# Applications of Mathematics 12

## June 2000 Provincial Examination

## ANSWER KEY / SCORING GUIDE

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**CURRICULUM:**

| Organizers | | Sub-Organizers | |
|---|---|---|---|
| 1. | Problem Solving | A | Problem Set |
| 2. | Number | B | Matrices |
| | | C | Financial Decision-Making |
| 3. | Patterns and Relations | D | Fractals |
| | | E | Linear Programming |
| | | F | Non-Linear Functions |
| 4. | Shape and Space | G | Periodic Functions |
| | | H | Geometry Applications |
| 5. | Statistics and Probability | I | Data Analysis |
| | | J | Applications of Probability |

**Part A: Multiple Choice**

| Q | K | C | CO | PLO | Q | K | C | CO | PLO |
|---|---|---|---|---|---|---|---|---|---|
| 1. | A | K | 2 | B1 | 24. | B | H | 4 | G2 |
| 2. | B | U | 2 | B2 | 25. | A | K | 4 | H2 |
| 3. | B | U | 2 | B2 | 26. | D | U | 4 | H2 |
| 4. | A | U | 2 | B3 | 27. | B | U | 4 | H1 |
| 5. | C | U | 2 | B2 | 28. | D | U | 4 | H1 |
| 6. | C | H | 2 | B2 | 29. | D | U | 4 | H2 |
| 7. | C | H | 2 | B3 | 30. | C | H | 4 | H1 |
| 8. | C | U | 2 | C1 | 31. | D | H | 4 | H1 |
| 9. | B | U | 2 | C2 | 32. | B | K | 5 | I1 |
| 10. | B | U | 2 | C2 | 33. | A | U | 5 | I2 |
| 11. | B | U | 2 | C2 | 34. | C | U | 5 | I3 |
| 12. | A | U | 2 | C1 | 35. | C | H | 5 | I2 |
| 13. | B | H | 2 | C2 | 36. | A | K | 5 | J1 |
| 14. | D | K | 3 | D2 | 37. | D | U | 5 | J5 |
| 15. | C | H | 3 | D4 | 38. | C | U | 5 | J4 |
| 16. | B | K | 3 | E4 | 39. | C | U | 5 | J1 |
| 17. | D | H | 3 | E1 | 40. | D | H | 5 | J1 |
| 18. | B | K | 3 | F2 | 41. | D | H | 5 | J6 |
| 19. | C | U | 3 | F3 | 42. | B | U | 1 | A2 |
| 20. | D | H | 3 | F3 | 43. | D | U | 1 | A1 |
| 21. | A | U | 4 | G3 | 44. | A | H | 1 | A2 |
| 22. | A | U | 4 | G4 | 45. | A | H | 1 | A1 |
| 23. | D | U | 4 | G3 | | | | | |

**Multiple Choice = 45 marks**

## Part B: Written Response

| Q | B | C | S | CO | PLO |
|---|---|---|---|---|---|
| 1. | 1 | U | 2 | 5 | I2 |
| 2. | 2 | U | 3 | 3 | F1, 3 |
| 3. | 3 | U | 3 | 1 | A2 |
| 4a. | 4 | U | 1 | 2 | B3 |
| 4b. | 5 | U | 1 | 2 | B3 |
| 4c. | 6 | U | 1 | 2 | B3 |
| 5a. | 7 | U | 2 | 3 | D3 |
| 5b. | 8 | U | 1 | 3 | D3 |
| 6a. | 9 | U | 1 | 3 | E1 |
| 6b. | 10 | U | 2 | 3 | E4 |
| 6c. | 11 | U | 1 | 3 | E4 |
| 7a. | 12 | U | 1 | 5 | J3 |
| 7b. | 13 | U | 1 | 5 | J3 |
| 7c. | 14 | U | 2 | 5 | J3 |
| 8a. | 15 | U | 2 | 4 | H2 |
| 8b. | 16 | U | 1 | 4 | H2 |

**Written Response = 25 marks**

Multiple Choice = 45 (45 questions)
Written Response = 25 (8 questions)
**EXAMINATION TOTAL = 70 marks**

**LEGEND:**

| | | |
|---|---|---|
| **Q** = Question Number | **K** = Keyed Response | **C** = Cognitive Level |
| **B** = Score Box Number | **S** = Score | **CO** = Curriculum Organizer |
| **PLO** = Prescribed Learning Outcome | | |

## PART B: WRITTEN RESPONSE

**Value: 25 marks** **Suggested Time: 45 minutes**

**INSTRUCTIONS:** Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

If, in a justification, you refer to information produced by the calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem, it is important to sketch the graph, showing its general shape and indicating the appropriate window dimensions.

