Applications of Mathematics 12

January 2001 Provincial Examination

ANSWER KEY / SCORING GUIDE

CURRICULUM:

Organizers		**Sub-Organizers**
1. Problem Solving	A	Problem Set
2. Number	B	Matrices
	C	Financial Decision-Making
3. Patterns and Relations	D	Fractals
	E	Linear Programming
	F	Non-Linear Functions
4. Shape and Space	G	Periodic Functions
	H	Geometry Applications
5. Statistics and Probability	I	Data Analysis
	J	Applications of Probability

Part A: Multiple Choice

Q	K	C	S	CO	PLO	Q	K	C	S	CO	PLO
1.	D	K	1	2	B1	24.	C	U	1	4	G3
2.	A	U	1	2	B3	25.	B	H	1	4	G1, G2
3.	A	U	1	2	B2	26.	A	K	1	4	H1
4.	C	U	1	2	B2	27.	D	U	1	4	H1
5.	A	U	1	2	B2	28.	D	U	1	4	H1
6.	D	U	1	2	B3	29.	C	U	1	4	H1
7.	C	H	1	2	B3	30.	D	H	1	4	H1
8.	D	U	1	2	C1	31.	A	H	1	4	H1
9.	D	U	1	2	C1	32.	B	K	1	5	I2
10.	C	H	1	2	C2	33.	A	U	1	5	I2
11.	D	K	1	3	D2	34.	B	H	1	5	I2
12.	B	U	1	3	D1	35.	B	K	1	5	J5
13.	A	U	1	3	D4	36.	B	U	1	5	J4
14.	A	U	1	3	D4	37.	A	U	1	5	J3
15.	C	H	1	3	D3	38.	C	U	1	5	J6
16.	C	K	1	3	E2	39.	C	U	1	5	J7
17.	D	H	1	3	E3	40.	B	H	1	5	J1, J2
18.	D	K	1	3	F2	41.	B	H	1	5	J5
19.	B	U	1	3	F3	42.	C	U	1	1	A1
20.	B	U	1	3	F3	43.	C	U	1	1	A1
21.	B	H	1	3	F2	44.	B	H	1	1	A2
22.	C	U	1	4	G4	45.	B	H	1	1	A2
23.	A	U	1	4	G4						

Multiple Choice = 45 marks

Part B: Written Response

Q	B	C	S	CO	PLO
1a.	1	U	1	5	I2
1b.	2	U	2	5	I2
2a.	3	U	2	2	B3
2b.	4	H	1	2	B3
3.	5	U	3	2	C2
4.	6	U	3	5	J5
5.	7	U	3	1	A2
6.	8	U	4	3	E4
7.	9	U	2	3	F3
8a.	10	U	3	4	H2
8b.	11	U	1	4	H2

Written Response = 25 marks

Multiple Choice = 45 (45 questions)
Written Response = 25 (8 questions)
EXAMINATION TOTAL = 70 marks

LEGEND:

Q = Question Number
K = Keyed Response
C = Cognitive Level
B = Score Box Number
S = Score
CO = Curriculum Organizer
PLO = Prescribed Learning Outcome

PART B: WRITTEN RESPONSE

Value: 25 marks | **Suggested Time: 45 minutes**

INSTRUCTIONS: Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

If, in a justification, you refer to information produced by the calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem, it is important to sketch the graph, showing its general shape and indicating the appropriate window dimensions.

When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.

Full marks will NOT be given for the final answer only.

1. A restaurant chain gathered data to relate advertising costs and sales. This is summarized in the following table:

Advertising Costs ($)	Sales ($)
1 000	19 000
4 000	44 000
10 000	52 000
14 000	53 000

a) What is the correlation coefficient for the least squares line of best fit? **(1 mark)**

Solution

Enter data in graphing calculator
$r = 0.8586601594$ } $\leftarrow$ **1 mark**

b) Determine the least squares linear regression equation and predict the expected sales if $20 000 was spent on advertising. (Give your answer to the nearest $1 000.) **(2 marks)**

Solution

Using linear regression:
$y = 2.326\ldots x + 25136.25\ldots$ ← **1 mark**

if $x = 20\,000$
$y = 71\,656.93$
$\approx \$72\,000$ ← **1 mark**

2. A survey revealed that 30% of a certain population smokes. Each year, 20% of the smokers quit and 5% of the non-smokers start.

