

# Applications of Mathematics 12

June 2002 Provincial Examination

## ANSWER KEY / SCORING GUIDE

### CURRICULUM:

Organizers	Sub-Organizers
A. Problem Solving	A Problem Set, Case Studies
B. Number	B Number Operations
C. Patterns and Relations	C Variables and Equations
D. Patterns and Relations	D Patterns
E. Shape and Space	E Measurement
F. Shape and Space	F 3-D Objects and 2-D Shapes
G. Statistics and Probability	G Chance and Uncertainty

### Part A: Multiple Choice

Q	K	C	S	CO	PLO	Q	K	C	S	CO	PLO
1.	A	K	1.5	B	B1	19.	D	H	1.5	D	D4
2.	A	U	1.5	B	B2	20.	C	U	1.5	E	E2, E4
3.	C	U	1.5	B	B2	21.	<b>DELETED</b>				
4.	D	U	1.5	B	B2	22.	A	H	1.5	E	E4
5.	B	H	1.5	B	B2	23.	C	K	1.5	F	F1
6.	C	H	1.5	B	B1	24.	A	U	1.5	F	F2, F3
7.	A	K	1.5	C	C1	25.	D	U	1.5	F	F2, F3
8.	B	U	1.5	C	C3	26.	C	U	1.5	F	F3, F4
9.	D	U	1.5	C	C2	27.	B	U	1.5	F	F3, F4
10.	B	U	1.5	C, A	C3, A1	28.	D	H	1.5	F	F3, F4
11.	C	H	1.5	C	C2	29.	B	K	1.5	G	G8
12.	B	K	1.5	D	D1	30.	C	U	1.5	G	G5
13.	D	U	1.5	D	D4	31.	C	U	1.5	G	G6
14.	D	U	1.5	D	D3	32.	A	U	1.5	G	G2
15.	A	U	1.5	D	D6	33.	B	U	1.5	G	G4
16.	D	U	1.5	D	D6	34.	D	H	1.5	G	G8
17.	B	U	1.5	D	D6	35.	A	H	1.5	G	G5
18.	B	H	1.5	D	D1	36.	B	H	1.5	G, A	G3, A1

**Multiple Choice = 54 marks**

**Part B: Written Response**

<b>Q</b>	<b>B</b>	<b>C</b>	<b>S</b>	<b>CO</b>	<b>PLO</b>
1a.	1	U	3	B	B2
1b.	2	U	2	B	B2
2a.	3	U	2	D	D3, D2
2b.	4	U	3	D, A	D3; A1
3a.	5	U	4	C	C4
3b.	6	U	1	C	C4
4.	7	U	6	F, A	F3; A1

**Written Response = 21 marks**

**Part C: Case Studies**

<b>Q</b>	<b>B</b>	<b>C</b>	<b>S</b>	<b>CO</b>	<b>PLO</b>
5a.	8	U	4	G, A	G1, 2, 3; A1
5b.	9	U	2	G, A	G1, 2, 3; A1
5c.	10	U	4	G, A	G1, 2, 3; A1
5d.	11	K	2	G, A	G1, 2, 3; A1
6a.	12	U	2	E, A	E1, 2, 3, 4; A2
6b.	13	U	3	E, A	E1, 2, 3, 4; A2
6c.	14	U	3	E, A	E1, 2, 3, 4; A2
6d.	15	U	3	E, A	E1, 2, 3, 4; A2
6e.	16	H	2	E, A	E1, 2, 3, 4; A2

**Case Studies = 25 marks**

Multiple Choice = 54 (36 questions)

Written Response = 21 (4 questions)

Case Studies = 25 (2 questions)