When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.

**Full marks will NOT be given for the final answer only.**

1. During an NHL hockey season, information comparing games played, $P$, to number of goals, $G$, was collected. The data for a player is shown in the table below.

| | | | |
|---|---|---|---|
| **Games played** *(P)* | 10 | 20 | 40 |
| **Number of goals** *(G)* | 13 | 25 | 37 |

Determine the least squares linear regression equation and predict the number of goals this player will score in the full 82-game season. **(2 marks)**

**Solution**

Using graphing calculator → $G = 0.7714\ldots\ P + 7$ ← **1 mark**

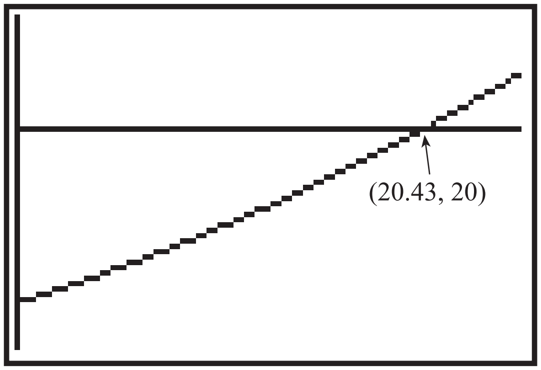
$(y = 0.7714x + 7)$

$G(82) = 70.3 \rightarrow$ 70 goals ← **1 mark**

2. The population of a certain country is 12 million and growing at a rate of 2.5% annually. Assuming that it is growing continuously, the population, $P$ (in millions), $t$ years from now is determined by the formula $P = 12e^{0.025t}$. Determine how long it will take the population to reach 20 million. **(3 marks)**

   If providing a graphical solution, state the function(s) used, sketch the graph, indicate appropriate window dimensions and clearly explain how your solution is derived from the graph.

## Solution



$$\left.\begin{array}{l} Y_1 = 20 \\ Y_2 = 12e^{0.025x} \end{array}\right\} \leftarrow \tfrac{1}{2}\ \textbf{mark}\ \text{for equations}$$

← **1 mark** for graph

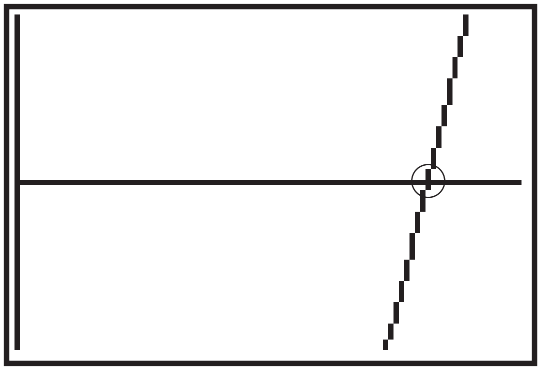
$x\ [0,\ 25]$ $y\ [10,\ 25]$ ← **$\frac{1}{2}$ mark** for window dimensions

Answer: 20.43 years ← **1 mark**

2. The population of a certain country is 12 million and growing at a rate of 2.5% annually. Assuming that it is growing continuously, the population, $P$ (in millions), $t$ years from now is determined by the formula $P = 12e^{0.025t}$. Determine how long it will take the population to reach 20 million. **(3 marks)**

   If providing a graphical solution, state the function(s) used, sketch the graph, indicate appropriate window dimensions and clearly explain how your solution is derived from the graph.

