1 mark)

Solution

The radius of the Ferris wheel is 12 m. ← 1 mark

6. For the first 25 s, determine all the times that the height of the seat will be 10 m. (2 marks)

Solution

$$5.07 \text{ s}, \quad 19.93 \text{ s},$$

↑ ↑
1 mark 1 mark

Use the following information to answer questions 7 to 9.

A network of agents has been created according to the following conditions:

- Each agent can contact himself/herself directly.
- Agent A is able to contact Agents B and C directly.
- Agent B is able to contact Agents C and D directly.
- Agent C is able to contact Agent B directly.
- Agent D is able to contact Agents A and B directly.

7. Determine a matrix to represent this network of agents.

(2 marks)

Solution

		To			
		A	B	C	D
From	A	1	1	1	0
	B	0	1	1	1
	C	0	1	1	0
	D	1	1	0	1

1 mark for correct diagonal entries.

1 mark for other correct entries.

8. Determine a matrix that shows the number of ways one agent can contact another by using at most one intermediate agent. **(2 marks)**

 Solution

$M + M^2$ ← 1 mark

		To			
		A	B	C	D
From	A	2	4	4	1
	B	1	4	3	3
	C	0	3	3	1
	D	3	4	2	3

1 mark for correct entries.

9. What is the least number of intermediate agents necessary for Agent C to contact Agent A? (Justify your answer.) **(1 mark)**

 Solution

The least number of intermediate agents is 2. }
 Matrix M^3 is the first time that an entry other than zero occurs. } 1 mark

OR

C can contact B who can contact D who can contact A. ← 1 mark

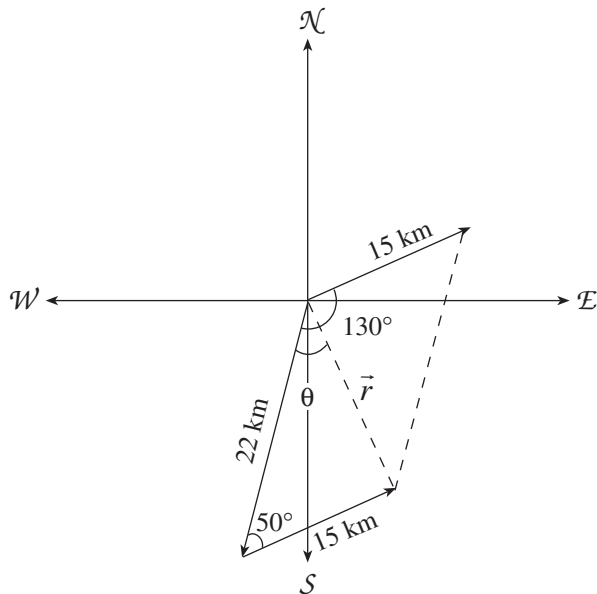
Use the following information to answer questions 10 to 12.

A ship travels 15 km on a bearing of $[070^\circ]$. It then changes direction and travels 22 km on a bearing of $[200^\circ]$.

10. Draw a vector diagram modelling this situation.

(1 mark)

Solution



$\frac{1}{2}$ mark for both vectors.

$\frac{1}{2}$ mark for correctly placing resultant.

11. How far is the ship from its starting point?

(2 marks)

Solution

Using Law of Cosines:

$$r^2 = 22^2 + 15^2 - 2(22)(15) \cos 50^\circ \quad \leftarrow 1 \text{ mark}$$

$$r = 16.87483\dots$$

$$= 16.87 \text{ km} \quad \leftarrow 1 \text{ mark}$$

12. Determine the direction the ship must head in order to return to its starting point.
Answer to the nearest degree.

(2 marks)

 Solution

Using Law of Cosines:

$$\cos \theta = \frac{r^2 + 22^2 - 15^2}{2(r)(22)}$$

$$\theta = 42.9167\dots \quad \leftarrow \mathbf{1 \text{ mark}}$$

\therefore The direction to return is

$$200 - \theta + 180^\circ = 337.0832\dots \quad \mathbf{OR} \quad [337^\circ] \quad \leftarrow \mathbf{1 \text{ mark}}$$

Note:  This question can also be solved using Law of Sines.

Use the following information to answer questions 13 to 15.

Ron wrote a History test and a Mathematics test. His scores, along with the mean and standard deviation for his class, are listed below. Assume that the class marks are normally distributed.

Subject	Ron's Score	Mean	Standard Deviation
History	73	70	6.2
Mathematics	67	64	5.3

13. On which test did Ron perform better in relation to the class? Justify your answer. (2 marks)

 Solution

$$Z_{\text{Math}} = \frac{67 - 64}{5.3} = 0.5660\dots$$

$$Z_{\text{History}} = \frac{73 - 70}{6.2} = 0.4838\dots$$

Since his math score was 0.57σ above the mean he performed better in Math than in History which was only 0.48σ above the mean.

} ← 1 mark
}
} ← 1 mark

14. What percentage of the class scored between 65 and 75 on the History test? (2 marks)

 Solution

$$\text{normalcdf}(65, 75, 70, 6.2) = 0.580017\dots \approx 58\%$$

↑
1 mark

↑
1 mark

15. If the top 18% of the class received an A or a B on the History test, determine the minimum mark for a B.

(1 mark)

 Solution

$$\text{invNorm}(0.82, 70, 6.2) = 75.675\dots$$

$$\begin{array}{cc} \uparrow & \uparrow \\ \frac{1}{2} \text{ mark} & \frac{1}{2} \text{ mark} \end{array}$$

minimum mark ≈ 76

Use the following information to answer questions 16 to 18.

A spherical weather balloon has diameter 3.6 m. An additional 20 m^3 of air is pumped into the balloon, increasing its volume and surface area.

16. Determine the new volume of the balloon.

(2 marks)

 **Solution**

Original Volume:

$$\begin{aligned}\frac{4}{3}\pi r^3 &= \frac{4}{3}\pi(1.8)^3 && \leftarrow \frac{1}{2} \text{ mark} \\ &= 24.43 \text{ m}^3 && \leftarrow \frac{1}{2} \text{ mark}\end{aligned}$$

New Volume:

$$\begin{aligned}&= 24.43 + 20 && \leftarrow \frac{1}{2} \text{ mark} \\ &= 44.43 \text{ m}^3 && \leftarrow \frac{1}{2} \text{ mark}\end{aligned}$$

17. Determine the new radius of the balloon.

(2 marks)

 **Solution**

$$\begin{aligned}V &= \frac{4}{3}\pi r^3 \\ \therefore 44.42902 &= \frac{4}{3}\pi r^3 && \leftarrow \mathbf{1 \text{ mark}} \\ r &= 2.197 && \leftarrow \mathbf{1 \text{ mark}}\end{aligned}$$

18. Determine the new surface area of the balloon.

(1 mark)

 Solution

$$= 4\pi r^2$$

$$= 4\pi(2.197)^2 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$= 60.66 \text{ m}^2 \quad \leftarrow \frac{1}{2} \text{ mark}$$

The new surface area is 60.66 m².

END OF KEY