**EXAMINATION TOTAL = 100 marks**

**LEGEND:**

**Q** = Question Number

**B** = Score Box Number

**PLO** = Prescribed Learning Outcome

**K** = Keyed Response

**S** = Score

**C** = Cognitive Level

**CO** = Curriculum Organizer

**PART B: WRITTEN RESPONSE**

**Value: 21 marks**

**Suggested Time: 30 minutes**

1. Commuters to a certain city either drive to work or use public transit. At present, two-thirds of commuters drive, while one-third use public transit. City planners have determined that 85% of commuters who use public transit will continue to use transit the following year, while 75% of car drivers will continue to drive.

a) After three years, what percent of commuters will be using public transit? **(3 marks)**

** Solution**

The required transition matrix for this problem is  $T$ , where

$$T = \begin{matrix} & \begin{matrix} \text{To} \\ \text{transit} & \text{car} \end{matrix} \\ \begin{matrix} \text{From} \\ \text{transit} \\ \text{car} \end{matrix} & \begin{bmatrix} 0.85 & 0.15 \\ 0.25 & 0.75 \end{bmatrix} \end{matrix} \quad \leftarrow 1 \text{ mark}$$

The initial state matrix is  $X$ , where  $X = \begin{matrix} & \begin{matrix} \text{transit} & \text{car} \end{matrix} \\ \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \end{bmatrix} \end{matrix} \quad \leftarrow \frac{1}{2} \text{ mark}$

Calculate  $XT^3 = [0.562 \quad 0.438] \quad \leftarrow 1 \text{ mark}$

$\therefore$  56% of commuters will be using public transit after three years.  $\leftarrow \frac{1}{2} \text{ mark}$

**Note:** The transition matrix can also be written:

$$T = \begin{matrix} & \text{To} \\ \text{From} & \begin{bmatrix} 0.75 & 0.25 \\ 0.15 & 0.85 \end{bmatrix} \end{matrix} \quad \text{or} \quad T = \begin{matrix} & \text{To} \\ \text{From} & \begin{bmatrix} 0.85 & 0.25 \\ 0.15 & 0.75 \end{bmatrix} \end{matrix} \quad \text{or} \quad T = \begin{matrix} & \text{To} \\ \text{From} & \begin{bmatrix} 0.75 & 0.15 \\ 0.25 & 0.85 \end{bmatrix} \end{matrix}$$

b) If the trend continues, what percent of commuters will eventually be using public transit?  
(2 marks)

** Solution**

To find the steady-state solution, solve the system:

$$XT = [x \quad y] \begin{bmatrix} 0.85 & 0.15 \\ 0.25 & 0.75 \end{bmatrix} = [x \quad y] \quad \leftarrow 1 \text{ mark}$$

$$x + y = 1$$

$$0.85x + 0.25y = x$$

$$0.15x + 0.75y = y$$

$$x = 0.625$$

$$y = 0.375 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$\therefore$  62.5% of commuters will eventually be using public transit.  $\leftarrow \frac{1}{2} \text{ mark}$

** Alternate Solution 1**

Multiply the initial state matrix by successively higher powers of the transition matrix until the steady state is reached.

$$\begin{bmatrix} \frac{1}{3} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} 0.85 & 0.15 \\ 0.25 & 0.75 \end{bmatrix}^{45} = [0.625 \quad .375]$$

$$\begin{bmatrix} \frac{1}{3} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} 0.85 & 0.15 \\ 0.25 & 0.75 \end{bmatrix}^{46} = [0.625 \quad .375] \quad \leftarrow 1 \text{ mark}$$

$\therefore$  62.5% of commuters will eventually be using public transit.  $\leftarrow 1 \text{ mark}$

** Alternate Solution 2**

Raise the transition matrix by a high enough power so that the row entries in the solution are identical.

$$\begin{bmatrix} 0.85 & 0.15 \\ 0.25 & 0.75 \end{bmatrix}^{50} = \begin{bmatrix} 0.625 & 0.375 \\ 0.625 & 0.375 \end{bmatrix} \quad \leftarrow 1 \text{ mark}$$

$\therefore$  62.5% of commuters will eventually be using public transit.  $\leftarrow 1 \text{ mark}$

2. The tides on the Pacific coast of BC follow a sinusoidal pattern. The depth of the water at a BC coast resort was measured eight times during one day by using times for a 24-hour clock. Data for these measurements is as follows.