## Alternate Solution



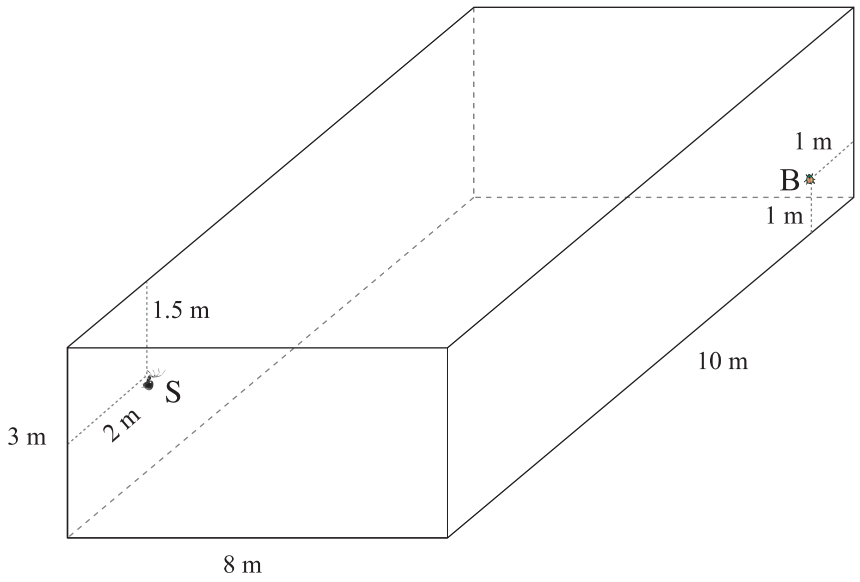
$Y_1 = 12e^{0.025x} - 20$ ← $\frac{1}{2}$ **mark** for equation

← **1 mark** for graph

$x\ [0,\ 25]$ $y\ [-1,\ 1]$ ← $\frac{1}{2}$ **mark** for window dimensions

Answer: 20.43 years ← **1 mark**

3. In the diagram below, a spider, S, is located on the wall 1.5 m from the ceiling and 2 m from the corner. The spider is heading to a spot on the opposite wall to eat a bug, B, located 1 m from the floor and 1 m from the corner. If the dimensions of the room are $8\text{ m} \times 10\text{ m} \times 3\text{ m}$, determine the shortest distance the spider has to crawl to get to the bug. **(3 marks)**



**Solution**

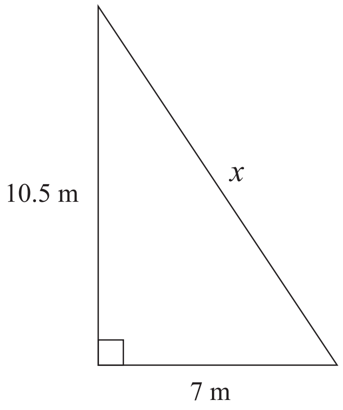
The spider must crawl:

$8\text{ m} + 1\text{ m} + 1.5\text{ m} = 10.5\text{ m}$ in one direction ← **1 mark**

and $10\text{ m} - 1\text{ m} - 2\text{ m} = 7\text{ m}$ in a perpendicular direction ← **1 mark**

Let $x$ be the shortest distance between S and B.

Using Pythagorean Theorem,



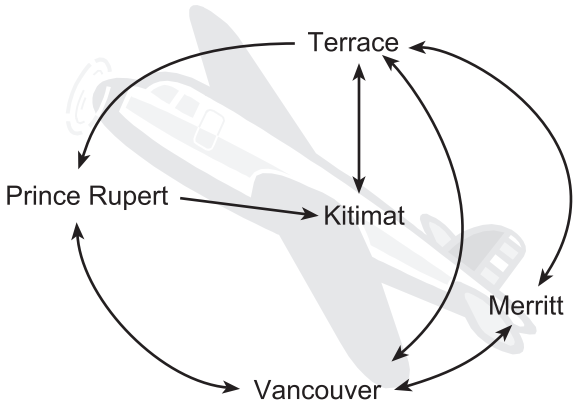
$$x^2 = (10.5)^2 + 7^2$$ ← **$\frac{1}{2}$ mark**

$$x^2 = 159.25$$

$$x = 12.62$$ ← **$\frac{1}{2}$ mark**

The shortest distance is 12.62 m

4. An airline has flights between five cities as shown in the diagram below.



a) Complete the flight matrix to summarize the data. **(1 mark)**

## Solution

**To**

$$F = \textbf{From} \begin{array}{cc} & \begin{array}{ccccc} \mathrm{V} & \mathrm{P} & \mathrm{T} & \mathrm{K} & \mathrm{M} \end{array} \\ \begin{array}{c} \mathrm{V} \\ \mathrm{P} \\ \mathrm{T} \\ \mathrm{K} \\ \mathrm{M} \end{array} & \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array} \quad \leftarrow \textbf{1 mark}$$