a) Determine the percentage of smokers after 2 years. **(2 marks)**

Solution

Transition matrix:

$$\textbf{From}\quad \begin{array}{c} \\ S \\ NS \end{array} \begin{array}{c} \textbf{To} \\ \begin{array}{cc} S & NS \end{array} \\ \begin{bmatrix} .8 & .2 \\ .05 & .95 \end{bmatrix} \end{array} \qquad \leftarrow \textbf{1 mark}$$

$$\underbrace{\begin{bmatrix} .3 & .7 \end{bmatrix} \begin{bmatrix} .8 & .2 \\ .05 & .95 \end{bmatrix}^2}_{\uparrow \ \frac{1}{2}\ \textbf{mark}} = \begin{bmatrix} .26 & .74 \end{bmatrix}$$

$= 26\%$ $\quad \leftarrow \frac{1}{2}$ **mark**

b) Determine the percentage of smokers in the long term. **(1 mark)**

Solution

$$\underbrace{\begin{bmatrix} .8 & .2 \\ .05 & .95 \end{bmatrix}^{100}}_{\uparrow\ \frac{1}{2}\text{ mark}} = \begin{bmatrix} .2 & .8 \\ .2 & .8 \end{bmatrix}$$

$= 20\%$ $\leftarrow$ **$\frac{1}{2}$ mark**

Alternate Solution

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} .8 & .2 \\ .05 & .95 \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix} \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$\begin{aligned} .8x + .05y &= x \\ .2x &= .05y \\ 20x &= 5y \\ 4x &= y \end{aligned}$$

$$\begin{aligned} x + y &= 1 \\ x + 4x &= 1 \\ 5x &= 1 \\ x &= \frac{1}{5} = 20\% \qquad \leftarrow \tfrac{1}{2}\textbf{ mark} \end{aligned}$$

3. Amy deposited \$2 000 into a savings fund earning 9% compounded annually on each of her 24^{th}, 25^{th} and 26^{th} birthdays. If she makes no additional deposits after these three, but leaves the accumulated amount in the account earning 9% compounded annually, how much will she have in the account when she retires on her 60^{th} birthday? **(3 marks)**

Solution

1 mark **1 mark**

$$2\,000(1.09)^{36} + 2\,000(1.09)^{35} + 2\,000(1.09)^{34} = \$122\,787.21$$

$\therefore$ Amy will have \$122 787.21 in the account when she retires at age 60.

1 mark

Alternate Solution 1

First find the amount on deposit after three years.

$\$2\ 000 + \$2\ 000(1.09) + \$2\ 000(1.09)^2 = \$6\ 556.20 \quad \leftarrow$ **1 mark**

Then find the amount when \$6 556.20 is left for an additional 34 years.

$\$6\ 556.20(1.09)^{34} = \$122\ 787.21$

1 mark **1 mark**

$\therefore$ Amy will have \$122 787.21 in the account when she retires at age 60.

3. Amy deposited \$2 000 into a savings fund earning 9% compounded annually on each of her 24th, 25th and 26th birthdays. If she makes no additional deposits after these three, but leaves the accumulated amount in the account earning 9% compounded annually, how much will she have in the account when she retires on her 60th birthday? **(3 marks)**

Alternate Solution 2

TVM Solver:

Enter:

$$\left.\begin{array}{r} N = 3 \\ I\% = 9 \\ PV = 0 \\ PMT = -2\ 000 \\ \blacksquare\ FV = 6\ 556.2 \\ P/Y = 1 \\ C/Y = 1 \end{array}\right\} \leftarrow \textbf{1 mark}$$

After three years Amy has \$6 556.20 on deposit. ← **½ mark**

Enter:

$$\left.\begin{array}{r} N = 34 \\ I\% = 9 \\ PV = -6\ 556.20 \\ PMT = 0 \\ \blacksquare\ FV = 122\ 787.2078 \\ P/Y = 1 \\ C/Y = 1 \end{array}\right\} \leftarrow \textbf{1 mark}$$

∴ at age 60 Amy will have \$122 787.21 on deposit. ← **½ mark**

4. A newly developed antibiotic causes harmful side effects in 25% of the patients treated with the drug. If this medication is given to 100 randomly selected patients, what is the probability that 30 or more of them will suffer from these side effects? **(3 marks)**