Time (hours)	Depth (metres)
1:00	4.38
3:00	7.63
5:00	8.40
7:00	5.98
9:00	2.64
11:00	1.51
13:00	3.67
15:00	7.07

a) Determine the sine regression equation for this data.

**(2 marks)**

### **Solution**

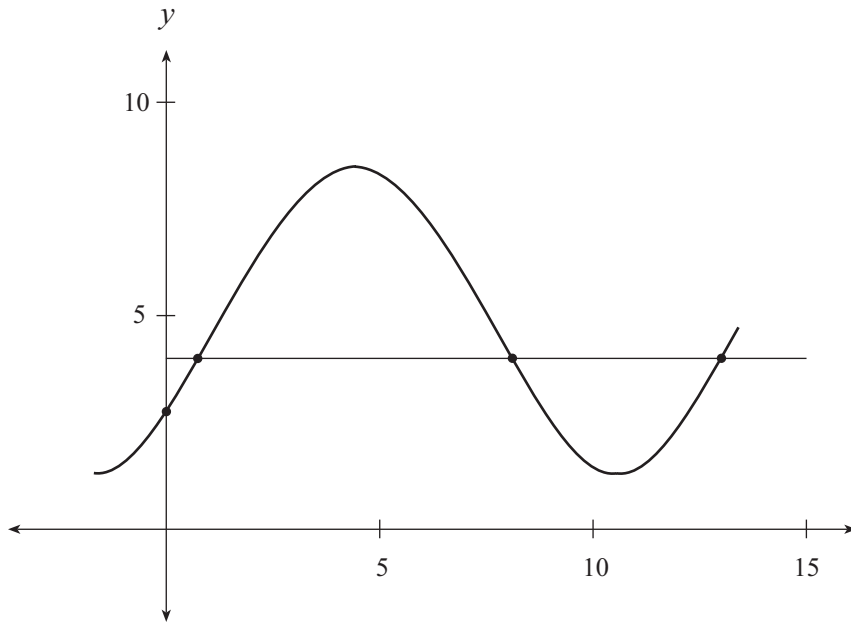
The sine regression equation is  $y = 3.5383\dots \sin(0.5060\dots x - 0.6814\dots) + 4.9995\dots$  or

$$y = 3.54 \sin(0.51x - 0.68) + 5.00$$

← **2 marks**

b) For how long was the depth of the water less than 4 m during the first 12 hours? (3 marks)

**Solution**



Graph the regression equation  
as  $y_1$

Determine its intersection  
with  $y_2 = 4$

This yields  $x$  values of  
0.781, 8.121 and 13.197 ← 1 mark  
so the depth of the water is  
less than 4 m in the first  
12 hours from

$$0 \text{ to } 0.78 = 0.78$$

$$8.12 \text{ to } 12 = 3.88 \quad \leftarrow 1 \text{ mark}$$

$$\underline{\quad\quad\quad} \\ 4.66 \text{ h} \quad \leftarrow 1 \text{ mark}$$

**1  $\frac{1}{2}$  marks** for 8.12 to 13.197

**2 marks** for 3.88 h for 8.121 to 12

**2  $\frac{1}{2}$  marks** for 5.86 h for 0 to 0.781 and 8.121 to 13.197

3. An investment portfolio of \$50 000 was held from Dec. 31, 1997 to Dec. 31, 2000. One-fourth of the portfolio was invested in guaranteed investment certificates (GICs) that paid 6.5% compounded annually. The remaining part of the portfolio was in Canadian stocks that traced the Toronto Stock Exchange (TSE) index. On Dec. 31, 1997, the TSE index was at 6 486. On Dec. 31, 2000, the TSE index was at 10 500.

a) What was the value of the total portfolio on Dec. 31, 2000?