**$\frac{1}{2}$ mark off for each wrong value.**

b) How many routes are there from Vancouver to Kitimat with exactly one stopover? **(1 mark)**

**Solution**

$\frac{1}{2}$ **mark**

$$\begin{array}{c} \frac{1}{2}\textbf{ mark} \\ \downarrow \\ F^2 = \textbf{From} \end{array} \begin{array}{cccccc} & & & \textbf{To} & & \\ & V & P & T & K & M \\ V & 3 & 1 & 1 & \textcircled{2} & 1 \\ P & 0 & 1 & 2 & 0 & 1 \\ T & 2 & 1 & 3 & 1 & 1 \\ K & 1 & 1 & 0 & 1 & 1 \\ M & 1 & 2 & 1 & 1 & 2 \end{array}\Bigg]$$

There are 2 routes from Vancouver to Kitimat with exactly 1 stopover.

**Note:** Students may reach this answer by inspecting the diagram.

c) How many routes are there from Prince Rupert to Merritt with no more than two stopovers? **(1 mark)**

**Solution**

$$\underbrace{F + F^2 + F^3 = \textbf{From} \begin{array}{c} \\ V \\ P \\ T \\ K \\ M \end{array} \begin{array}{c} \textbf{To} \\ \begin{array}{ccccc} V & P & T & K & M \end{array} \\ \begin{bmatrix} 6 & 6 & 8 & 4 & 6 \\ 5 & 3 & 3 & 4 & \textcircled{3} \\ 8 & 7 & 7 & 6 & 7 \\ 3 & 2 & 4 & 2 & 2 \\ 7 & 4 & 6 & 4 & 4 \end{bmatrix} \end{array}}_{\begin{array}{c}\uparrow \\ \frac{1}{2}\textbf{ mark}\end{array}}$$

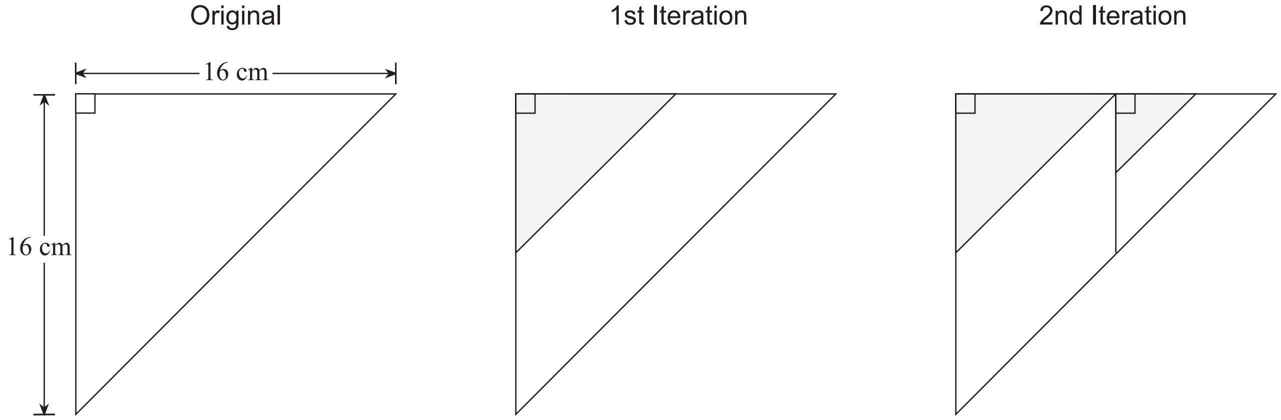
There are 3 routes from Prince Rupert to Merritt with no more than 2 stopovers.
↑
$\frac{1}{2}$ **mark**

$\frac{1}{2}$ **mark for $F^3$ only.**

**Note:** Students may reach this answer by inspecting the diagram.

5. The fractal shown in the diagram below is created as follows:

- A shaded triangle is formed by joining the midpoints of the vertical and horizontal sides.
- A vertical line is drawn from the midpoint of the horizontal side, creating a new isosceles right triangle.
- The process is continued.



a) What is the total **unshaded** area in the 4th iteration? **(2 marks)**

**Solution**

$\frac{1}{2}(16)^2 - \frac{1}{2}(8)^2 - \frac{1}{2}(4)^2 - \frac{1}{2}(2)^2 - \frac{1}{2}(1)^2$ ← **$1\frac{1}{2}$ marks**