Solution

$n = 100$

$p = 0.25$

$q = 0.75$

$\mu = 100(0.25) = 25$ ← $\frac{1}{2}$ **mark**

$\sigma = \sqrt{100(0.25)(0.75)} = 4.33$ ← $\frac{1}{2}$ **mark**

Using continuity correction ← $\frac{1}{2}$ **mark**

$z = \dfrac{29.5 - 25}{4.33} = 1.039$ ← $\frac{1}{2}$ **mark**

Area under curve is $0.3508 + 0.5 = 0.8508$ ← $\frac{1}{2}$ **mark**

Required probability is $1 - 0.8508 = 0.1492$ ← $\frac{1}{2}$ **mark**

Alternate Solution

Using the binomial cdf on a graphing calculator,

$$\underset{\substack{\uparrow \\ \frac{1}{2}\ \textbf{mark}}}{1 -} \text{binomcdf}\underbrace{(100,\ 0.25,\ 29)}_{\substack{\uparrow \\ \textbf{2 marks}}} \underset{\substack{\uparrow \\ \frac{1}{2}\ \textbf{mark}}}{= 0.1495}$$

5. A car rental company has 200 cars. The company can rent out all the cars if the price is \$36 per day. With each \$2 increase per day, 5 fewer cars are rented out. What is the maximum net revenue? **(3 marks)**

Solution

Revenue = (no. of cars)(price per car) ← $\frac{1}{2}$ **mark**

Let x represent the number of \$2 increases ← $\frac{1}{2}$ **mark**

Revenue = $(200-5x)(36+2x)$ ← $\frac{1}{2}$ **mark**

$$R = 7\,200 + 220x - 10x^2$$

Graph produces maximum at $(11,\ 8\,410)$
or completing the square shows vertex at $(11,\ 8\,410)$ ← **1 mark**

Maximum net revenue is \$8 410 ← $\frac{1}{2}$ **mark**

Alternate Solution

No. of cars	Cost \$	Revenue \$
200	36	7 200
195	38	7 410
190	40	7 600
.	.	.
.	.	.
.	.	.
150	56	8 400
145	58	8 410
140	60	8 400

← **1 $\frac{1}{2}$ marks** for chart

By symmetry of quadratic function ← **1 mark**
Maximum revenue occurs at \$8 410 ← $\frac{1}{2}$ **mark**

6. Fertilizer requirements and costs for maintaining the grass at a golf course are outlined in the table below.

	Type A	Type B	Minimum required
	kilograms per m^3		
Phosphoric acid	20	10	690
Nitrogen	30	30	1 440
Potash	5	10	330
Cost per m^3	\$30	\$35	

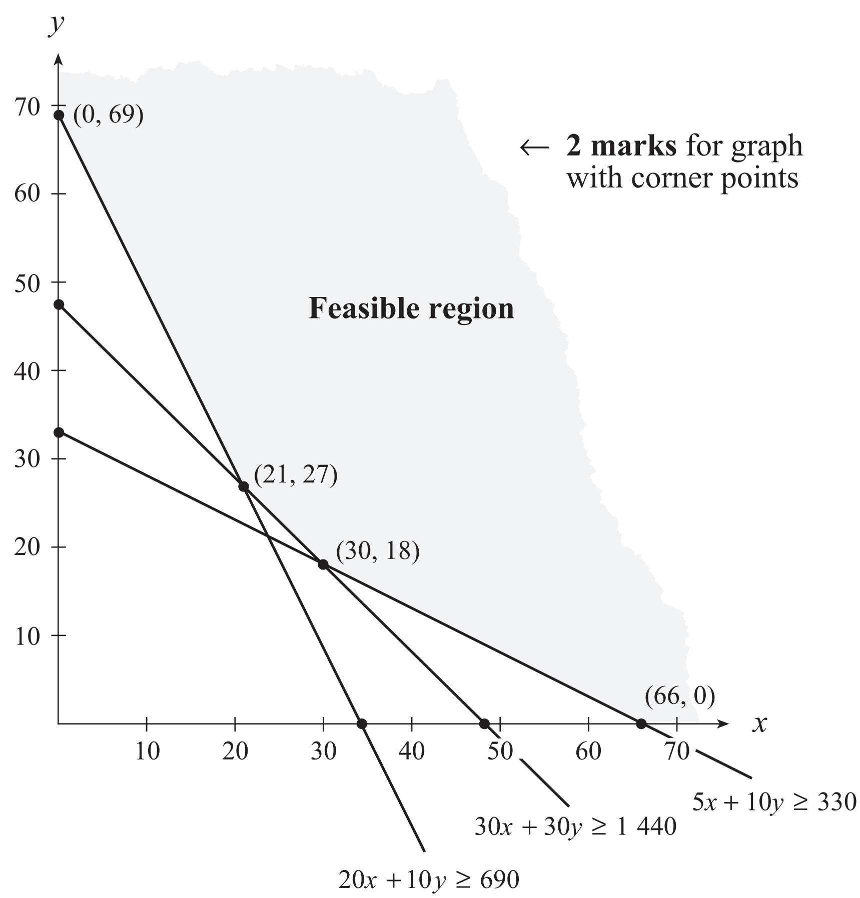
Let x represent the number of m^3 of Type A fertilizer and y represent the number of m^3 of Type B fertilizer. List the constraints and objective function, then solve the linear programming problem to determine the minimum cost of fertilizer for the golf course.