**(4 marks)**

### **Solution**

$$\text{Value of GIC} = 12\,500(1.065)^3 = 15\,099.37$$

$$\begin{array}{cccc} \uparrow & \uparrow & \uparrow & \uparrow \\ \underbrace{\hspace{10em}} & & & \\ \frac{1}{2} \text{ mark each} & & & \end{array}$$

$$\text{Value of stocks} = 37\,500 \left( \frac{10\,500}{6\,486} \right) = 60\,707.68$$

$$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ \frac{1}{2} \text{ mark} & \frac{1}{2} \text{ mark} & \frac{1}{2} \text{ mark} \end{array}$$

$$\text{Total} = \$75\,807.05 \quad \leftarrow \frac{1}{2} \text{ mark}$$

### **Alternate Solution**

Using TVM Solver on the TI-83 calculator:

Enter:

$$N = 3 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$I\% = 6.5 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$PV = -12\,500 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$PMT = 0$$

■  $FV = 15\,099.37 \quad \leftarrow \frac{1}{2} \text{ mark}$

$$P/Y = 1$$

$$C/Y = 1$$

The solver gives the solution \$15 099.37

b) What is the percentage increase in the value of the portfolio after three years?  
(Give your answer as a percentage accurate to one decimal place.)

**(1 mark)**

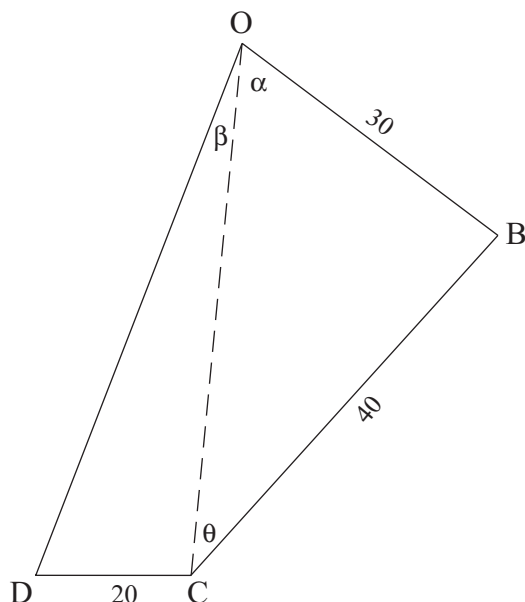
** Solution**

$$\frac{75\,807.05 - 50\,000}{50\,000} \times 100 = 51.6\% \quad \leftarrow \mathbf{1 \text{ mark}}$$



4. A ship leaves Ocean Harbour on a practice mission. It has orders to first go 30 km on a bearing of  $135^\circ$  to Birch Island, then 40 km on a bearing of  $225^\circ$  to Port Cod and finally 20 km on a bearing of  $270^\circ$  to Destination. What is the distance and bearing of Destination from Ocean Harbour? (Give distance to the nearest tenth and bearing to the nearest degree.) **(6 marks)**

**Solution**



$$OC = \sqrt{30^2 + 40^2} \quad (\text{because } \angle OBC = 90^\circ) \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$OC = 50 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$\theta = \tan^{-1}\left(\frac{30}{40}\right)$$

$$\theta = 36.869\dots^\circ \quad \leftarrow 1 \text{ mark}$$

$$\alpha = 53.13\dots^\circ$$

$$\angle OCD = 180^\circ - 45^\circ - 36.869\dots$$

$$\angle OCD = 98.131\dots^\circ \quad \leftarrow 1 \text{ mark}$$

$$(OD)^2 = 20^2 + 50^2 - 2(20)(50)\cos 98.131\dots^\circ$$

$$OD = 56.41\dots$$

Distance of Ocean Harbour to Destination is 56.4 km  $\leftarrow 1 \text{ mark}$

$$\frac{\sin 98.13}{56.41\dots} = \frac{\sin \beta}{20}$$

$$\angle \beta = 20.54^\circ \quad \leftarrow 1 \text{ mark}$$

Bearing of Destination from Ocean Harbour is  $135^\circ + 53.13^\circ + 20.54^\circ = 209^\circ \leftarrow 1 \text{ mark}$