$= 85.5 \text{ cm}^2$ ← **$\frac{1}{2}$ mark**

b) If the process is continued without end, what area of the fractal is unshaded? **(1 mark)**

**Solution**

$\frac{1}{2}(16)^2 - (32 + 8 + 2 + \dots) = 128 - \frac{32}{1 - \frac{1}{4}}$ ← **$\frac{1}{2}$ mark**

$= 128 - \frac{128}{3} = \frac{256}{3}$

$= 85.33 \text{ cm}^2$ ← **$\frac{1}{2}$ mark**

6. A snowboard manufacturer makes a standard model snowboard and a deluxe competition model.

   - Each standard snowboard requires 6 hours for fabricating and 1 hour for finishing.
   - Each competition snowboard requires 8 hours for fabricating and 3 hours for finishing.
   - There is a maximum of 120 hours available per week for fabricating, and 30 hours for finishing.
   - The manufacturer makes a \$70 profit on each standard snowboard and \$110 on each competition snowboard.

   a) Letting $x$ represent the number of standard model snowboards produced each week and $y$ represent the number of competition boards, list the constraints and the objective function needed to determine the maximum weekly profit. **(1 mark)**

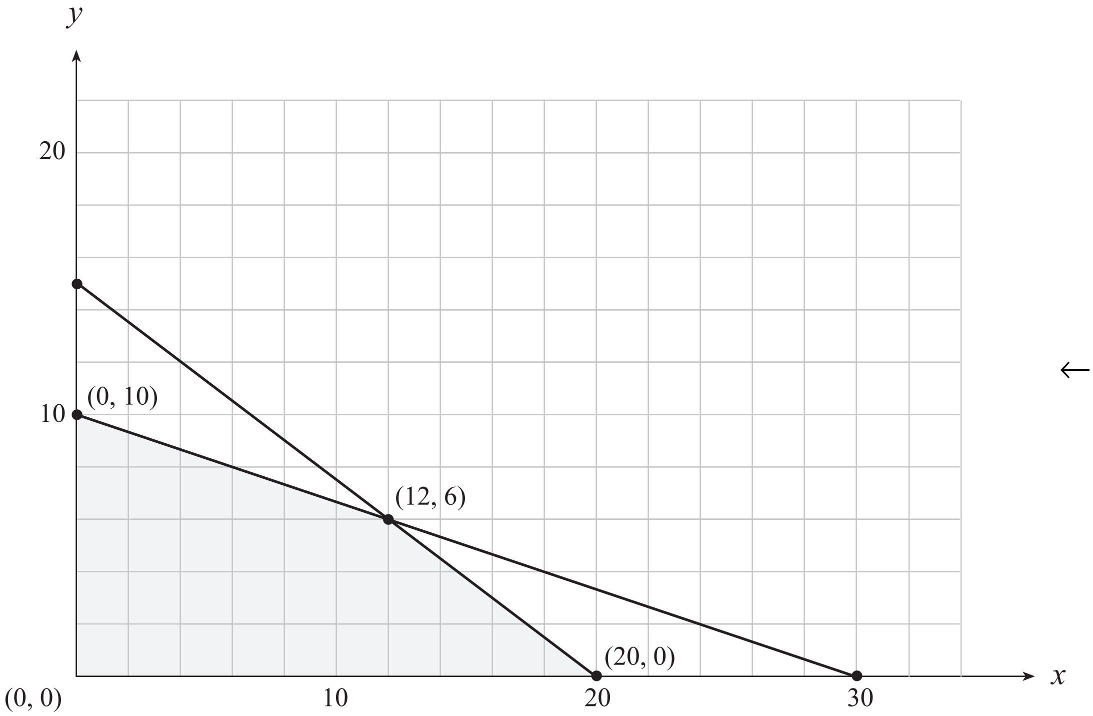
## Solution

$$\left.\begin{aligned} 6x+8y &\le 120 \\ x+3y &\le 30 \\ x &\ge 0 \\ y &\ge 0 \end{aligned}\right\} \text{constraints} \quad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$P = 70x + 110y \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

b) How many of each type of snowboard should be manufactured per week for maximum weekly profit? **(2 marks)**