(4 marks)

Solution

$$20x + 10y \geq 690$$
$$30x + 30y \geq 1\,440$$
$$5x + 10y \geq 330$$
$$x \geq 0$$
$$y \geq 0$$
$$C = 30x + 35y$$

1 mark for constraints



← **2 marks** for graph with corner points

Test corner points

$(0,\ 69) \rightarrow C = \$2\,415$

$(21,\ 27) \rightarrow C = \$1\,575$

$(30,\ 18) \rightarrow C = \$1\,530$

$(66,\ 0) \rightarrow C = \$1\,980$

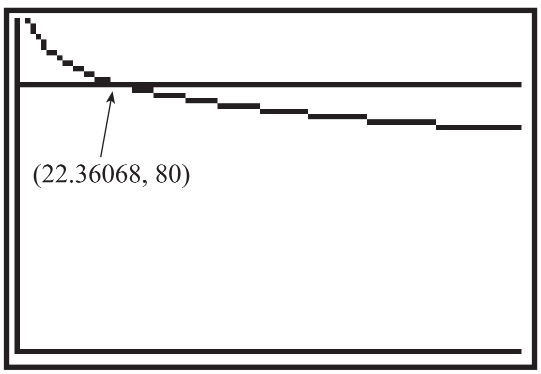
$\therefore$ minimum cost is \$1 530

1 mark for values at corner points and answer

7. The formula $D = 10\log\left(\dfrac{5\times10^{10}}{r^2}\right)$ gives the decibel level, D, of sound at a rock concert at r rows back from the stage. How many rows back can Jen sit and still hear the music at a level of at least 80 decibels? **(2 marks)**

If providing a graphical solution, state the function(s) used, sketch the graph, indicate appropriate window dimensions and clearly explain how your solution is derived from the graph.

Solution



$$\left.\begin{aligned} Y_1 &= 10\log\left(\frac{5\times10^{10}}{x^2}\right) \\ Y_2 &= 80 \end{aligned}\right\} \leftarrow \textbf{1 mark} \text{ for equations}$$

$\leftarrow \tfrac{1}{2}$ **mark** for graph

$x\ [0,\ 100]$ $\quad$ $y\ [0,\ 100]$

$\therefore$ Jen can sit 22 rows back and still hear 80 decibels of sound. $\leftarrow \tfrac{1}{2}$ **mark**

7. The formula $D = 10\log\left(\frac{5\times10^{10}}{r^2}\right)$ gives the decibel level, D, of sound at a rock concert at r rows back from the stage. How many rows back can Jen sit and still hear the music at a level of at least 80 decibels? **(2 marks)**

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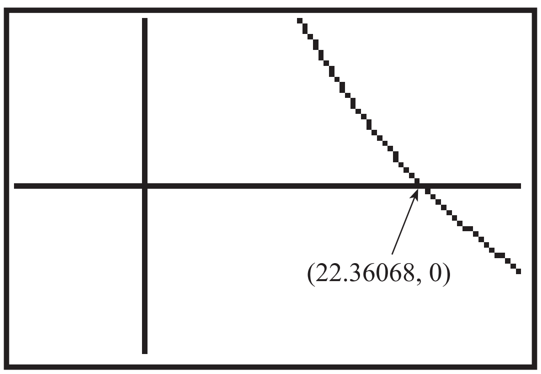
Alternate Solution 1



x $[-10,\ 30]$ y $[-5,\ 5]$

$\mathrm{Y}_1 = 10\log\left(\frac{5\times10^{10}}{x^2}\right) - 80$ $\leftarrow$ **1 mark** for equation