## PART C: CASE STUDIES

Value: 25 marks

Suggested Time: 30 minutes

5. As part of its yearly plan, a recreation centre surveyed its community and gathered the following data. A frequency distribution of the number of children under 18 years of age from 100 families selected at random is shown below.

$x$	$f$
0	6
1	13
2	20
3	23
4	19
5	12
6	7

a) Calculate the mean and standard deviation for the number of children in each family. (4 marks)

 **Solution**


$$\bar{x} = \frac{6(0) + 13(1) + 20(2) + 23(3) + 19(4) + 12(5) + 7(6)}{100} = 3 \quad \leftarrow 2 \text{ marks}$$

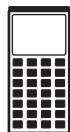
$$\sigma = \sqrt{\frac{1}{100} (6 \times 0^2 + 13 \times 1^2 + 20 \times 2^2 + 23 \times 3^2 + 19 \times 4^2 + 12 \times 5^2 + 7 \times 6^2) - 3^2} = 1.6 \quad \leftarrow 2 \text{ marks}$$

 **Alternate Solution** for the standard deviation.

$$\sigma = \sqrt{\frac{6(0-3)^2 + 13(1-3)^2 + 20(2-3)^2 + 23(3-3)^2 + 19(4-3)^2 + 12(5-3)^2 + 7(6-3)^2}{100}} = 1.6$$

↑  
2 marks

 **Alternate Solution** for the mean and the standard deviation.



Press   .

List values under  $L_1$  and  $L_2$

1 - Vars Stats  $L_1, L_2$

$$\bar{x} = 3$$

← 2 marks

$$\sigma = 1.6$$

← 2 marks

b) What is the probability of a family having from 2 to 4 children under 18 years of age?  
(2 marks)

 **Solution**

$$P(2 \text{ or } 3 \text{ or } 4 \text{ children}) = \frac{20 + 23 + 19}{100} = 0.62 \quad \leftarrow \text{2 marks}$$



d) Why are the answers to part b) and part c) slightly different?

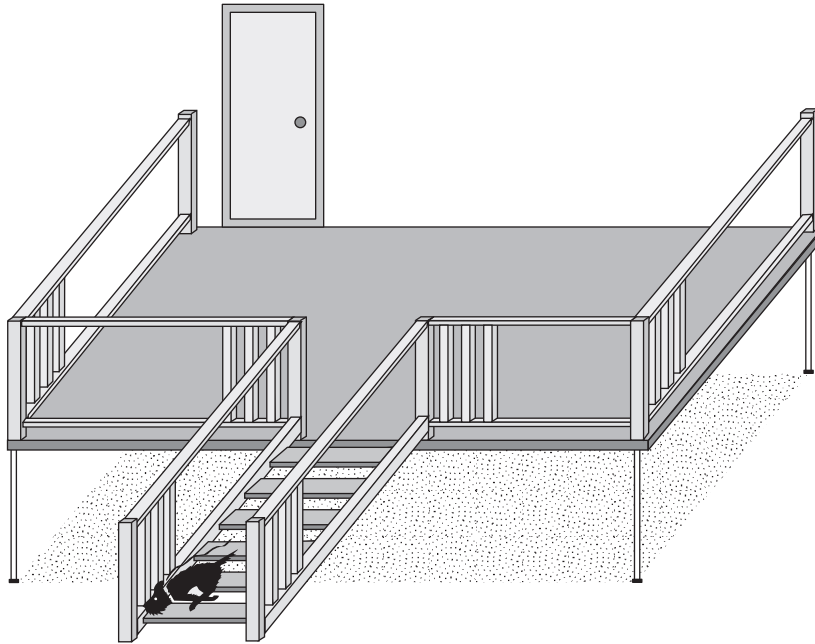
(2 marks)

 **Solution**

The binomial distribution is calculated from discrete (counting) data, while the normal distribution assumes continuous data. This is why the normal distribution is an approximation of the exact binomial value.