### Solution



← $\frac{1}{2}$ **mark** for graph

| Corner points: | $P = 70x + 110y$ |
|---|---|
| $(0,\ 0)$ | $P = 0$ |
| $(0,\ 10)$ | $P = 1\,100$ |
| $(12,\ 6)$ | $P = 1\,500$ |
| $(20,\ 0)$ | $P = 1\,400$ |

← **1 mark**

∴ 12 standard snowboards and 6 competition snowboards should be manufactured per week for maximum weekly profit. ← $\frac{1}{2}$ **mark**

c) Determine the maximum weekly profit. **(1 mark)**

### Solution

The maximum weekly profit is the maximum value of the objective function, $1 500. ← **1 mark**

7. A communications company is researching the length of long distance telephone calls before marketing its new long distance service. The table below summarizes the data gathered for calls under 40 minutes.

| **Length of calls** (minutes) | **Number of calls** |
|---|---|
| $0 \le t < 10$ | 320 |
| $10 \le t < 20$ | 260 |
| $20 \le t < 30$ | 250 |
| $30 \le t < 40$ | 170 |

**Total 1 000**

a) Determine the mean for the length of telephone calls. **(1 mark)**

## Solution

Enter graphed data in graphing calculator:

$\bar{x} = 17.7$ ← **1 mark**

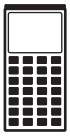
## Alternate Solution

$$\bar{x} = \frac{320(5) + 260(15) + 250(25) + 170(35)}{1\,000} \quad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$= 17.7 \quad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

b) Determine the standard deviation for the length of telephone calls. **(1 mark)**

### Solution



$$\begin{aligned}\sigma &= 10.84942395\\ &= 10.85 \quad \leftarrow \textbf{1 mark}\end{aligned}$$

### Alternate Solution

$$\sigma = \sqrt{\frac{1}{1000}\left[320\cdot 5^2 + 260\cdot 15^2 + 250\cdot 25^2 + 170\cdot 35^2\right] - 17.7^2} \quad \leftarrow \textbf{1 mark}$$

**OR**

$$\sigma = \sqrt{\frac{320(5-17.7)^2 + 260(15-17.7)^2 + 250(25-17.7)^2 + 170(35-17.7)^2}{1000}} = 10.85 \quad \leftarrow \textbf{1 mark}$$

c) Determine a 90% confidence interval for the mean length of a call under 40 minutes.

**(2 marks)**

## Solution

Enter results in graphing calculator
with $n = 1\,000$ and confidence interval 90% } ← **1 mark**

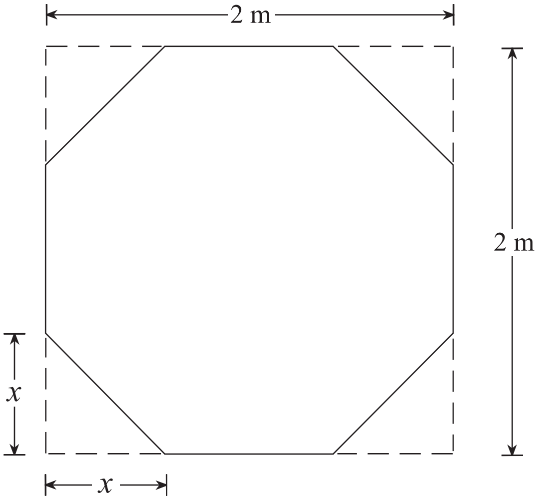
Answer: 17.14 to 18.26 ← **1 mark**

## Alternate Solution

$$\left.\begin{aligned} 17.7 - (1.64)\frac{(10.85)}{\sqrt{1\,000}} < \mu < 17.7 + (1.64)\frac{(10.85)}{\sqrt{1\,000}} \\ 17.7 - 0.563 < \mu < 17.7 + 0.563 \end{aligned}\right\} \leftarrow \textbf{1 mark}$$

$$17.14 < \mu < 18.26 \qquad \leftarrow \textbf{1 mark}$$

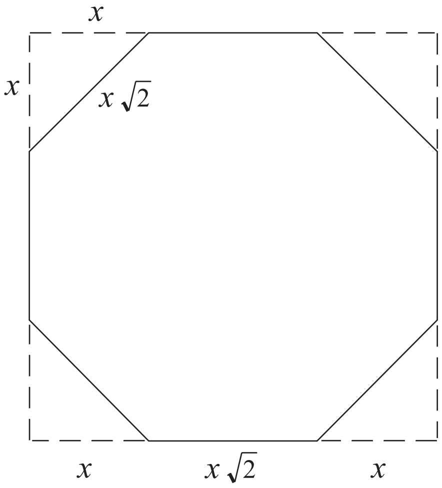
8. For a design project, triangular pieces are cut off the corners of a $2\text{ m} \times 2\text{ m}$ square sheet of cardboard to form a regular octagon, as shown in the diagram below.



a) Determine $x$, the distance from each corner that the cuts should be made. **(2 marks)**

## Solution

**Algebraically:**



Let $x$ be the length to be cut off.