$\leftarrow$ **½ mark** for graph

Zero occurs at 22.36068

$\therefore$ Jen can sit 22 rows back and still hear 80 decibels of sound. $\leftarrow$ **½ mark**

7. The formula $D = 10\log\left(\frac{5\times10^{10}}{r^2}\right)$ gives the decibel level, D, of sound at a rock concert at r rows back from the stage. How many rows back can Jen sit and still hear the music at a level of at least 80 decibels? **(2 marks)**

If providing a graphical solution, state the function(s) used, sketch the graph, indicate appropriate window dimensions and clearly explain how your solution is derived from the graph.

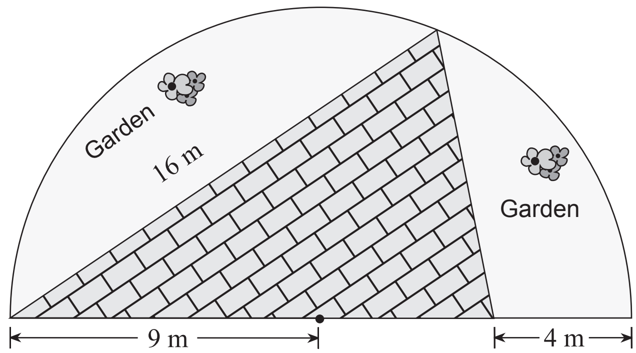
Alternate Solution 2

Algebraically:

$$\left.\begin{aligned}
10\log\left(\frac{5\times10^{10}}{r^2}\right) &= 80\\
\log\left(\frac{5\times10^{10}}{r^2}\right) &= 8\\
\frac{5\times10^{10}}{r^2} &= 10^8\\
r^2 &= 500\\
r &= \sqrt{500}\\
&= 22.36
\end{aligned}\right\} \leftarrow \textbf{1 mark}$$

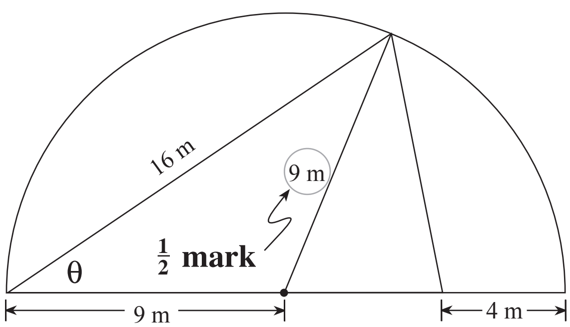
$\therefore$ Jen can sit 22 rows back and still hear 80 decibels of sound. $\leftarrow$ **1 mark**

8. A solarium is semicircular in shape with a radius of 9 m. It consists of a triangular-tiled area and gardens in the non-tiled area, as shown in the diagram.



a) Determine the area of the garden. **(3 marks)**

Solution



$$\left.\begin{aligned}\cos\theta &= \frac{16^2+9^2-9^2}{2(16)(9)} = 0.\overline{8}\\ \theta &= 27.2660445^\circ\end{aligned}\right\} \leftarrow \textbf{1 mark}$$

$$\left.\begin{aligned}\therefore\ A_t &= \frac{1}{2}(16)(14)\sin\theta\\ &= 51.3097589\ \text{m}^2\end{aligned}\right\} \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$\left.\begin{aligned}A_s - A_t &= \frac{\pi 9^2}{2} - A_t\\ &= 75.9247\ \text{m}^2\end{aligned}\right\} \leftarrow \textbf{1 mark}$$

b) If the topsoil in the garden is 0.2 m deep and costs $\$25/\text{m}^3$, determine the cost of the topsoil. **(1 mark)**

Solution

$$\begin{aligned} V &= 75.9247 \times 0.2 \quad && \leftarrow \tfrac{1}{2} \textbf{ mark} \\ &= 15.1849 \text{ m}^3 \\ \text{Cost} &= 15.1849 \times 25 \\ &= \$379.62 && \leftarrow \tfrac{1}{2} \textbf{ mark} \end{aligned}$$

END OF KEY