} ← 2 marks

6. A patio attached to the back of a house is going to be rebuilt by replacing the deck of the patio, putting up new railings and replacing the stairs leading to the deck of the patio.



- a) The deck of the patio is 24 feet by 8 feet. Sheets of plywood are used to form the deck of the patio. Each 8 foot by 4 foot sheet of plywood costs \$23.95. Determine the cost of plywood for the deck. Do not include taxes in your calculations. **(2 marks)**

**Solution**

$$6 \times 23.95 = \$143.70$$

↑

**1 mark**

↑

**1 mark**

- b) The deck of the patio needs a waterproof coating. The coating will cost \$9.75 for each 12 square feet of deck to be covered. Determine the cost of waterproofing the surface of the deck. Do not include taxes in your calculations. **(3 marks)**

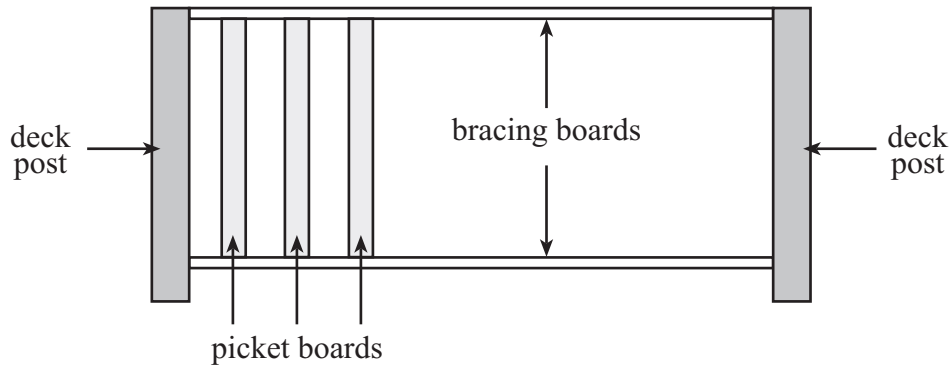
**Solution**

$$24 \times 8 = 192 \text{ ft}^2 \quad \leftarrow \text{1 mark}$$

$$\frac{192}{12} = 16 \quad \leftarrow \text{1 mark}$$

$$16 \times \$9.75 = \$156.00 \quad \leftarrow \text{1 mark}$$

c) Sections of the deck railings are built as shown below.



The total amount of deck railing, excluding the stairs, is 36 feet. There are three picket boards for every foot. The costs of the materials are as follows.

- cost of each deck post is \$5.95
- cost of each bracing board is \$6.50
- cost of each picket board is \$2.10

Determine the cost of the materials needed to rebuild the deck railings. Do not include taxes in your calculations.