Then $x\sqrt{2}$ is the length of a side of the octagon. $\Big\}\leftarrow$ **$\frac{1}{2}$ mark**

Since a side of the square is 2 m,

$x + x\sqrt{2} + x = 2$ $\leftarrow$ **$\frac{1}{2}$ mark**

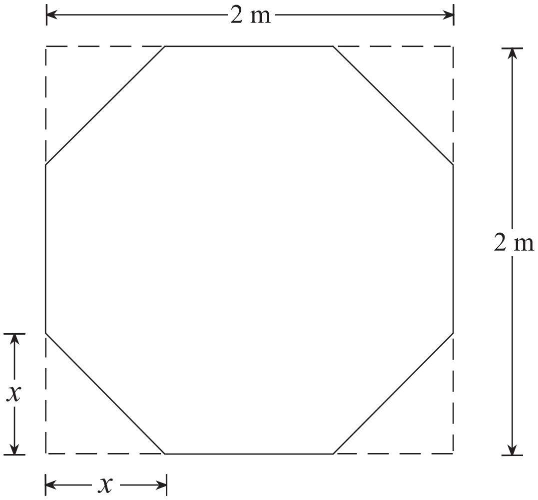
$2x + x\sqrt{2} = 2$

$x\left(2 + \sqrt{2}\right) = 2$ $\leftarrow$ **$\frac{1}{2}$ mark**

$$x = \frac{2}{2 + \sqrt{2}}$$

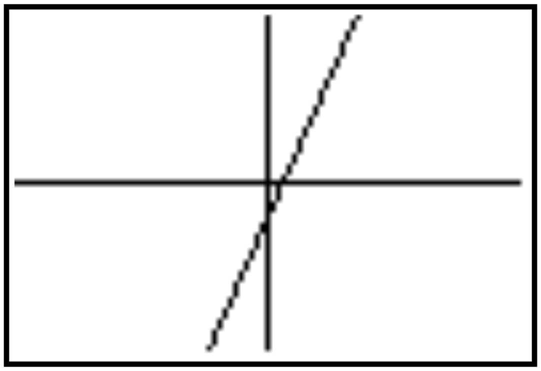
or $x = 0.59$ m $\leftarrow$ **$\frac{1}{2}$ mark**

8. For a design project, triangular pieces are cut off the corners of a $2 \text{ m} \times 2 \text{ m}$ square sheet of cardboard to form a regular octagon, as shown in the diagram below.



a) Determine $x$, the distance from each corner that the cuts should be made. **(2 marks)**

## Alternate Solution



$Y_1 = 2x + x\sqrt{2} - 2$ ← $\frac{1}{2}$ **mark** for equation

← $\frac{1}{2}$ **mark** for graph

$x \; [-10,\ 10]$ $\qquad$ $y \; [-10,\ 10]$ ← $\frac{1}{2}$ **mark** for window dimensions

The zero of the function occurs when $x = 0.59$ ← $\frac{1}{2}$ **mark**

∴ the cuts should be made 0.59 m from the corners.

b) Determine the area of the resulting octagon. **(1 mark)**

## Solution

Area of octagon = area of original square – area of triangles.

$$= 2 \times 2 - 4\left(\frac{0.5857\ldots \times 0.5857\ldots}{2}\right) \quad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$= 3.31\ \text{m}^2 \quad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

## Alternate Solution

The octagon is made up of 8 triangular areas with base $x\sqrt{2}$ and height 1.

$$\therefore\ \text{total area is } 8\left(\frac{x\sqrt{2}}{2}\cdot 1\right) = 8\left(\frac{(0.5857\ldots)\sqrt{2}}{2}\cdot 1\right) \quad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$= 3.31\ \text{m}^2 \quad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

**END OF KEY**