**(3 marks)**

**Solution**

$$6 \times \$5.95 + 8 \times \$6.50 + 36 \times 3 \times \$2.10 = \$314.50$$

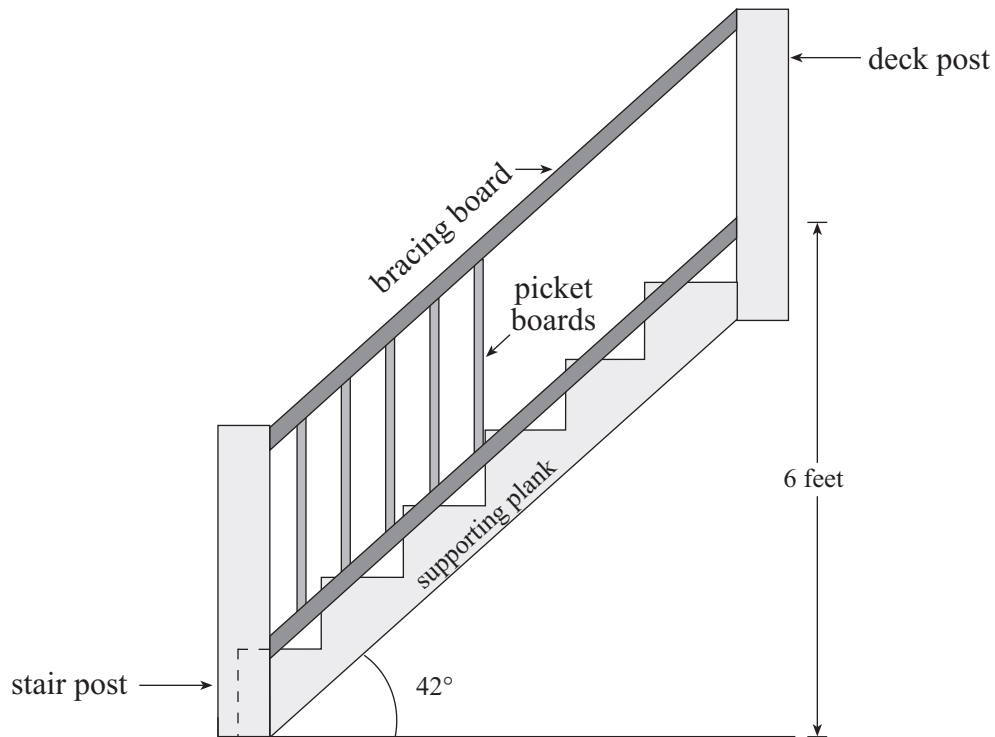


↑  
**2 marks**

↑  
**1 mark**



- d) A side view of the stairs leading up to the deck of the patio is shown below. There are six steps each measuring four feet in length.



The costs of the materials are as follows.

- a supporting plank is needed on each side; these are sold in 8 ft., 10 ft., 12 ft., etc. lengths and cost \$1.45 a linear foot
- bracing boards to hold picket boards on each side of the stairs cost \$6.50 for each board
- two stair posts at the bottom of the stairs cost \$5.95 each
- picket boards are \$2.10 each with 27 picket boards required for each stair railing
- planks for the steps can be purchased for \$1.45 a linear foot

Determine the cost of the materials needed to build the stairs. Do not include taxes in your calculations.

**(3 marks)**

### **Solution**

$$10 \times 2 \times \$1.45 + 4 \times \$6.50 + 2 \times \$5.95 + 27 \times 2 \times \$2.10 + 6 \times 4 \times \$1.45 = \$215.10$$

$$\underbrace{\quad\quad\quad}_{\uparrow} \quad \underbrace{\quad\quad\quad}_{\uparrow} \quad \underbrace{\quad\quad\quad}_{\uparrow} \quad \underbrace{\quad\quad\quad}_{\uparrow} \quad \underbrace{\quad\quad\quad}_{\uparrow} \quad \underbrace{\quad\quad\quad}_{\uparrow}$$

$\frac{1}{2}$  mark     $\frac{1}{2}$  mark     $\frac{1}{2}$  mark     $\frac{1}{2}$  mark     $\frac{1}{2}$  mark     $\frac{1}{2}$  mark

e) What other expenses might a homeowner have if he or she rebuilds this patio?

**(2 marks)**

** Solution**

**Any TWO of the following for 1 mark each**

- cost of nails, metal braces, sandpaper, and other miscellaneous items
- cost of transporting materials
- renting tools, etc.
- tools for spreading the waterproof coating
- municipal approval fees
- PST, GST

**END OF KEY**